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New solution of Maxwell's equations for spherical wave

Contents

Annotation

It is noted that the known solution for a spherical electromagnetic wave does not satisfy the law of conservation of energy (it is retained only on the average), the electric and magnetic intensities of the same name (by coordinates) are in phase, only one from system of Maxwell's equations is satisfied, the solution is not wave solution, there is no flow of energy with real value. A solution is offered that is free from these shortcomings.

1. Introduction

In [1] a solution of the Maxwell equations for a spherical wave in the far field was proposed. Next, we consider the solution of Maxwell's equations for a spherical wave in the entire region of existence of a wave (without splitting into bands). Such a problem arises in the solution of the equations of electrodynamics for an elementary electric dipole-vibrator. The solution of this problem is known and it is on the basis of this solution that the antennas are constructed. However, this solution has a number of shortcomings, in particular [2],

1. the energy conservation law is satisfied only on the average,
2. The solution is inhomogeneous and it is practically necessary to divide it into separate zones (as a rule, near, middle and far), in which the solutions turn out to be completely different,
3. In the near zone there is no flow of energy with the real value
4. The magnetic and electrical components are in phase,

5. In the near zone, the solution is not wave (i.s. the distance is not an argument of the trigonometric function),
6. The known solution does not satisfy Maxwell's system of equations (a solution that satisfies a single equation of the system can not be considered a solution of the system of equations).

In practice, these drawbacks of the known solution mean that they (mathematical solutions) do not strictly describe the real characteristics of technical devices. A more rigorous solution, when applied in the design systems of such devices, must certainly improve their quality.

2. Solution of the Maxwell's equations

So, we will use spherical coordinates. Fig. 1 shows the spherical coordinate system (ρ, θ, φ). Next, we will place the formulas in tables and use the following notation:

T (table_number) - (column_number) - (line_number)

Table 1-3 lists the expressions for the rotor and the divergence of the vector E in these coordinates [3]. Here and below

E - electrical intensities,

H - magnetic intensities,

μ - absolute magnetic permeability,

ε - absolute dielectric constant.

Next, we will look for the solution in the form of the functions $E,\ H$, presented in Table 2-2, where the actual functions of the form $g(\theta)$ and the complex functions of the form $e(\rho),\ h(\rho)$ are to be calculated, and the coefficients $\chi,\ \alpha,\ \omega$ are known.

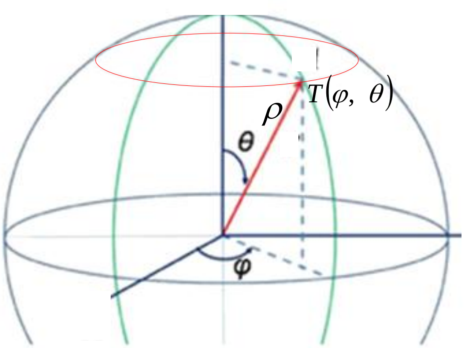

Fig. 1.

Under these conditions, we transform the formulas (T1-3) into (T1-4), where the following notations are adopted:

$$e'_\rho = \frac{\partial(e_\rho(\rho))}{\partial\rho}, \tag{1}$$

$$\hat{g} = \frac{\partial(g(\theta))}{\partial\theta}, \tag{2}$$

$$\Psi(E_\rho) = \psi(e_\rho(\rho)) \cdot g(\theta) \cdot \exp(...), \tag{3}$$

$$T(E_\rho) = \Gamma_\rho(\theta) \cdot e_\rho(\rho) \cdot \exp(...), \tag{4}$$

where

$$\psi(e_\rho(\rho)) = \left(\frac{e_\rho}{\rho} + e'_\rho + i\chi e_\rho\right), \tag{5}$$

$$\Gamma_\rho(\theta) = \left(\frac{g(\theta)}{\mathrm{tg}(\theta)} + \hat{g}(\theta)\right). \tag{6}$$

The function (3) is formed from a function of the form $\left(\frac{E_\rho}{\rho} + \frac{\partial(E_\rho)}{\partial\rho}\right)$

The Maxwell equations in spherical coordinates in the absence of charges and currents have the form given in Table 3-2. Next, we substitute the rotors and divergences from Table 2-4 and the functions $E,\ H$ from Table 2 (after differentiation with respect to time) in Table 3-3. Next, we rewrite the equations from Table 3-3 in Table 4-2. In this case, we also reduce the common factors of the form $\exp(...)$ and use the formulas (1-6).

As a result of these transformations, we obtained an **over**determined system of 8 partial differential equations with respect to 6 unknown functions with two arguments ρ and θ.

