Calculating the day of the week – Direct Substitution
Cariño's dw-Algorithm

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January 7, 2018
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Abstract. This study is an algorithm of calculating the day of the week for any given date in Gregorian & Julian calendar using simplified formula. It consists of eight algebraic (6 for Julian) expression, five of which are integer function by substituting the year, month and day. This formula will calculate the modulo 7 which gives a number from 0 to 6, i.e., 0=Saturday, 1=Sunday, and so on, that determines the exact day of the week. This algorithm has no condition even during leap-year and 400-year cycle.

1 Introduction

1.1 This algorithm is devised using basic mathematics, without any condition or modification to the formula, it will provide a direct substitution to the formula.

1.2 For any calendar date, m denotes for month, d for day and y for year; m is the number of months in the calendar year, i.e., $m = 1$ for the month of January, $m = 2$ for the month of February and $m = 12$ for the last month of the year which is December; d on the other hand, is the day in a given calendar date, i.e., 1 until 31. Lastly, y is the calendar year in either Gregorian & Julian calendar.

2 The Formula

Formula for Gregorian calendar in original form,

$$dw = \left[\, 3m + 4 + d - \left\lfloor \frac{3m}{7} \right\rfloor - 2\left\lfloor \frac{(m+7)}{10} \right\rfloor + \left\lfloor \frac{(50y+m-3)}{40} \right\rfloor - \left\lfloor \frac{(12y+m-3)}{1200} \right\rfloor + \left\lfloor \frac{(12y+m-3)}{4800} \right\rfloor \,\right] mod\ 7$$

where

- dw is the day of the week (0 = Saturday, 1 = Sunday, …., 6 = Friday)
- m is the month (1 = January, 2 = February, ………, 12 = December)
- d is the day of the month
- y is the Gregorian year

3 Simplified Formula

3.1 Original form,

$$dw = \left[3m + 4 + d - \left\lfloor \frac{3m}{7} \right\rfloor - 2\left\lfloor \frac{(m+7)}{10} \right\rfloor + \left\lfloor \frac{(50y+m-3)}{40} \right\rfloor - \left\lfloor \frac{(12y+m-3)}{1200} \right\rfloor + \left\lfloor \frac{(12y+m-3)}{4800} \right\rfloor \right] mod\ 7$$

3.2 Simplified form,

$$dw = \left[3m + 4 + d - \left\lfloor \frac{3m}{7} \right\rfloor - 2\left\lfloor \frac{(m+7)}{10} \right\rfloor + \left\lfloor \frac{(50y+m-3)}{40} \right\rfloor - \lfloor J \rfloor + \left\lfloor \frac{J}{4} \right\rfloor \right] mod\ 7$$

where

- $J = \frac{(12y+m-3)}{1200}$

4 Examples

Several examples are presented/shown to illustrate the algorithm.

4.1 October 15, 1582, first day of Gregorian calendar.

$m = 10, \qquad d = 15, \qquad y = 1582$

$$J = \frac{(12\{1582\} + 10 - 3)}{1200}$$

$$= \frac{18991}{1200}$$

$$= 15.8258\overline{3}$$

$$dw = \left[3(10) + 4 + 15 - \left\lfloor \frac{3(10)}{7} \right\rfloor - 2\left\lfloor \frac{(10+7)}{10} \right\rfloor + \left\lfloor \frac{(50\{1582\}+10-3)}{40} \right\rfloor - \lfloor 15.8258\overline{3} \rfloor + \left\lfloor \frac{15.8258\overline{3}}{4} \right\rfloor\right] mod\ 7$$

$$= [30 + 4 + 15 - \lfloor 4.2857 \rfloor - 2\lfloor 1.7 \rfloor + \lfloor 1977.675 \rfloor - \lfloor 15.8258\overline{3} \rfloor + \lfloor 3.956 \rfloor] mod\ 7$$

