Conjecture on a relation between smaller numbers of amicable pairs and Poulet numbers divisible by 5

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Abstract. In a previous paper I presented seven sequences of numbers of the form 2*k*P – (30 + 290*n)*k – 315, where P is Poulet number, and I conjectured that two of them have all the terms odd abundant numbers and the other five have an infinity of terms odd abundant numbers. Because it is known that all the smaller numbers of amicable pairs are abundant numbers (see A002025 in OEIS), in this paper I revert the relation from above and I conjecture that all Poulet numbers P divisible by 5 can be written as P = (A + 315 + (30 + 290*n)*k)/(2*k), where A is a smaller of an amicable pair and n and k naturals. For example: 645 = (12285 + 315 + 30*10)/(2*10); also 1105 = (12285 + 315 + 2060*84)/(2*84) or 1105 = (69615 + 315 + 320*37)/(2*37). Note that for the first 17 such Poulet numbers there exist at least a combination [n, k] for A = 12285, the first smaller of an amicable pair divisible by 5!

Conjecture: All Poulet numbers P divisible by 5 can be written as P = (A + 315 + (30 + 290*n)*k)/(2*k), where A is a smaller of an amicable pair and n and k naturals.

Note: for Poulet numbers divisible by 5 see the sequence A216023 submitted by me in OEIS.

Verifying the conjecture:
(for the first twenty such Poulet numbers)

: 645 = (12285 + 315 + 30*10)/(2*10), so
[A, n, k] = [12285, 0, 10];

: 1105 = (12285 + 315 + 2060*84)/(2*84), so
[A, n, k] = [12285, 7, 84];

: 1905 = (12285 + 315 + 3510*42)/(2*42), so
[A, n, k] = [12285, 12, 42];

: 2465 = (12285 + 315 + 4090*15)/(2*15), so
[A, n, k] = [12285, 14, 15];

: 10585 = (12285 + 315 + 20330*15)/(2*15), so
[A, n, k] = [12285, 70, 15];

: 11305 = (12285 + 315 + 19460*4)/(2*4), so
[A, n, k] = [12285, 67, 4];

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:    16705 = (12285 + 315 + 32510*14)/(2*14), so
          [A, n, k] = [12285, 112, 14]; also
     16705 = (12285 + 315 + 33380*420)/(2*420), so
          [A, n, k] = [12285, 115, 420];

:    18705 = (12285 + 315 + 36570*15)/(2*15), so
          [A, n, k] = [12285, 126, 15];

:    34945 = (12285 + 315 + 69050*15)/(2*15), so
          [A, n, k] = [12285, 238, 15];

:    39865 = (12285 + 315 + 78330*9)/(2*9), so
          [A, n, k] = [12285, 270, 9];

:    41665 = (12285 + 315 + 81230*14)/(2*6), so
          [A, n, k] = [12285, 280, 6]; also
     41665 = (12285 + 315 + 82970*35)/(2*35), so
          [A, n, k] = [12285, 286, 35]; also
     41665 = (12285 + 315 + 83260*180)/(2*180), so
          [A, n, k] = [12285, 287, 180];

:    55245 = (12285 + 315 + 109650*15)/(2*15), so
          [A, n, k] = [12285, 378, 15];

:    62745 = (12285 + 315 + 124740*12)/(2*12), so
          [A, n, k] = [12285, 429, 12]; also
     62745 = (12285 + 315 + 125310*70)/(2*70), so
          [A, n, k] = [12285, 432, 70];

:    72885 = (12285 + 315 + 145320*28)/(2*28), so
          [A, n, k] = [12285, 501, 28];

:    74665 = (12285 + 315 + 147930*9)/(2*9), so
          [A, n, k] = [12285, 510, 9];

:    83665 = (12285 + 315 + 166490*15)/(2*15), so
          [A, n, k] = [12285, 574, 15];

:    107185 = (12285 + 315 + 213470*14)/(2*14), so
          [A, n, k] = [12285, 736, 14]; also
     107185 = (12285 + 315 + 213470*70)/(2*420), so
          [A, n, k] = [12285, 739, 420];

:    121465 = (100485 + 315 + 142130*1)/(2*1), so
          [A, n, k] = [100485, 490, 1]; also
     121465 = (100485 + 315 + 239570*30)/(2*30), so
          [A, n, k] = [100485, 826, 30];

:    208465 = (100485 + 315 + 316130*1)/(2*1), so
          [A, n, k] = [100485, 1090, 1]; also
     208465 = (100485 + 315 + 413570*30)/(2*30);
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:     215265 = (12285 + 315 + 426330*3)/(2*3), so
           [A, n, k] = [12285, 1470, 3]; also
      215265 = (12285 + 315 + 430390*90)/(2*90), so
           [A, n, k] = [12285, 1484, 90]; also
      215265 = (67095 + 315 + 408060*3)/(2*3), so
           [A, n, k] = [67095, 1407, 3]; also
      215265 = (69615 + 315 + 427200*21)/(2*21), so
           [A, n, k] = [69615, 1473, 21]; also
      215265 = (100485 + 315 + 426330*24)/(2*24), so
           [A, n, k] = [100485, 1470, 24]; also
      215265 = (100485 + 315 + 429810*140)/(2*140), so
           [A, n, k] = [100485, 1482, 140]; also
      215265 = (122265 + 315 + 423720*18)/(2*18), so
           [A, n, k] = [122265, 1461, 18]; also [...]
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Note that for the first 17 such Poulet numbers there exist at least a combination [n, k] for A = 12285, the first smaller of an amicable pair divisible by 5!

Note that for the Poulet number 215265 there exist at least a combination [n, k] for the first 11 smaller numbers of amicable pairs!