

EINSTEIN FIELD EQUATIONS OBTAINED ONLY WITH GAUSS CURVATURE AND ZOOM UNIVERSE MODEL CHARACTERISTICS

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Abstract

Demonstration how to obtain the Einstein Field Equations without using the Stress-Energy Tensor, without using the Bianchi Identities and without using the Energy Conservation to obtain it.

Demonstration how to obtain the Einstein Field Equations only using the Gauss Curvature and the zoom universe model characteristics.

Gravity in zoom universe model

Sphere and Hypersphere example to understand it better

Zoom universe model characteristics and zoom special relativity

First we can see:

[1] All Special Relativity Equations Obtained with Galilean Transformation <http://vixra.org/abs/1712.0638>

Gauss-Codazzi equations & Riemann Tensor

For a Surface of 2 dimensions embedded in a 3 dimensions space $S : R^2 \rightarrow R^3$ expressed by:

$$\vec{s} = (x(u_1, u_2), y(u_1, u_2), z(u_1, u_2))$$

If we do this:

$$d\vec{s} = \frac{\partial \vec{s}}{\partial u_1} du_1 + \frac{\partial \vec{s}}{\partial u_2} du_2 = \vec{s}_1 du_1 + \vec{s}_2 du_2 = \vec{e}_1 du^1 + \vec{e}_2 du^2$$

$$\vec{N} = \frac{\vec{e}_1 \wedge \vec{e}_2}{|\vec{e}_1 \wedge \vec{e}_2|}$$

We have base vectors and a normal vector in a point of the surface

And we have:

$$I = ds^2 = d\vec{s} \cdot d\vec{s} = \vec{e}_1 \cdot \vec{e}_1 (du^1)^2 + \vec{e}_2 \cdot \vec{e}_2 (du^2)^2 + 2 \vec{e}_1 \cdot \vec{e}_2 (du^1)(du^2) = g_{11}(du^1)^2 + g_{22}(du^2)^2 + 2 g_{12}(du^1)(du^2)$$

$$II = d\vec{s} \cdot d\vec{N} = \vec{e}_1 \cdot \vec{N}_1 (du^1)^2 + \vec{e}_2 \cdot \vec{N}_2 (du^2)^2 + 2 \vec{e}_1 \cdot \vec{N}_2 (du^1)(du^2) = l_{11}(du^1)^2 + l_{22}(du^2)^2 + 2 l_{12}(du^1)(du^2)$$

The first and second fundamental form

And we have:

$$\vec{N}_1 = -l_1^i \vec{e}_1 - l_1^2 \vec{e}_2$$

$$\vec{N}_2 = -l_2^1 \vec{e}_1 - l_2^2 \vec{e}_2$$

The derivation of the normal vector \vec{N} expressed in a base vector coordinates

And the Gauss curvature K:

$$K = \frac{l_{11} l_{22} - l_{12}^2}{g_{11} g_{22} - g_{12}^2} = \frac{L}{g}$$

The Gauss-Codazzi equations for 2 dimensions can be written as [5]:

$$\frac{\partial \Gamma_{ab}^i}{\partial u_j} - \frac{\partial \Gamma_{ab}^j}{\partial u_i} + \Gamma_{ab}^m \Gamma_{mj}^i - \Gamma_{aj}^m \Gamma_{mb}^i = l_{ab} l_j^i - l_{aj} l_b^i, \quad i = 1, 2$$

The Riemann Tensor definition it is:

$$R_{ajb}^i = \frac{\partial \Gamma_{ab}^i}{\partial u_j} - \frac{\partial \Gamma_{ab}^j}{\partial u_i} + \Gamma_{ab}^m \Gamma_{mj}^i - \Gamma_{aj}^m \Gamma_{mb}^i$$