$$= [30 + 4 + 15 - 4 - 2 + 1977 - 15 + 3] mod\ 7$$

$$= [2008] mod\ 7$$

$$= 6\ ; \boldsymbol{Friday}$$

So, October 15, 1582 is Friday

4.2 February 28, 1900, latest centennial that is not a leap-year

$m = 2, \qquad d = 28, \qquad y = 1900$

$$J = \frac{(12\{1900\} + 2 - 3)}{1200}$$

$$= \frac{22799}{1200}$$

$$= 18.9991\overline{6}$$

$$dw = \left[3(2) + 4 + 28 - \left\lfloor\frac{3(2)}{7}\right\rfloor - 2\left\lfloor\frac{(2+7)}{10}\right\rfloor + \left\lfloor\frac{(50\{1900\}+2-3)}{40}\right\rfloor - \lfloor 18.9991\bar{6}\rfloor + \left\lfloor\frac{18.9991\bar{6}}{4}\right\rfloor\right] mod\ 7$$

$$= [6 + 4 + 28 - \lfloor 0.857\rfloor - 2\lfloor 0.9\rfloor + \lfloor 2374.975\rfloor - \lfloor 18.9991\bar{6}\rfloor + \lfloor 4.75\rfloor] mod\ 7$$

$$= [6 + 4 + 28 - 0 - 0 + 2374 - 18 + 4] mod\ 7$$

$$= [2398] mod\ 7$$

$$= 4\ ; \boldsymbol{Wednesday}$$

So, February 28, 1900 is Wednesday

4.3 March 30, 1000000, first year of a 7-digit calendar year

$m = 3, \quad d = 30, \quad y = 1x10^6$

$$J = \frac{(12\{1x10^6\} + 3 - 3)}{1200}$$

$$= \frac{12x10^6}{1200}$$

$$= 10x10^3$$

$$dw = \left[3(3) + 4 + 30 - \left\lfloor\frac{3(3)}{7}\right\rfloor - 2\left\lfloor\frac{(3+7)}{10}\right\rfloor + \left\lfloor\frac{(50\{1x10^6\}+3-3)}{40}\right\rfloor - \lfloor 10x10^3\rfloor + \left\lfloor\frac{10x10^3}{4}\right\rfloor\right] mod\ 7$$

$$= [9 + 4 + 30 - \lfloor 1.29\rfloor - 2\lfloor 1\rfloor + \lfloor 1.25x10^6\rfloor - \lfloor 10x10^3\rfloor + \lfloor 2500\rfloor] mod\ 7$$

$$= [9 + 4 + 30 - 1 - 2 + 1.25x10^6 - 10x10^3 + 2500] mod\ 7$$

$$= [1242540] mod\ 7$$

$$= 5\ ; \boldsymbol{Thursday}$$

So, March 30, 1000000 is Thursday

5 The Algorithms

- $J = \frac{(12y+m-3)}{1200}$

5.1 Gregorian Calendar:

$$dw = \left[3m + 4 + d - \left\lfloor\frac{3m}{7}\right\rfloor - 2\left\lfloor\frac{(m+7)}{10}\right\rfloor + \left\lfloor\frac{(50y+m-3)}{40}\right\rfloor - \lfloor J\rfloor + \left\lfloor\frac{J}{4}\right\rfloor\right] mod\ 7$$

5.2 Julian Calendar:

$$dw = \left[3m + 2 + d - \left\lfloor\frac{3m}{7}\right\rfloor - 2\left\lfloor\frac{(m+7)}{10}\right\rfloor + \left\lfloor\frac{(50y+m-3)}{40}\right\rfloor\right] mod\ 7$$

Acknowledgements

This work is dedicated to my family especially to my wife Melanie and two sons, Milan and Mileone.

References

1 https://en.wikipedia.org/wiki/Determination_of_the_day_of_the_week

2 https://en.wikipedia.org/wiki/Gregorian_calendar

3 https://en.wikipedia.org/wiki/Julian_calendar