The solution of the system of Maxwell equations, in addition to the natural requirement of the feasibility of all equations of the system, must satisfy the basic physical laws:

1. the law of conservation of energy (not on average in time, but at each moment of time),
2. The phase shift experimentally established in electrical engineering between electric and magnetic intensities,
3. experimentally established wave character of the propagation of electric and magnetic intensities in space,

4. The solution should not allow the existence of an infinite value of any intensity.

Mathematically, these patterns should not be a consequence of solving the system of Maxwell equations, but additional conditions that transform the overdetermined system of Maxwell's equations into a strictly defined system. However, a solution can also be found without taking these conditions into account, since even a certain (and even more so, overdetermined) system of partial differential equations can have many solutions. In this set of solutions, there is only one that satisfies the above laws. The greatness of Maxwell's system of equations is that there is always a solution that describes reality. But how does nature find such a solution? The answer, perhaps, lies in the fact that there exists a functional (with a saddle point) relative to the intensities, in which the first variations in the intensities, when converted to zero, coincide with the Maxwell equations. The descent along the functional in the direction of these variations is equivalent to the solution of these equations [5].

The wave character of the solution is provided by the factors $\exp(...)$ of the species in the determination of the electric and magnetic intensities in Table 2. Sufficient conditions for phase displacement between electric and magnetic strains are the following:

$$E_\rho = -iH_\rho, H_\rho = iE_\rho \tag{7}$$

$$E_\varphi = iH_\varphi, H_\varphi = -iE_\varphi \tag{8}$$

$$E_\theta = -iH_\theta, H_\theta = iE_\theta \tag{9}$$

Denote by:

$$E^{sumH}_{\rho,\varphi,\theta} = E_{\rho,\varphi,\theta} + H_{\rho,\varphi,\theta} \tag{11}$$

$$E^{\min H}_{\rho,\varphi,\theta} = E_{\rho,\varphi,\theta} - H_{\rho,\varphi,\theta} \tag{12}$$

First we will seek a solution for vacuum, where in the CGS system

$$\varepsilon = \mu = 1. \tag{13}$$

and denote by

$$q = \omega/c \tag{14}$$

We summarize the equations from Table T4-2 in pairs and write the resulting equations into Table T4-3, using the notation (11, 12, 14). As a result of these transformations, we obtained an **<u>under</u>**definished system of 4 partial differential equations with respect to 6 unknown functions with two arguments ρ and θ.

It follows from (7-12):

$$E^{sumH}_\rho = e_\rho + h_\rho = e_\rho(1+i) = -ih_\rho(1+i) = h_\rho(1-i) \tag{15}$$

$$E_{\rho}^{\min H} = e_{\rho} - h_{\rho} = e_{\rho}(1-i) = -ih_{\rho}(1-i) = -h_{\rho}(1+i) \tag{16}$$

$$E_{\varphi}^{sumH} = e_{\varphi} + h_{\varphi} = e_{\varphi}(1-i) = ih_{\varphi}(1-i) = h_{\varphi}(1+i) \tag{17}$$

$$E_{\varphi}^{\min H} = e_{\varphi} - h_{\varphi} = e_{\varphi}(1+i) = ih_{\varphi}(1+i) = h_{\varphi}(-1+i) \tag{18}$$

$$E_{\theta}^{sumH} = e_{\theta} + e_{\theta} = e_{\theta}(1+i) = -ih_{\theta}(1+i) = h_{\theta}(1-i) \tag{19}$$

$$E_{\theta}^{\min H} = e_{\theta} - h_{\theta} = e_{\theta}(1-i) = -ih_{\theta}(1-i) = -h_{\theta}(1+i) \tag{20}$$

$$E_{r,f,\theta}^{sumH} + E_{r,f,\theta}^{\min H} = 2e_{r,f,\theta},\ H_{r,f,\theta}^{sumH} + H_{r,f,\theta}^{\min H} = 2h_{r,f,\theta} \tag{21}$$

We now rewrite the equations from Table T4-3 into Table T5-2, replacing variables $E_{\rho,\varphi,\theta}^{sumH}$, $E_{\rho,\varphi,\theta}^{\min H}$ with variables $e_{\rho,\varphi,\theta}$ according to (15-20).

