Ramanujan's cubic continued fraction
Experimental mathematics
Maple experiment , identify command

Edgar Valdebenito

abstract

In this note we briefly explore the Ramanujan's cubic continued fraction

1. Ramanujan's cubic continued fraction (Particular case).

$$\frac{\sqrt{6\sqrt{3}}-\left(1+\sqrt{3}\right)}{4}=\cfrac{e^{-2\pi/3}}{1+\cfrac{e^{-2\pi}+e^{-4\pi}}{1+\cfrac{e^{-4\pi}+e^{-8\pi}}{1+\cfrac{e^{-6\pi}+e^{-12\pi}}{1+\ldots}}}} \qquad (1)$$

2. Experiment: fixed-point method and maple identify command.

- The function $f(x)$:

$$f(x)=1+\cfrac{x^3+x^6}{1+\cfrac{x^6+x^{12}}{1+\cfrac{x^9+x^{18}}{1+\cfrac{x^{12}+x^{24}}{1+\ldots}}}} \quad |x|<1 \qquad (2)$$

- Equation:

$$x=R\,f(x) \qquad (3)$$

where $R=\frac{\sqrt{6\sqrt{3}}-\left(1+\sqrt{3}\right)}{4}$.

- Iterative method:

$$x_{n+1} = R\ f\left(x_n\right) \quad , x_1 = 0 \tag{4}$$

$$x_n^* = identify\left(x_n, all = \ false, FuncPoly = true\right) \tag{5}$$

n	x_n	$x_n^* = identify\left(x_n, all = \ false, FuncPoly = true\right)$
1	0.1229147470	0.1229147470
2	0.1231434228	0.1231434228
3	0.1231447039	0.1231447039
4	0.1231447111	$e^{-2\pi/3}$
5	0.1231447111	$e^{-2\pi/3}$

Table 1. Maple-identify-command

- Remark 1:

$$x_n \to e^{-2\pi/3} \quad , n \to \infty \tag{6}$$

$$e^{-2\pi/3} = R\ f\left(e^{-2\pi/3}\right) \tag{7}$$

- Remark 2: $F\left(x\right) = R\ f\left(x\right)$ is contractive.

3. Formula for $e^{-2\pi/3}$:

$$e^{-2\pi/3} = R\ f\left(R\ f\left(R\ f\left(R...\right)\right)\right) \tag{8}$$

References

1. C. Adiga, B.C. Berndt, S. Bhargava, and G.N. Watson, Chapter 16 of Ramanujan's second notebook: Theta-functions and q-series, Mem. Amer. Math. Soc. , 53(no. 315), Amer. Math. Soc. , Providence, Rhode Island (1985).
2. G.E. Andrews, An Introduction to Ramanujan's lost Notebook, Amer.Math. Monthly 86 (1979) 89-108.
3. G.E. Andrews and B.C. Berndt, The Lost Notebook, Part I, Springer Verlag,New York,2005.
4. H.H. Chan, On Ramanujan's cubic continued fraction, Acta Arith. 73 (1995),343-355.