Bell's Inequality Leaks Like a Sieve: Nonrecurrence and Bell-like Inequalities

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https://doi.org/10.1515/phys-2017-0089
Received September 12, 2017; revised November 5, 2017; accepted November 6, 2017

Abstract: The general class, Λ, of Bell hidden variables is composed of two subclasses Λ_R and Λ_N such that $\Lambda_R \cup \Lambda_N = \Lambda$ and $\Lambda_R \cap \Lambda_N = \{\}$. The class Λ_N is very large and contains random variables whose domain is the continuum, the reals. There are an uncountable infinite number of reals. Every instance of a real random variable is unique. The probability of two instances being equal is zero, exactly zero. Λ_N induces sample independence. All correlations are context dependent but not in the usual sense. There is no "spooky action at a distance". Random variables, belonging to Λ_N , are independent from one experiment to the next. The existence of the class Λ_N makes it impossible to derive any of the standard Bell inequalities used to define quantum entanglement.

Keywords: Bell, inequalities, nonrecurrent, variables, quantum
PACS: 03.65.Ud

1 Introduction

John S. Bell [1] derived an inequality claiming it holds for all local hidden variable models of quantum mechanics (of the singlet state). Bell's formulation is incomplete. It does not hold for all possible hidden variables even though the class, Λ, of his hidden variables is general. Bell writes

Let this more complete specification be effected by means of parameters λ. It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous . However, we write as if λ were a single continuous parameter.

Bell does not make use of the properties of continuous hidden variables. Instances of random variables belonging to the continuum (reals) [2] do not repeat and are members of Λ_N. Every instance of a real random variable is unique. The probability of two instances being equal is zero, exactly zero [3].

His correlations restrict Λ to a subset Λ_R consisting of hidden variables that repeat under different measurement device orientations. That implies Bell's inequality does not govern the behavior of correlations derived from nonrecurrent hidden variables, Λ_N. This suggests Bell's formulation is not correct for nonrecurrent hidden variables.

Consider experiments. We write the sample average with a bar over variables x (instances of a random variable X) and note the sample average approaches the theoretical expected value for large sample size N (law of large numbers[1])

$$\bar{x}_N = \frac{1}{N}\sum_{n=0}^{N-1} x_n$$

$$\lim_{N\to\infty} \bar{x}_N = \langle X \rangle$$

When the random variable is a function of nonrecurrent hidden variables the sample average of the k^{th} experiment is written

$$\overline{(AB)}_{k,N} = \frac{1}{N}\sum_{n=0}^{N-1} A(\vec{a}_k, \lambda_{kn}) B(\vec{b}_k, \lambda_{kn})$$

Due to nonrecurrence each instance has its own unique hidden variable λ_{kn} specific to the k^{th} experiment and n^{th} occurrence. An experiment labeled k has measurement device orientations $(\vec{a}_k, \vec{b}_k)$ called the configuration.

In vector notation

$$\left(\vec{A}_k\right)_n = A(\vec{a}_k, \lambda_{kn})$$

$$\left(\vec{B}_k\right)_n = B(\vec{b}_k, \lambda_{kn})$$

The sample average is then the inner product of the two vectors

$$\overline{(AB)}_{k,N} = \frac{1}{N}\vec{A}_k \cdot \vec{B}_k$$

The probability density $\rho_{\mathcal{N}}(\lambda)$ specifies hidden variables are nonrecurrent. To take into account experiment independence write the correlation as

$$r_k = \int d\lambda \rho_N(\lambda) A_k(\vec{a}_k, \lambda) B_k(\vec{b}_k, \lambda)$$

Each experiment, k, has its own random variables A_k and B_k satisfying

$$\langle A_i A_j \rangle = \delta_{i,j}$$

(and similarly for B). The A_i are independent of the A_j. Their means are zero and hence their correlation is zero. This formulation is local. Each function is dependent only on its local orientation and the hidden variable, λ.

[1]Wikipedia states "A 'law of large numbers' is one of several theorems expressing the idea that as the number of trials of a random process increases, the percentage difference between the expected and actual values goes to zero."