It is seen that the equations T6-2-2 and T6-2-3 are compatible only if the following two conditions are met:

$$\alpha = 0 \tag{22}$$

$$e_{\theta} = i \cdot e_{\varphi} \tag{23}$$

$$g_{\theta} = g_{\varphi} \tag{23a}$$

Taking these conditions into account, we rewrite the equations from Table T6-2 in Table T6-3. It is seen that the equations T6-3-2 and T6-3-3 are the same, and the term $\frac{e_{\varphi}}{\rho}\hat{g}_{\varphi}$ can be deleted from the equations T6-3-1 and T6-3-4. The two equations that we got are written in Table T-7-2. After simple transformations, these equations are rewritten in Table T-7-3. We now write these equations with allowance for the formula (2.5):

$$\frac{e_{\rho}g_{\rho}}{\rho} + e'_{\rho}g_{\rho} + i\chi e_{\rho}g_{\rho} + iqe_{\rho}g_{\rho} + \frac{\cos}{\rho\sin}(1+i)e_{\varphi}g_{\varphi} = 0 \tag{24}$$

$$\frac{e_{\varphi}}{\rho} + e'_{\varphi} + i\chi e_{\varphi} + iqe_{\varphi} = 0 \tag{25}$$

Equation (25) splits into two equations:

$$\frac{e_{\varphi}}{\rho} + e'_{\varphi} = 0 \tag{26}$$

$$i\chi e_{\varphi} + iqe_{\varphi} = 0 \tag{27}$$

from which it follows that

$$\chi = -q \tag{28}$$

$$e_\varphi = \frac{A}{\rho} \tag{29}$$

where A is a constant. Substituting (28, 29) into (24), we find:

$$e'_\rho g_\rho = -\frac{e_\rho g_\rho}{\rho} - \frac{\cos}{\sin}\frac{A(1+i)}{\rho^2} g_\varphi \tag{30}$$

or

$$e'_\rho = -\frac{e_\rho}{\rho} - \frac{\cos}{\sin}\frac{A(1+i)}{\rho^2}\frac{g_\varphi}{g_\rho} \tag{31}$$

Let

$$g_\varphi = \sin, \ \ g_\rho = \cos \tag{32}$$

From (31, 32) we find:

$$e'_\rho = -\frac{e_\rho}{\rho} - \frac{A\cdot(1+i)}{\rho^2} \tag{33}$$

An analysis of this equation is given in Section 4.

As a result of the above calculations, complex functions $e_\rho(\rho)$, $e_\varphi(\rho)$, $e_\theta(\rho)$ are defined. For these functions $g(\theta)$, the functions E_ρ, E_φ, E_θ are determined from Table 2.

For these functions E_ρ, E_φ, E_θ the functions H_ρ, H_φ, H_θ are determined from (7-8), from which it follows that

$$h_\rho = ie_\rho \tag{34}$$

$$h_\varphi = -ie_\varphi \tag{35}$$

$$h_\theta = ie_\theta \tag{36}$$

The functions H_ρ, H_φ, H_θ are also listed in Table 2.

3. Energy Flows

Density of electromagnetic energy flow - Poynting vector

$$S = \eta E \times H, \tag{1}$$

where

$$\eta = c/4\pi. \tag{2}$$

In the SI system formula (1) takes the form:

$$S = E \times H. \tag{3}$$

In spherical coordinates φ, θ, ρ the flux density of electromagnetic energy has three components $S_\varphi, S_\theta, S_\rho$, directed along the radius, along the circumference, along the axis, respectively. It was shown in [4] that they are determined by the formula

$$S=\begin{bmatrix} S_\varphi \\ S_\theta \\ S_\rho \end{bmatrix}=\eta(E\times H)=\eta\begin{bmatrix} E_\theta H_\rho-E_\rho H_\theta \\ E_\rho H_\varphi-E_\varphi H_\rho \\ E_\varphi H_\theta-E_\theta H_\varphi \end{bmatrix}. \tag{4}$$

Taking into account (2.7-2.9) from (4) we find:

$$S_\rho=E_\varphi H_\theta-E_\theta H_\varphi=E_\varphi iE_\theta+E_\theta iE_\varphi \tag{4a}$$

or

$$S_\rho=2iE_\theta E_\varphi, \tag{5}$$

$$S_\theta=E_\rho H_\varphi-E_\varphi H_\rho=iH_\rho H_\varphi-iH_\varphi H_\rho=0 \tag{6}$$

$$S_\varphi=E_\theta H_\rho-E_\rho H_\theta=-iH_\theta H_\rho+iH_\rho H_\theta=0. \tag{7}$$

It follows from (6, 7) that there is no flow of energy along the circles of the sphere.

In Appendix 1 it is shown that the energy flux density, passing through a sphere with a radius ρ,

$$\overline{S_\rho}=8\eta\pi^2A^2. \tag{8}$$

and does not depend on time, i.e. this flux has the same value on a spherical surface of any radius at any instant of time. In other words, the energy flux directed along the radius retains its value with increasing radius and does not depend on time, which corresponds to the law of conservation of energy.