With what for 2 dimensions we can write:

$$R_{ajb}^i = l_{ab} l_j^i - l_{aj} l_b^i, \quad i = 1, 2$$

Ricci Tensor & Gauss curvature

The Ricci Tensor definition it is (for 2 dimensions):

$$R_{ab} = R_{aib}^i = R_{a1b}^1 + R_{a2b}^2$$

With what for 2 dimensions we can write:

$$R_{ab} = l_{ab} l_i^i - l_{ai} l_b^i, \quad i = 1, 2$$

And we can do:

$$R_{ab} = g^{ir} (l_{ab} l_{ir} - l_{ai} l_{br}), \quad i = 1, 2$$

The inverse of the metric tensor, we can obtain with the cofactor of g, in 2 dimensions we have:

$$g^{ij} = \frac{G_j^i}{|g_{ij}|} \quad \rightarrow \quad g^{11} = \frac{g_{22}}{g_{11} g_{22} - g_{12}^2}, \quad g^{12} = \frac{-g_{12}}{g_{11} g_{22} - g_{12}^2}, \quad g^{22} = \frac{g_{11}}{g_{11} g_{22} - g_{12}^2}$$

$$\begin{aligned} R_{ab} = g^{ir} (l_{ab} l_{ir} - l_{ai} l_{br}) &= g_{ba} \frac{(l_{ab} l_{ir} - l_{ai} l_{br})}{|g_{ij}|} \quad \rightarrow \text{If } a = b, \quad i = 1, 2 \\ &= g_{ba} \frac{(l_{aa} l_{br} - l_{ab} l_{ar})}{|g_{ij}|} \quad \rightarrow \text{If } a \neq b, \quad i = 1, 2 \end{aligned}$$

We can see if $(a = b) \rightarrow (i = r)$ and if $(a \neq b) \rightarrow (i \neq r)$ with what, knowing the Gauss curvature definition:

$$R_{ab} = g_{ba} [K]_{\text{plane } a-b} \quad (\text{in 2 dimensions})$$

We can see that this expression would also be useful for more dimensions **as long as we have a diagonal metric** and add the curvatures in common

$$g = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix}$$

$$R_{11} = g_{11} ([K]_{\text{plane } 1-2} + [K]_{\text{plane } 1-3} + [K]_{\text{plane } 1-4})$$

$$R_{22} = g_{22} ([K]_{\text{plane } 2-1} + [K]_{\text{plane } 2-3} + [K]_{\text{plane } 2-4})$$

$$R_{33} = g_{33} ([K]_{\text{plane } 3-2} + [K]_{\text{plane } 3-1} + [K]_{\text{plane } 3-4})$$

$$R_{44} = g_{44} ([K]_{\text{plane } 4-2} + [K]_{\text{plane } 4-3} + [K]_{\text{plane } 4-1})$$

Ricci Scalar & Gauss curvature

The Ricci Scalar definition it is (for 2 dimensions):

$$R = g^{ab}R_{ab} = g^{11}R_{11} + g^{12}R_{12} + g^{21}R_{21} + g^{22}R_{22}$$

With what for 2 dimensions we can write ($\delta =$ Kronecker delta):

$$R = g^{ab}R_{ab} = g^{ab}g_{ba} [K]_{\text{plane } a-b} = \delta_a^a [K]_{\text{plane } a-b} = 2 [K]_{\text{plane } a-b}$$

We can see that this expression would also be useful for more dimensions **as long as we have a diagonal metric** and add all curvatures

$$g = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix}$$

$$R = 2 [K]_{\text{plane } 1-2} + 2 [K]_{\text{plane } 1-3} + 2 [K]_{\text{plane } 1-4} + 2 [K]_{\text{plane } 2-3} + 2 [K]_{\text{plane } 2-4} + 2 [K]_{\text{plane } 3-4}$$

Einstein Tensor & Gauss curvature

The Einstein tensor definition it is:

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R$$

With what, always **as long as we have a diagonal metric**

$$g = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix}$$

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = g_{ba} \sum [K]_{\text{plane } a-b} - g_{ab} \sum [K]_{\text{all planes}} = -g_{ab} \sum [K]_{\text{planes except plane } a-b}$$

$$G_{11} = R_{11} - \frac{1}{2} g_{11} R = -g_{11} ([K]_{\text{plane } 2-3} + [K]_{\text{plane } 2-4} + [K]_{\text{plane } 4-3})$$

$$G_{22} = R_{22} - \frac{1}{2} g_{22} R = -g_{22} ([K]_{\text{plane } 1-3} + [K]_{\text{plane } 1-4} + [K]_{\text{plane } 4-3})$$

$$G_{33} = R_{33} - \frac{1}{2} g_{33} R = -g_{33} ([K]_{\text{plane } 2-1} + [K]_{\text{plane } 2-4} + [K]_{\text{plane } 4-2})$$

$$G_{44} = R_{44} - \frac{1}{2} g_{44} R = -g_{44} ([K]_{\text{plane } 2-3} + [K]_{\text{plane } 2-1} + [K]_{\text{plane } 1-3})$$