It appears this formulation is context sensitive, and it is but not in the usually sense. There is no "spooky action at a distance". Alice's data at the time of recording is not a function of Bob's orientation and Bob's data at the time of recording is not a function of Alice's orientation. But note, when the data are *correlated* they are brought to a common point and the joint orientations are revealed. An *experiment* consists of the set of data for which the configuration (joint orientations) is constant. For example, with the CHSH [4] experiments the configurations consist of

Table 1: CHSH configurations

$$K_1 = \left(\vec{a}_1, \vec{b}_1\right) = \left(\vec{a}, \vec{b}\right)$$
$$K_2 = \left(\vec{a}_2, \vec{b}_2\right) = \left(\vec{a}, \vec{b}\,'\right)$$
$$K_3 = \left(\vec{a}_3, \vec{b}_3\right) = \left(\vec{a}\,', \vec{b}\right)$$
$$K_4 = \left(\vec{a}_4, \vec{b}_4\right) = \left(\vec{a}\,', \vec{b}\,'\right)$$

When Alice collects her data she only knows her own orientations $\vec{a}$, $\vec{a}\,'$ and similarly for Bob's orientations $\vec{b}$,$\vec{b}\,'$. It is the job of the *correlator* to segment Alice's and Bob's data into sequences of constant configuration, K_k. That makes the sequences correlation context sensitive, but the data are unchanged by segmentation only the partitions are created. Hence the label k for the random variables in this formulation reflects segmentation and not data dependency. The data in each segment are independent of every other segment when the hidden variables belong to Λ_N.

2 Inequalities

The existence of the class Λ_N makes it impossible to derive any of the standard inequalities: Bell [1], CHSH [4], CH [5] or the GHZ [6] constraint, used to define quantum entanglement [7,8]

Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles are generated or interact in ways such that the quantum state of each particle cannot be described independently of the others, even when the particles are separated by a large distance—instead, a quantum state must be described for the system as a whole.

The class Λ_N generates new predictions as specified in the following table comparing the inequalities and their equivalent nonrecurrent form. The derivation of each form is presented below.

Table 2: Recurrent vs. nonrecurrent constraints

Recurrent

Bell	$\lvert r_1 - r_2 \rvert - r_3 \le 1$
CHSH	$\lvert r_1 - r_2 \rvert + \lvert r_3 + r_4 \rvert \le 2$
CH	$xy - xy' + x'y + x'y' - x' - y \le 0$
GHZ	$A_\gamma(\pi) = A_\gamma(0), A_\gamma(\pi) = -A_\gamma(0)$

Nonrecurrent

Bell	$\lvert r_1 - r_2 \rvert + r_1 r_2 \le 1$
CHSH	$\lvert r_1 - r_2 \rvert + \lvert r_3 + r_4 \rvert + r_1 r_2 - r_3 r_4 \le 2$
CH	$x_1 y_1 - x_2 y_2 + x_3 y_3 + x_4 y_4 - x_3 - y_1 \le 1$
GHZ	$A_4(\pi) \ne A_5(\pi)$

2.1 Bell's inequality

The Bell inequality is composed of 3 correlations. In Bell's notation the r_3 correlation is $r_3 = P(\vec{b}, \vec{c})$. That correlation never occurs for the class Λ_N because $P(\vec{b}, \vec{c})$ is obtained by assuming the A of $A(\vec{a}, \lambda) B(\vec{b}, \lambda)$ is the same as the A for $A(\vec{a}, \lambda') B(\vec{c}, \lambda')$. That assumption is false for the class Λ_N since each configuration (a,b) and (a,c) happens at a different time and hence have different hidden variables. The A's do not multiply to 1 and $P(\vec{b}, \vec{c})$ does not occur. The nonrecurrent inequality holds for all r_1 and r_2. It places no constraint on those correlations.