4. About the longitudinal wave

We consider in more detail the equation (2.33). It has a solution of the following form [8, p. 12]:

$$e_\rho=-A\cdot(1+i)\frac{\ln(\rho)}{\rho^2} \tag{1}$$

It determines the electric intensities of the longitudinal electromagnetic field - see Table 2. The magnetic intensities of the longitudinal electromagnetic field also follows from Table 2. The electric intensity of the longitudinal electromagnetic field is also present in the known solution for a spherical wave in the near zone, but there is no magnetic intensity of the longitudinal electromagnetic field, which (of course) contradicts Maxwell's equations. In addition, in the proposed solution, the electric intensity has a different description. In general, the solution

does not exist in the absence of longitudinal intensities - one can easily verify that the equations of T6-3 are not compatible, when $e_\rho(\rho)=0$.
In [1] a solution was given for the far zone, where $e_\rho(\rho)=0$. But in solution from [1] there are cases when there are infinite values of any intensity - this makes that decision practically inapplicable.

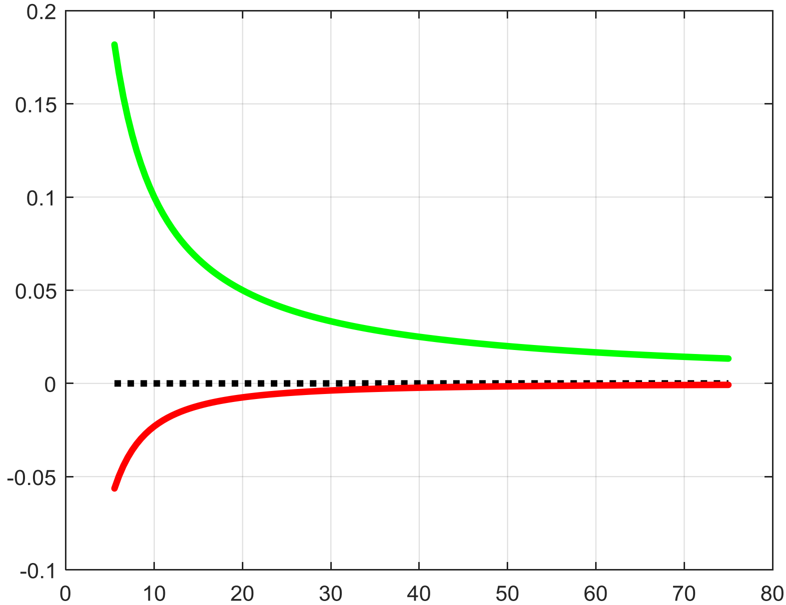

Fig. 2 (AntObr.m)

In Fig. 2 shows the form of the solution of equation (1) at $A=1$, where the real part e_ρ of the function (1) is shown (see the lower curve) and the function (2.29) $e_\varphi = A/\rho$ (see the upper curve). It is important to note that the function (1) always has a negative value (with respect to the constant A). When $A=-1$ the <u>longitudinal wave is directed away from the source</u>, i.e. coincides in the direction of the energy flow. The energy from the main energy flux of the transverse wave (3.8) is transmitted to the longitudinal wave. In this case, the main energy flux decreases (a comparative estimate of the energy of the longitudinal and transverse waves is not given here). Thus, the energy of the transverse wave is converted into the energy of the longitudinal wave. Simultaneously, the intensity of the transverse wave decreases and the propagation of the wave stops (indeed, it is difficult to imagine an unbounded spherical wave in space).

5. Conclusion

1. A rigorous solution of Maxwell's equations, shown in Table. 1 and free from the above disadvantages, is presented in Table. 2, where

$$\varepsilon = \mu = 1. \tag{1}$$

$$q = \omega / c \tag{2}$$

$$\chi = -q \tag{3}$$

$$\alpha = 0 \tag{4}$$

$$E_\rho = -iH_\rho, H_\rho = iE_\rho \tag{5}$$

$$E_\varphi = iH_\varphi, H_\varphi = -iE_\varphi \tag{6}$$

$$E_\theta = -iH_\theta, H_\theta = iE_\theta \tag{7}$$

$$g_\rho = \cos(\theta) \tag{8}$$

$$g_\theta(\theta) = g_\varphi(\theta) = \sin(\theta) \tag{9}$$

$$e_\varphi = \frac{A}{\rho} \tag{10}$$

$$e_\theta = i \cdot e_\varphi \tag{11}$$

$$e_\rho = -A \cdot (1+i) \frac{\ln(\rho)}{\rho^2} \tag{12}$$

$$h_\rho = ie_\rho \tag{13}$$

$$h_\varphi = -ie_\varphi \tag{14}$$

$$h_\theta = ie_\theta \tag{15}$$

2. The solution found is complex. It is known that the real part of the complex solution is also a solution. Therefore, as a solution, instead of the functions presented in Table. 2, you can take their real parts. Taking into account this remark and the above formulas, we rewrite Table 2 in Table 8, where the real values of the intensities are shown. In Fig. 3 shows the intensities vectors in a spherical coordinate system.