Zoom universe model & Gauss curvature

Now first let's calculate how would be the curvature of each plane if we do the zoom level fixed (R) we would have a hypersphere with 3 dimensions on its surface (α, θ, ϕ) and a radius R .

But for each 2 dimensions plane (plane $\alpha-\theta$) (plane $\alpha-\phi$) (plane $\phi-\theta$) we have a 2 dimensions sphere with same radius R and same spherical Gauss curvature $\frac{1}{R^2}$

$$[K]_{\text{plane } \alpha-\theta} \text{ or } [K]_{\text{plane } \alpha-\phi} \text{ or } [K]_{\text{plane } \phi-\theta} = \frac{1}{R^2}$$

We know for [1] that: $U = -F R$ (potential energy, work done on an object is found by multiplying force and distance)

$$[K]_{\text{plane } \alpha-\theta} \text{ or } [K]_{\text{plane } \alpha-\phi} \text{ or } [K]_{\text{plane } \phi-\theta} = \frac{1}{R^2} = \frac{FF}{UU}$$

Multiplying both sides for Area and radius ($A R$):

$$[K]_{\text{plane } \alpha-\theta} \text{ or } [K]_{\text{plane } \alpha-\phi} \text{ or } [K]_{\text{plane } \phi-\theta} = \frac{1}{R^2} = \frac{FF}{UU} = \frac{FFAR}{UUAR}$$

Now we know [1] that $U = -F R = mc^2$, we know that the sphere area it's $A = 4\pi R^2$, we know that the Pressure it's $P = \frac{E}{A} = \frac{ER}{AR} = \frac{-U}{\text{Vol}}$, (

Vol = volume) and in one volume we can have n particles $P = \frac{E}{A} = \frac{ER}{AR} = \frac{-U}{Vol} = -\frac{nm^2c^2}{Vol}$, with what:

$$[K]_{\text{plane } \alpha-\theta} \text{ or } [K]_{\text{plane } \alpha-\phi} \text{ or } [K]_{\text{plane } \phi-\theta} = \frac{1}{R^2} = \frac{FF}{UU} = \frac{FEAR}{UUAR} = -\frac{mc^2 4\pi R^2 n mc^2}{U^2 R Vol}$$

We know that the mass density $\rho_m = \frac{nm}{Vol}$, we know the light escape velocity $c^2 = \frac{2Gm}{R}$, (G it is the gravitational constant) and we know [1] that: $v = \frac{c}{\gamma}$

$$[K]_{\text{plane } \alpha-\theta} \text{ or } [K]_{\text{plane } \alpha-\phi} \text{ or } [K]_{\text{plane } \phi-\theta} = \frac{1}{R^2} = \frac{FF}{UU} = \frac{FEAR}{UUAR} = -\frac{mc^2 4\pi R^2 n mc^2}{U^2 R Vol} = -\frac{m 2 G m 4 \pi R^2 \rho_m c^2}{m^2 c^4 R R} = -\frac{8 \pi G}{c^4} \rho_m \gamma^2 v^2$$

We can see what all mass and mass density in this equation it's refer to all mass and mass density into all universe plane sphere

The rest of planes in Zoom universe model & Gauss curvature

For calculating the rest of planes we can do one of this dimensions fixed (α, θ, ϕ) and r would be part of the surface:

with α fixed (A) $\rightarrow [K]_{\text{plane } r-\theta}, [K]_{\text{plane } r-\phi}$

with θ fixed (Θ) $\rightarrow [K]_{\text{plane } r-\alpha}, [K]_{\text{plane } r-\phi}$

with ϕ fixed (Φ) $\rightarrow [K]_{\text{plane } r-\theta}, [K]_{\text{plane } r-\alpha}$