John S Bell's original inequality [1] is modified for nonrecurrent hidden variables as follows. We write r_k for the correlation of the k^{th} experiment

$$r_k = \frac{1}{N} \sum_{n=0}^{N-1} A(\vec{a}_k, \lambda_{kn}) B(\vec{b}_k, \lambda_{kn})$$

where each λ_{kn} is unique. Following Bell the difference of two such correlations is written

$$r_i - r_j = \frac{1}{N} \sum_{n=0}^{N-1} A(\vec{a}_i, \lambda_{in}) B(\vec{b}_i, \lambda_{in}) - \frac{1}{N} \sum_{n=0}^{N-1} A(\vec{a}_j, \lambda_{jn}) B(\vec{b}_i, \lambda_{jn})$$

$$r_i - r_j = \frac{1}{N} \left[\sum_{n=0}^{N-1} A(\vec{a}_i, \lambda_{in}) B(\vec{b}_i, \lambda_{in}) - A(\vec{a}_j, \lambda_{jn}) B(\vec{b}_i, \lambda_{jn}) \right]$$

which in the limit of large N becomes

$$r_i - r_j = \left\langle A_i(\vec{a}_i) B_i(\vec{b}_i) - A_j(\vec{a}_j) B_j(\vec{b}_j) \right\rangle$$

As with Bell factor that expression

$$r_i - r_j = \left\langle A_i(\vec{a}_i) B_i(\vec{b}_i) \left[1 - A_i(\vec{a}_i) B_i(\vec{b}_i) A_j(\vec{a}_j) B_j(\vec{b}_j) \right] \right\rangle$$

and take the absolute value using

$$\left| A_i(\vec{a}_i) B_i(\vec{b}_i) \right| = 1$$

to obtain the inequality

$$\left| r_i - r_j \right| \le \left\langle \left| 1 - A_i(\vec{a}_i) B_i(\vec{b}_i) A_j(\vec{a}_j) B_j(\vec{b}_j) \right| \right\rangle$$

which is

$$\left|r_i - r_j\right| \le 1 - \left\langle A_i\left(\vec{a}_i\right) B_i\left(\vec{b}_i\right) A_j\left(\vec{a}_j\right) B_j\left(\vec{b}_j\right)\right\rangle,$$

Now, unlike Bell, the product

$$A\left(\vec{a}_i, \lambda_{in}\right) A\left(\vec{a}_j, \lambda_{jn}\right)$$

for

$$\vec{a}_i = \vec{a}_j$$

is not equal to 1 since, for hidden variables belonging to Λ_N,

$$(i \ne j)\, \lambda_{in} \ne \lambda_{jn}.$$

That fact is the crucial difference between Bell's Λ_R and Λ_N . One can say, in general, that Bell's assumption $A\left(\vec{a}, \lambda_{in}\right) A\left(\vec{a}, \lambda_{jn}\right) = 1$ does not hold for hidden variables belonging to Λ_N.

For Λ_N different samples are independence and so the expectation of the product equals to the product of the expectations (a step not true for Bell)

$$\left|r_i - r_j\right| \le 1 - \left\langle A_i\left(\vec{a}_i\right) B_i\left(\vec{b}_i\right)\right\rangle \left\langle A_j\left(\vec{a}_j\right) B_j\left(\vec{b}_j\right)\right\rangle,$$

or

$$\left|r_i - r_j\right| \le 1 - r_i r_j,$$

and

$$\left|r_i - r_j\right| + r_i r_j \le 1.$$

Λ_N leads to an inequality that differs from Bell's. It places no constraints on r_i and r_j. They hold for all $-1 \le r_i \le 1$ and $-1 \le r_j \le 1$. To show that, define $\max_{ij}=\mathrm{MAX}(r_i,r_j)$ and $\min_{ij}=\mathrm{MIN}(r_i,r_j)$ and note that for all r_i and r_j

$$0 \le \left(1 + \min_{ij}\right),$$

$$0 \le \left(1 - \max_{ij}\right),$$

and hence

$$0 \le \left(1 + \min_{ij}\right)\left(1 - \max_{ij}\right).$$

Expand that product to give

$$\max_{ij} - \min_{ij} + \max_{ij} * \min_{ij} \le 1$$

which is equivalent to

$$\left|r_i - r_j\right| + r_i r_j \le 1.$$

Hence the Λ_N inequality is true for all r_i and r_j.