3. The electric and magnetic intensities of the same name (according to coordinates ρ, φ, θ) are phase shifted by a <u>quarter of a period</u>.

4. There is a longitudinal electromagnetic wave having electric and magnetic components.

5. In a transverse electromagnetic wave, the energy flux passing through the spheres along the radius remains constant with increasing radius and <u>does NOT change with time</u>.

6. The energy of the transverse wave is converted into the energy of the longitudinal wave. In this case, the intensity of the transverse wave decreases and the wave propagation ceases.

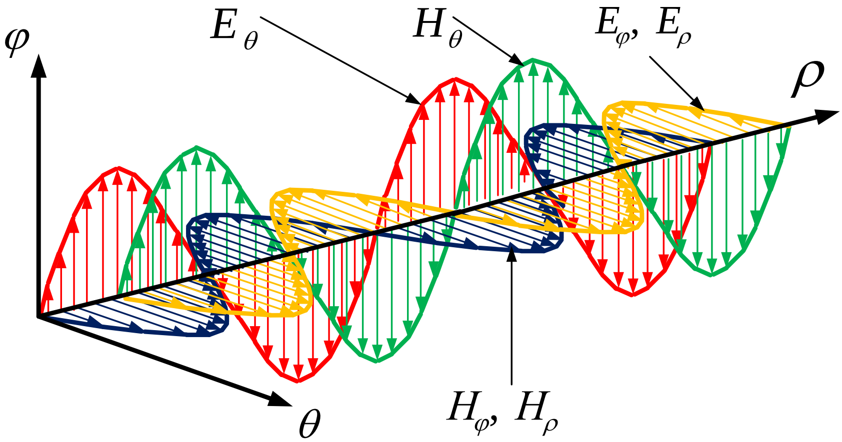

Fig. 3.

Appendix 1

Рассматривая табл. 2 и формулы (2.22, 2.23, 2.23а, 29, 32) находим:

$$E_\varphi = \frac{A}{\rho}\sin(\theta)\exp(i\cdot(\chi\rho+\omega t)) = \frac{A}{\rho}\sin(\theta)[\cos((\chi\rho+\omega t))+i\sin((\chi\rho+\omega t))] \tag{1}$$

$$\begin{aligned} E_\theta &= i\cdot E_\varphi = i\frac{A}{\rho}\sin(\theta)\exp(i\cdot(\chi\rho+\omega t)) = \frac{A}{\rho}\sin(\theta)\exp\left(i\cdot\left((\chi\rho+\omega t)+\frac{\pi}{2}\right)\right) = \\ &= \frac{A}{\rho}\sin(\theta)\left[\cos\left((\chi\rho+\omega t)+\frac{\pi}{2}\right)+i\sin\left((\chi\rho+\omega t)+\frac{\pi}{2}\right)\right] = \\ &= \frac{A}{\rho}\sin(\theta)[-\sin(\chi\rho+\omega t)-i\cos(\chi\rho+\omega t)] \end{aligned} \tag{2}$$

From (1, 2, 4.5) we find:

$$S_\rho = 2iE_\theta E_\varphi = \frac{2iA^2}{\rho^2}\sin^2(\theta)[\cos(...)+i\sin(...)][-\sin(...)-i\cos(...)] =$$

$$= \frac{2iA^2}{\rho^2}\sin^2(\theta)[-i\cos^2(...)-i\sin^2(...)] = \frac{2iA^2}{\rho^2}\sin^2(\theta)(-i)$$

or

$$S_\rho = \frac{2A^2}{\rho^2}\sin^2(\theta). \tag{3}$$

Note also that the surface area of a sphere with a radius ρ is $4\pi\rho^2$. Then the flow of energy passing through a sphere with a radius ρ is

$$\overline{S_\rho} = \eta \int_\theta 4\pi\rho^2 S_\rho d\theta = \eta 4\pi\rho^2 \frac{2A^2}{\rho^2} \int_\theta \sin^2(\theta) d\theta$$

or

$$\overline{S_\rho} = 8\eta\pi^2 A^2. \tag{4}$$

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Tables

Table 1.

