Time Transformation between Inertial Reference Frames

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(Dated: January 24, 2018)

Time in an inertial reference frame can be obtained from the definition of velocity in that inertial reference frame. Velocity depends on coordinate and time. Therefore, coordinate transformation and velocity transformation between inertial reference frames can lead to time transformation. Based on this approach, the time transformation between two arbitrary inertial reference frames in one dimensional space is derived. The result shows that the elapsed time is identical in all inertial reference frames.

I. INTRODUCTION

Two identical inertial reference frames can be seperated by the application of a temporary acceleration to one of these two frames. Such acceleration relates coordinate and velocity from one inertial reference frame to the other inertial reference frame. According to the definition of velocity, elapsed time can be identified if both velocity and coordinate translation can be determined.

First step is to derive the coordinate transformation between two inertial reference frames. Second step is to derive the velocity transformation. Finally, time transformation can be derived from the definition of velocity.

II. COORDINATE TRANSFORMATION

Consider one-dimensional motion.

A. Acceleration

Based on the definition of acceleration, a stationary object put under constant acceleration A for a duration T will move a distance D and increase its velocity to V.

$$D = X_f - X_i \tag{1}$$

$$V = A * T \tag{2}$$

$$X_f = X_i + \frac{A*T^2}{2} \tag{3}$$

X_i is the initial position of the object before application of constant acceleration A.

X_f is the final position of the object after application of constant acceleration A for a duration T.

T is the total elapsed time for the application of acceleration

V is the final velocity of the object.

Place two identical objects at two different locations, $X1_i$ and $X2_i$. Both objects are at rest initially. Put both objects under identical constant acceleration A at the same time for a duration T.

Their final locations, $X1_f$ and $X2_f$, can be calculated according to the definition of acceleration. From equation (3),

$$X1_f = X1_i + \frac{A*T^2}{2} \tag{4}$$

$$X2_f = X2_i + \frac{A*T^2}{2} \tag{5}$$

Both objects will move at the same velocity of V at the end of duration T.

$$V = A * T \tag{6}$$

The distance between these two objects is R. From equation (4) and (5),

$$R = X2_f - X1_f = X2_i - X1_i \tag{7}$$

R remains constant during acceleration.

The acceleration is terminated at the end of duration T. Therefore, for any time t greater than T,

$$X1_f = X1_i + \frac{A*T^2}{2} + (t - T) * V \tag{8}$$

$$X2_f = X2_i + \frac{A*T^2}{2} + (t - T) * V \tag{9}$$

$$X2_f - X1_f = X2_i - X1_i = R \tag{10}$$

R remains constant after acceleration is terminated.

B. Reference Frame

Both objects are stationary to each other at all time. They form a reference frame F_2 that moves at the velocity V relative to a reference frame F_1 in which both objects are initially at rest.

$$V = A * T \tag{11}$$

The speed of F_2 relative to F_1 is V.

Let the initial location of object 1 be the origin of both F_1 and F_2. The location of object 2 becomes a representation of the coordinate in both F_1 and F_2.

Let x' be the location of object 2 in F_2. Let x be the location of object 2 in F_1.

$$x' = X2_i \quad (12)$$

$$x = X2_f \quad (13)$$

Therefore, the coordinate transformation between F_1 and F_2 is, from equation (9),

$$x = x' + \frac{A*T^2}{2} + (t - T) * V \quad (14)$$

C. Conservation of Length

Place a stationary object of length L in F_2. The positions of both ends of this object in F_2 are x'_a and x'_b.

$$L = x'_b - x'_a \quad (15)$$

Based on coordinate transformation between F_1 and F_2, equation (14),

$$x_a = x'_a + \frac{A*T^2}{2} + (t - T) * V \quad (16)$$

$$x_b = x'_b + \frac{A*T^2}{2} + (t - T) * V \quad (17)$$

x_a and x_b are the positions of both ends of this object in F_1. The length of this object in F_1 is x_b-x_a.

$$x_b - x_a = x'_b - x'_a = L \quad (18)$$

The length of this object is L in both F_1 and F_2. The length is independent of the relative motion between F_1 and F_2.

III. VELOCITY TRANSFORMATION

Consider one-dimensional motion

A. Identical Reference Frames

Let an inertial reference frame F_3 be stationary relative to inertial reference frame F_1. Let an object W_1 in F_3 moves at a speed of v'.

The speed of W_1 in F_1 is v'

The speed of W_1 in F_3 is v'

B. Acceleration

Put F_3 under constant acceleration A relative to F_1 for a duration T.

According to the definition of acceleration, this temporary acceleration produces a difference in the relative speed between F_1 and F_3 and accelerates all objects in F_3 by A*T in F_1.

The speed of W_1 in F_1 is v' + A*T

The speed of W_1 in F_3 is v'

The speed of F_3 relative to F_1 is A*T. The speed of F_2 relative to F_1 is also A*T. Therefore, F_3 is stationary relative to F_2

The speed of W_1 in F_1 is v' + A*T

The speed of W_1 in F_3 is v'

The speed of W_1 in F_2 is v'

Let the speed of W_1 in F_1 be v. The velocity transformation between F_1 and F_2 is

$$v = v' + A * T \quad (19)$$

Therefore, a moving object in F_2 will move in F_1 at a speed equal to the sum of its speed in F_2 and the relative speed between F_2 and F_1. This is the velocity transformation from F_2 to F_1. It is independent of the speed of light.

IV. TIME TRANSFORMATION

Let the time in F_1 be t. Let the time in F_2 be t'. According to the definition of velocity,

$$v = \frac{dx}{dt} \quad (20)$$

in F_1. While in F_2,

$$v' = \frac{dx'}{dt'} \quad (21)$$

From equation (14),

$$dx = dx' + (dt) * V \quad (22)$$

$$\frac{dx}{dt} = \frac{dx'}{dt} + V \quad (23)$$

From equations (19), (11), (20), (21),

$$\frac{dx}{dt} = \frac{dx'}{dt'} + V \quad (24)$$

From equations (23) and (24),

$$dt' = dt \quad (25)$$

$$t' = t + C \quad (26)$$

C is a constant in time.

V. CONCLUSION

Elapsed time is identical in all inertial reference frames. Time does not run slower nor faster in any particular inertial reference frame. Two simultaneous events in one inertial reference frame are simultaneous in all other inertial reference frames.

The velocity transformation between two inertial reference frames exclusively depends on the relative speed between two inertial reference frames. It is independent of the speed of light.

The coordinate transformation between two inertial reference frames also exclusively depends on the relative speed between two inertial reference frames. It is also independent of the speed of light.

For more than a century, there have been speculation that the speed of light is a factor in transformation of time, coordinate, and velocity. This is clearly incorrect as in the proof of this paper.

Therefore, any proposed transformation that incorporates the speed of light is invalid in physics. One particular example is Lorentz Transformation[1][2] which is based on the assumption that the speed of light is independent of inertial reference frame.

As a result of its incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation. Therefore, Lorentz Transformation is not a proper transformation in physics.

Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[4][8]

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