Bell's procedure does not bound the correlations formed for nonrecurrent hidden variables.

2.2 CHSH inequality

The CHSH form arises from a double application of the Bell form. It places no constraints on the correlations r_1, r_2, r_3, r_4 .

The same steps as with Bell can be performed for the derivation of the CHSH [4] inequality which is written as

$$\left|r_1 - r_2\right| + \left|r_3 + r_4\right| \le 2$$

Start from

$$\left|r_1 - r_2\right| + \left|r_3 + r_4\right| = \left|\left\langle A_1B_1 - A_2B_2\right\rangle\right| + \left|\left\langle A_3B_3 + A_4B_4\right\rangle\right|$$
$$\leq \left\langle\left|A_1B_1\left(1 - A_1B_1A_2B_2\right)\right|\right\rangle + \left\langle\left|A_3B_3\left(1 + A_3B_3A_4B_4\right)\right|\right\rangle$$
$$\leq \left\langle\left|1 - \left(A_1B_1\right)\left(A_2B_2\right)\right|\right\rangle + \left\langle\left|1 + \left(A_3B_3\right)\left(A_4B_4\right)\right|\right\rangle$$
$$\leq \left\langle 1 - \left(A_1B_1\right)\left(A_2B_2\right)\right\rangle + \left\langle 1 + \left(A_3B_3\right)\left(A_4B_4\right)\right\rangle$$
$$\leq 1 - \left\langle A_1B_1\right\rangle\left\langle A_2B_2\right\rangle + 1 + \left\langle A_3B_3\right\rangle\left\langle A_4B_4\right\rangle$$
$$\leq 2 - r_1r_2 + r_3r_4$$

so

$$\left|r_1 - r_2\right| + \left|r_3 + r_4\right| + r_1r_2 - r_3r_4 \leq 2$$

We have already shown that for all r_1 and r_2

$$\left|r_1 - r_2\right| + r_1r_2 \leq 1$$

so correspondingly

$$\left|r_3 - \left(-r_4\right)\right| + r_3\left(-r_4\right) \leq 1$$

Hence that inequality holds for all r_1, r_2, r_3, and r_4. No constraints are placed on those correlations.

2.3 CH inequality

The CH inequality again assumes the hidden variables recur under different configurations. When the expressions are written taking into account the independence of each configuration the upper bound increases to 1. Experiments that violate the upper bound of 0 do not violate the upper bound of 1 and hence a local model is not ruled out by 0 violation.

Clauser [5] in his "Bells theorem: experimental tests and implications" writes

Following our discussion of §3.3, we assumed that, given λ, a and b, the probabilities $p_1(\lambda,a)$ and $p_2(\lambda,b)$ are independent. Thus we write the probabilities of detecting both components as

$$p_{12}(\lambda,a,b) = p_1(\lambda,a)\,p_2(\lambda,b) \qquad (3.15)$$

The ensemble average probabilities of equations (3.14) are then given by:

$$p_1(a) = \int_\Lambda p_1(\lambda,a)\,d\rho$$
$$p_2(b) = \int_\Lambda p_2(\lambda,b)\,d\rho \qquad (3.16)$$
$$p_{12}(a,b) = \int_\Lambda p_{12}(\lambda,\mathrm{a},b)\,d\rho$$

To proceed, CH introduce the following lemma, the proof of which may be found in their paper [11]: if x, x', y, y', X, Y are real numbers such that $0 \le x,x' \le X$ and $0 \le y,y' \le Y$, then the inequality

$$-XY \leq xy - xy' + x'y + x'y' - Yx' - Xy \leq 0 \qquad (3.17)$$

holds. Inequality (3.17) and equation (3.15) yield:

$$-1 \leq p_{12}(\lambda,a,b) - p_{12}(\lambda,a,b') + p_{12}(\lambda,a',b) + p_{12}(\lambda,a',b') - p_1(\lambda,a') - p_2(\lambda,b) \leq 0 \qquad (3.18)$$

Integrating inequality (3.18) over λ with distribution ρ, and using equation (3.16), one obtains the result:

$$-1 \leq p_{12}(a,b) - p_{12}(a,b') + p_{12}(a',b) + p_{12}(a',b') - p_1(a') - p_2(b) \leq 0 \qquad (3)$$