1	**2**	**3**	**4**
1	$\mathrm{rot}_{\rho}(E)$	$\frac{E_{\varphi}}{\rho \mathrm{tg}(\theta)}+\frac{\partial E_{\varphi}}{\rho\partial\theta}-\frac{\partial E_{\theta}}{\rho\sin(\theta)\partial\varphi}$	$\frac{T(E_{\varphi})}{\rho}-\frac{i\alpha E_{\theta}}{\rho\sin(\theta)}$
5	$\mathrm{rot}_{\rho}(H)$	$\frac{H_{\varphi}}{\rho \mathrm{tg}(\theta)}+\frac{\partial H_{\varphi}}{\rho\partial\theta}-\frac{\partial H_{\theta}}{\rho\sin(\theta)\partial\varphi}$	$\frac{T(H_{\varphi})}{\rho}-\frac{i\alpha H_{\theta}}{\rho\sin(\theta)}$
2	$\mathrm{rot}_{\theta}(E)$	$\frac{\partial E_{\rho}}{\rho\sin(\theta)\partial\varphi}-\frac{E_{\varphi}}{\rho}-\frac{\partial E_{\varphi}}{\partial\rho}$	$\frac{i\alpha E_{\rho}}{\rho\sin(\theta)}-\psi(E_{\varphi})$
3	$\mathrm{rot}_{\varphi}(E)$	$\frac{E_{\theta}}{\rho}+\frac{\partial E_{\theta}}{\partial\rho}-\frac{\partial E_{\rho}}{\rho\partial\varphi}$	$\psi(E_{\theta})-\frac{i\alpha E_{\rho}}{\rho}$
6	$\mathrm{rot}_{\theta}(H)$	$\frac{\partial H_{\rho}}{\rho\sin(\theta)\partial\varphi}-\frac{H_{\varphi}}{\rho}-\frac{\partial H_{\varphi}}{\partial\rho}$	$\frac{i\alpha H_{\rho}}{\rho\sin(\theta)}-\psi(H_{\varphi})$
7	$\mathrm{rot}_{\varphi}H$	$\frac{H_{\theta}}{\rho}+\frac{\partial H_{\theta}}{\partial\rho}-\frac{\partial H_{\rho}}{\rho\partial\varphi}$	$\psi(H_{\theta})-\frac{i\alpha H_{\rho}}{\rho}$
4	$\mathrm{div}(E)$	$\frac{E_{\rho}}{\rho}+\frac{\partial E_{\rho}}{\partial\rho}+\frac{E_{\theta}}{\rho \mathrm{tg}(\theta)}+$ $+\frac{\partial E_{\theta}}{\rho\partial\theta}+\frac{\partial E_{\varphi}}{\rho\sin(\theta)\partial\varphi}$	$\psi(E_{\rho})+\frac{T(E_{\theta})}{\rho}+\frac{i\alpha E_{\varphi}}{\rho\sin(\theta)}$
8	$\mathrm{div}(H)$	$\frac{H_{\rho}}{\rho}+\frac{\partial H_{\rho}}{\partial\rho}+\frac{H_{\theta}}{\rho \mathrm{tg}(\theta)}+$ $+\frac{\partial H_{\theta}}{\rho\partial\theta}+\frac{\partial H_{\varphi}}{\rho\sin(\theta)\partial\varphi}$	$\psi(H_{\rho})+\frac{T(H_{\theta})}{\rho}+\frac{i\alpha H_{\varphi}}{\rho\sin(\theta)}$

Table 2.

1	2
	$E_\theta = e_\theta(\rho) g_\theta(\theta) \exp(i \cdot (\chi\rho + \alpha\varphi + \omega t))$
	$E_\varphi = e_\varphi(\rho) g_\varphi(\theta) \exp(i \cdot (\chi\rho + \alpha\varphi + \omega t))$
	$E_\rho = e_\rho(\rho) g_\rho(\theta) \exp(i \cdot (\chi\rho + \alpha\varphi + \omega t))$
	$H_\theta = h_\theta(\rho) g_\theta(\theta) \exp(i \cdot (\chi\rho + \alpha\varphi + \omega t))$
	$H_\varphi = h_\varphi(\rho) g_\varphi(\theta) \exp(i \cdot (\chi\rho + \alpha\varphi + \omega t))$
	$H_\rho = h_\rho(\rho) g_\rho(\theta) \exp(i \cdot (\chi\rho + \alpha\varphi + \omega t))$

Table 3.