Implicit in the derivation of (3) is the assumption the same λ can appear with different configurations (a,b), (a,b'), (a',b), and (a',b'). That assumption is false for nonrecurrent random variables. Each configuration has its own unique set of hidden variables. Hence equation *(3.18)* does not hold for those hidden variables. That equation lacks generality. If λ appears in $p_{12}(\lambda,a,b)$ it does not appear in any of the other terms. For nonrecurrent hidden variables the context in *(3.15)* must be specified. The CH recurrent form *(3.17)* has 4 variables: x, x', y, y'. The corresponding nonrecurrent form has 8 variables:

$$0 \le x_i \le X,\ 0 \le y_i \le Y,\ \mathrm{i} = 1,2,3,4$$

We repeat *(3.17)* with the *recurrent* "kernel" specified as

$$K_R = xy - xy' + x'y + x'y' - Yx' - Xy$$

and then write the corresponding *nonrecurrent* "kernel"

$$K_N = x_1 y_1 - x_2 y_2 + x_3 y_3 + x_4 y_4 - Yx_3 - Xy_1$$

The choice of $-Yx_3 - Xy_1$ is arbitrary. They could just as well have been set to $-Yx_5 - Xy_5$ in a fifth experiment but we seek the least upper bound and assume the x and y are taken from the experiments as written. Doing so increases the constraints on the variables. Regroup the nonrecurrent kernel and write

$$K_N = (x_1 - X)\,y_1 - x_2 y_2 + x_3\,(y_3 - Y) + x_4 y_4$$

Each term in that expression is independent of every other term. As such the max of K_N is obtained by maximizing each term separately.

$$MAX\,(K_N) = MAX\,((x_1 - X)\,y_1) + MAX\,(-x_2 y_2) + MAX\,(x_3\,(y_3 - Y)) + MAX\,(x_4 y_4)$$

$$MAX\,(K_N) = 0 + 0 + 0 + XY = XY$$

The minimum is likewise determined

$$MIN\,(K_N) = MIN\,((x_1 - X)\,y_1) + MIN\,(-x_2 y_2) + MIN\,(x_3\,(y_3 - Y)) + MIN\,(x_4 y_4)$$

$$MIN\,(K_N) = -XY - XY - XY + 0 = -3XY$$

Setting XY=1 leads to

$$-3 \le K_N \le 1$$

as compared to

$$-1 \le K_R \le 0$$

The upper bound for experimental results is 1 and not 0. Hence any experiment that exceeds 0 does not imply nonlocality but rather that the hidden variables are nonrecurrent (and local).

. A recent paper by Giustina et al. [9] purports to violate the CH-Eberhard upper bound of 0 and hence simultaneously close several loopholes. The nonrecurrent upper bound is 1 for CH and hence there is no difficulty explaining those experimental results with a local model. The CH derivation is invalidated by nonrecurrent hidden variables. In general the CH inequality is false.

2.4 GHZ constraint

The GHZ constraint again assumes the hidden variables recur under different configurations. When the independence of those configurations is taken into account the GHZ contradiction does not occur and the EPR [8] program is maintained.

The GHZ [6] paper derives a condition that does not use inequalities. They introduce the expressions

$$\begin{aligned} &\mathit{If}\ \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 = 0, \\ &\mathit{then}\ E^{\psi}(\vec{n}_1,\vec{n}_2,\vec{n}_3,\vec{n}_4) = -1 && (10a) \\ &\mathit{If}\ \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 = \pi, \\ &\mathit{then}\ E^{\psi}(\vec{n}_1,\vec{n}_2,\vec{n}_3,\vec{n}_4) = 1 && (10b) \end{aligned}$$

and then list 4 premises

(i) Perfect correlations: With four Stern-Gerlach analyzers set at angles satisfying the conditions of either (10a) or (10b), knowledge of the outcomes for any three particles enables a prediction with certainty of the outcome for the fourth.
(ii) Locality: Since at the time of measurement the four particles are arbitrarily far apart, they presumably do not interact, and hence no real change can ake place in any one of them in consequence of what is done to the other three.
(iii) Reality: same as in Sec. II.
(iv) Completeness: Same as in Sec.II.

We now reproduce their salient arguments with the corresponding nonrecurrent form. Consider the meaning for Λ_N. Each case is a different configuration. As such each case must be tagged with a different hidden variable. For notation simplicity we write A_i rather than A_{λ_i} We also suppress the arguments of the functions since they are recoverable from the configuration table (see Table 3 below).

They state

$$\text{If } \varphi_1+\varphi_2-\varphi_3-\varphi_4=0,$$

$$\text{then } A_\gamma(\varphi_1)B_\gamma(\varphi_2)C_\gamma(\varphi_3)D_\gamma(\varphi_4)=-1 \tag{11a}$$

$$\text{If } \varphi_1+\varphi_2-\varphi_3-\varphi_4=\pi,$$

$$\text{then } A_\gamma(\varphi_1)B_\gamma(\varphi_2)C_\gamma(\varphi_3)D_\gamma(\varphi_4)=1 \tag{11b}$$

Let us now consider some implications of just one of (11a) and (11b), say, the first. Four instances of (11a) are

$$A_\gamma(0)B_\gamma(0)C_\gamma(0)D_\gamma(0)=-1 \tag{12a}$$

$$A_1B_1C_1D_1=-1 \tag{N12a}$$

$$A_\gamma(\phi)B_\gamma(0)C_\gamma(\phi)D_\gamma(0)=-1 \tag{12b}$$

$$A_2B_2C_2D_2=-1 \tag{N12b}$$

$$A_\gamma(\phi)B_\gamma(0)C_\gamma(0)D_\gamma(\phi)=-1 \tag{12c}$$

$$A_3B_3C_3D_3=-1 \tag{N12c}$$

$$A_\gamma(2\phi)B_\gamma(0)C_\gamma(\phi)D_\gamma(\phi)=-1 \tag{12d}$$

$$A_4B_4C_4D_4=-1 \tag{N12d}$$

The configurations are specified by the angles used in (12a-d)

Table 3: GHZ Configurations

$K_1=(0,0,0,0)$
$K_2=(\phi,0,\phi,0)$
$K_3=(\phi,0,0,\phi)$
$K_4=(2\phi,0,\phi,\phi)$
$K_5=(\theta+\pi,0,\theta,0)$ *(see below)*

From Eqs. (12a) and (12b) we obtain

$$A_\gamma(0)C_\gamma(0)=A_\gamma(\phi)C_\gamma(\phi), \tag{13a}$$

$$A_1B_1C_1D_1=A_2B_2C_2D_2 \tag{N13a}$$

There is no cancellation of factors because the hidden variables are different. The $B_1(0)$ and $B_2(0)$ do not cancel nor do the $D_1(0)$ and $D_2(0)$.

and from Eqs. (12a) and (12c) we obtain

$$A_\gamma(\phi)D_\gamma(\phi)=A_\gamma(0)D_\gamma(0). \tag{13b}$$

$$A_1B_1C_1D_1=A_3B_3C_3D_3 \tag{N13b}$$

A consequence of these is

$$C_\gamma(\phi)/D_\gamma(\phi)=C_\gamma(0)/D_\gamma(0), \tag{14a}$$

$$[A_2A_3B_2B_3]C_2/D_3=C_3/D_2 \tag{N14a}$$

which can be rewritten as

$$C_\gamma(\phi)D_\gamma(\phi)=C_\gamma(0)D_\gamma(0), \tag{14b}$$

$$[A_2A_3B_2B_3]C_2D_3=C_3D_2 \tag{N14b}$$

because $D_\gamma(\varphi)=\pm 1$ and hence equals its inverse, and the same for $D_\gamma(0)$.