1	2	3
1.	$\mathrm{rot}_\rho E + \frac{\mu}{c}\frac{\partial H_\rho}{\partial t} = 0$	$\frac{T(E_\varphi)}{\rho} - \frac{i\alpha E_\theta}{\rho\sin(\theta)} + \frac{i\omega\mu}{c} H_\rho = 0$
5.	$\mathrm{rot}_\rho H - \frac{\varepsilon}{c}\frac{\partial E_\rho}{\partial t} = 0$	$\frac{T(H_\varphi)}{\rho} - \frac{i\alpha H_\theta}{\rho\sin(\theta)} - \frac{i\omega\varepsilon}{c} E_\rho = 0$
2.	$\mathrm{rot}_\theta E + \frac{\mu}{c}\frac{\partial H_\theta}{\partial t} = 0$	$\frac{i\alpha E_\rho}{\rho\sin(\theta)} - \Psi(E_\varphi) + \frac{i\omega\mu}{c} H_\theta = 0$
3.	$\mathrm{rot}_\varphi E + \frac{\mu}{c}\frac{\partial H_\varphi}{\partial t} = 0$	$\Psi(E_\theta) - \frac{i\alpha E_\rho}{\rho} + \frac{i\omega\mu}{c} H_\varphi = 0$
6.	$\mathrm{rot}_\theta H - \frac{\varepsilon}{c}\frac{\partial E_\theta}{\partial t} = 0$	$\frac{i\alpha H_\rho}{\rho\sin(\theta)} - \Psi(H_\varphi) - \frac{i\omega\varepsilon}{c} E_\theta = 0$
7.	$\mathrm{rot}_\varphi H - \frac{\varepsilon}{c}\frac{\partial E_\varphi}{\partial t} = 0$	$\Psi(H_\theta) - \frac{i\alpha H_\rho}{\rho} - \frac{i\omega\varepsilon}{c} E_\varphi = 0$
4.	$\mathrm{div}(E) = 0$	$\Psi(E_\rho) + \frac{T(E_\theta)}{\rho} + \frac{i\alpha E_\varphi}{\rho\sin(\theta)} = 0$
8.	$\mathrm{div}(H) = 0$	$\Psi(H_\rho) + \frac{T(H_\theta)}{\rho} + \frac{i\alpha H_\varphi}{\rho\sin(\theta)} = 0$

Table 4.

1	**2**	**3**
1	$\frac{e_\varphi g_\varphi}{\rho \mathrm{tg}} + \frac{e_\varphi \hat{g}_\varphi}{\rho} - \frac{i\alpha}{\rho \sin} e_\theta g_\theta + \frac{i\omega\mu}{c} h_\rho g_\rho = 0$	$\frac{E_\varphi^{sumH}}{\rho \mathrm{tg}} g_\varphi + \frac{E_\varphi^{sumH}}{\rho} \hat{g}_\varphi -$ $- \frac{i\alpha}{\rho \sin} E_\theta^{sumH} g_\theta -$ $- iqE_\rho^{\min H} g_\rho = 0$
5	$\frac{h_\varphi g}{\rho \mathrm{tg}} + \frac{h_\varphi \hat{g}_\varphi}{\rho} - \frac{i\alpha}{\rho \sin} h_\theta g_\theta - \frac{i\omega\varepsilon}{c} e_\rho g_\rho = 0$	
2	$\frac{i\alpha}{\rho \sin} e_\rho g_\rho - \psi(e_\varphi) g_\varphi + \frac{i\omega\mu}{c} h_\theta g_\theta = 0$	$\frac{i\alpha}{\rho \sin} E_\rho^{sumH} g_\rho - \psi\left(E_\varphi^{sumH}\right) g_\varphi$ $- iqE_\theta^{\min H} g_\theta = 0$
6	$\psi(e_\theta) g_\theta - \frac{i\alpha}{\rho} e_\rho g_\rho + \frac{i\omega\mu}{c} h_\varphi g_\varphi = 0$	
3	$\frac{i\alpha}{\rho \sin} h_\rho g_\rho - \psi(h_\varphi) g_\varphi - \frac{i\omega\varepsilon}{c} e_\theta g_\theta = 0$	$\psi\left(E_\theta^{sumH}\right) g_\theta - \frac{i\alpha}{\rho} E_\rho^{sumH} g_\rho -$ $- iqE_\varphi^{\min H} g_\varphi = 0$
7	$\psi(h_\theta) g_\theta - \frac{i\alpha}{\rho} h_\rho g_\rho - \frac{i\omega\varepsilon}{c} e_\varphi g_\varphi = 0$	
4	$\psi(e_\rho) g_\rho + \frac{e_\theta}{\rho \mathrm{tg}} g_\theta + \frac{e_\theta \hat{g}_\theta}{\rho} + \frac{i\alpha}{\rho \sin} e_\varphi g_\varphi = 0$	$\psi\left(E_\rho^{sumH}\right) g_\rho + \frac{E_\theta^{sumH}}{\rho \mathrm{tg}} g_\theta +$ $+ \frac{E_\theta^{sumH}}{\rho} \hat{g}_\theta + \frac{i\alpha}{\rho \sin} E_\varphi^{sumH} g_\varphi = 0$
8	$\psi(h_\rho) g_\rho + \frac{h_\theta g}{\rho \mathrm{tg}} + \frac{h_\theta \hat{g}_\theta}{\rho} + \frac{i\alpha}{\rho \sin} h_\varphi g_\varphi = 0$	