We then obtain from (12d) and (14b)

$$A_\gamma(2\phi)B_\gamma(0)C_\gamma(0)D_\gamma(0)=-1 \tag{15}$$

$$A_4B_3C_3D_2[A_2A_3B_2B_4C_2C_4D_3D_4]=-1 \tag{N15}$$

which in combination with Eq. (12a) yields

$$A_\gamma(2\phi)=A_\gamma(0)=const\ for\ all\ \phi. \tag{16}$$

$$A_4(2\phi)=A_1(0)f(\phi), \tag{N16}$$

$$f(\phi)=A_2(\phi)A_3(\phi)B_1(0)B_2(0)B_3(0)B_4(0)C_1(0)C_2(\phi)C_3(0)C_4(\phi)D_1(0)D_2(0)D_3(\phi)D_4(\phi).$$

Equation (16) is a quite surprising preliminary result. By itself, this equation is not mathematically contradictory, but physically it is very troublesome: For if $A_\gamma(\varphi)$ is intended as EPR's program suggest, to represent an intrinsic spin quantity, then $A_\gamma(0)$ and $A_\gamma(\pi)$ would be expected to have opposite signs. The trouble becomes manifest, and an actual contradiction emerges, when we use (11b)--which until now has not been brought into play--to obtain

$$A_\gamma(\theta+\pi)B_\gamma(0)C_\gamma(\theta)D_\gamma(0)=1 \tag{17}$$

$$A_5(\theta+\pi)B_5(0)C_5(\theta)D_5(0)=1 \tag{N17}$$

which in combination with Eq. (12b) yields

$$A_\gamma(\theta+\pi) = -A_\gamma(\theta) \tag{18}$$

$$A_5(\theta+\pi) = -A_2(\theta)\, g(\theta), \tag{N18}$$

$$g(\theta) = B_2(0)\, B_5(0)\, C_2(\theta)\, C_5(\theta)\, D_2(0)\, D_5(0).$$

This result confirms *the sign change that we anticipated on physical grounds in EPR's program, but it also contradicts the earlier result of Eq. (16)(let φ=π/2,θ=0). We have thus brought to the surface an inconsistency hidden in premises (i)-(iv).*

Using the suggested values for φ we obtain

$$A_\gamma(\pi) = A_\gamma(0) \tag{16}$$

$$A_4(\pi) = A_1(0)\, f(\pi) \tag{N16}$$

$$A_\gamma(\pi) = -A_\gamma(0) \tag{18}$$

$$A_5(\pi) = -A_2(0)\, g(0) \tag{N18}$$

We immediately see that $A_4(\pi)$ can not be set equal to $A_5(\pi)$. Those functions occur under different experimental configurations

$$K_4 = (\pi, 0, \pi/2, \pi/2)$$

$$K_5 = (\pi, 0, 0, 0)$$

and hence, for Λ_N, they have different hidden values. In general they are not equal. Because they are not equal there is no contradiction. Because there is no contradiction the premises (i)-(iv) hold. Because those premises hold the EPR hidden variable program is maintained. Quantum theories based on hidden variables remain possible.

3 Conclusion

We have presented cases for four inequalities and one condition based on the existence of the hidden variable class Λ_N which are capable of violating the inequalities and condition. Experimental violation does not discriminate between local hidden variable models and standard Hilbert space based quantum mechanics. The state of the local hidden variable model takes one and only one value at each point in time. In contrast the state of standard quantum mechanics takes all possible values at each point in time. Leonard Susskind mentioned in a recent lecture that Richard Feynman said "Hilbert space is so damn big!". We now see that such excess is unnecessary. Real local single valued hidden variables can violate Bell's inequalities. Complex nonlocal multivalued quantum mechanics can violate Bell's inequality. Inequality violation does not determine which model is correct.

The literature is vast on the implications of inequality violation. Much of that literature is made suspect by the hidden variable class Λ_N.

The abstract of the EPR paper [10] says

In a complete theory there is an element corresponding to each element of reality. A suffiient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the kowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these quantities can not have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality given by a wave function is not complete.

John S Bell states in his paper [1]

THE paradox of Einstein, Podolsky and Rosen [8] *was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore the theory causality and locality.*

We have presented evidence that Bell's formulation fails for nonrecurrent hidden variables. As such Einstein's program of completing quantum mechanics with hidden variables remains viable.

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