Table 5.

1	**2**
1.	$\frac{(1-i)e_\varphi}{\rho \mathrm{tg}} g_\varphi + \frac{(1-i)e_\varphi}{\rho} \hat{g}_\varphi - \frac{i\alpha}{\rho \sin} (1+i) e_\theta g_\theta - (1-i) \frac{i\omega\mu}{c} e_\rho g_\rho = 0$
2.	$\frac{i\alpha}{\rho \sin} (1+i) e_\rho g_\rho - (1-i) \psi(e_\varphi) g_\varphi - (1-i) \frac{i\omega\mu}{c} e_\theta g_\theta = 0$
3.	$(1+i) \psi(e_\theta) g_\theta - (1+i) \frac{i\alpha}{\rho} e_\rho g_\rho - (1-i) \frac{i\omega\mu}{c} e_\varphi g_\varphi = 0$
4.	$(1+i) \psi(e_\rho) g_\rho + (1+i) \frac{e_\theta}{\rho \mathrm{tg}} g_\theta + (1+i) \frac{e_\theta}{\rho} \hat{g}_\theta + (1-i) \frac{i\alpha}{\rho \sin} e_\varphi g_\varphi = 0$

Table 6.

1	2	3
1	$\frac{1}{\rho \sin}\left(e_\theta \alpha g_\theta - e_\varphi g_\varphi \cos\right) + iqe_\rho g_\rho - -\frac{e_\varphi}{\rho}\hat{g}_\varphi = 0$	$\frac{e_\varphi}{\rho}\hat{g} = \frac{g \cos}{\rho \sin} e_\varphi + iqe_\rho g_\rho$
2	$\psi(e_\varphi) g_\varphi + qe_\theta g_\theta + \frac{\alpha}{\rho \sin} e_\rho g_\rho = 0$	$\psi(e_\varphi) g_\varphi + iqe_\varphi g_\varphi = 0$
3	$\psi(e_\theta) g_\theta - qe_\varphi g_\varphi - \frac{i\alpha}{\rho} e_\rho g_\rho = 0$	$i \cdot \psi(e_\varphi) g_\varphi - qe_\varphi g_\varphi = 0$
4	$\psi(e_\rho) g_\rho + \frac{1}{\rho \sin}\left(e_\theta g_\theta \cos + e_\varphi \alpha g_\varphi\right) + +\frac{e_\theta}{\rho}\hat{g}_\theta = 0$	$\psi(e_\rho) g_\rho + \frac{ig_\varphi \cos}{\rho \sin} e_\varphi + +\frac{ie_\varphi}{\rho}\hat{g}_\varphi = 0$

Table 7.

1	2
1.	$-\psi(e_\rho) g_\rho - \frac{i \cos}{\rho \sin} e_\varphi g_\varphi - \frac{\cos}{\rho \sin} e_\varphi g_\varphi - iqe_\rho g_\rho = 0$
2.	$\psi(e_\varphi) g_\varphi + iqe_\varphi g_\varphi = 0$

Table 8.

1	2
	$E_\theta = e_\varphi(\rho)\sin(\theta)\sin(\chi\rho + \omega t)$
	$E_\varphi = e_\varphi(\rho)\sin(\theta)\cos(\chi\rho + \omega t)$
	$E_\rho = e_\rho(\rho)\cos(\theta)\cos(\chi\rho + \omega t)$
	$H_\theta = -e_\varphi(\rho)\sin(\theta)\cos(\chi\rho + \omega t)$
	$H_\varphi = e_\varphi(\rho)\sin(\theta)\sin(\chi\rho + \omega t)$
	$H_\rho = e_\rho(\rho)\cos(\theta)\sin(\chi\rho + \omega t)$