All Gauss curvature will be $\frac{1}{(\text{radius})^2} = \frac{1}{A^2}$ or $\frac{1}{\Theta^2}$ or $\frac{1}{\Phi^2}$ and all plane sphere contains all universe plane sphere mass and mass density regardless of the dimension used as a radio

$$[K]_{\text{rest of planes}} = \frac{1}{(\text{radius})^2} = \frac{1}{A^2} \text{ or } \frac{1}{\Theta^2} \text{ or } \frac{1}{\Phi^2} = \frac{FF}{UU} = \frac{FEA(\text{radius})}{UUA(\text{radius})}$$

Now we can see the energy used to be in a position on the surface of the sphere it is the Kinetic energy (K),

$$U = -F(\text{radius}) = K = \frac{mv^2}{2}, \text{ and the Pressure } P = \frac{E}{A} = \frac{E(\text{radius})}{A(\text{radius})} = \frac{-K}{Vol} = -\frac{nmv}{2 Vol}$$

$$[K]_{\text{rest of planes}} = \frac{1}{(\text{radius})^2} = \frac{1}{A^2} \text{ or } \frac{1}{\Theta^2} \text{ or } \frac{1}{\Phi^2} = \frac{FF}{UU} = \frac{FEA(\text{radius})}{KKA(\text{radius})} = -\frac{mv^2 \pi (\text{radius})^2 n mcv^2}{K^2(\text{radius}) Vol}$$

We know that the mass density $\rho_m = \frac{nm}{Vol}$, we know the escape velocity $v^2 = \frac{2Gm}{R}$, (G it is the gravitational constant) and we know for [1] $v = \frac{c}{\gamma}$

$$[K]_{\text{rest of planes}} = \frac{1}{(\text{radius})^2} = \frac{1}{A^2} \text{ or } \frac{1}{\Theta^2} \text{ or } \frac{1}{\Phi^2} = \frac{FF}{UU} = \frac{FEA(\text{radius})}{KKA(\text{radius})} = -\frac{mv^2 \pi (\text{radius})^2 n mcv^2}{K^2(\text{radius}) Vol} = -\frac{4m 2 G m \pi (\text{radius})^2 \rho_m v^2}{m^2 v^4 (\text{radius}) (\text{radius})} = -\frac{8 \pi G \rho_m v^2 \gamma^4}{m^2 c^4} = -\frac{8 \pi G}{c^4} \rho_m \gamma^2 c^2$$

We can see that the velocity of r dimension either as a radius or as a surface should be c, that agrees with the exposed in [1] [light transmission medium and waves velocity traveling in it (c)]

Join Einstein Tensor & Zoom universe model

Now if we join all equations (the sphere have a diagonal metric tensor) (index 1 = r, index 2 = α , index 3 = θ , index 4 = ϕ):

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = -g_{ab} \sum [K]_{\text{planes except plane } a-b} = g_{ab} \frac{8 \pi G}{c^4} \rho_m \gamma^2 (\sum^2 \text{velocity planes})$$

We can see that the sum of 3 bi-dimensional velocity components squared gives us a velocity squared in 3 dimensions

$$G_{11} = R_{11} - \frac{1}{2} g_{11} R = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{11} (v_{\text{plane } 2-3}^2 + v_{\text{plane } 2-4}^2 + v_{\text{plane } 3-4}^2) = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{11} v_{\text{cube planes } 2-3-4}^2$$

$$G_{22} = R_{22} - \frac{1}{2} g_{22} R = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{22} (c_{\text{plane } 1-3}^2 + c_{\text{plane } 1-4}^2 + v_{\text{plane } 3-4}^2) = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{22} v_{\text{cube planes } 1-3-4}^2$$

$$G_{33} = R_{33} - \frac{1}{2} g_{33} R = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{33} (c_{\text{plane } 2-1}^2 + c_{\text{plane } 2-4}^2 + v_{\text{plane } 2-4}^2) = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{33} v_{\text{cube planes } 1-2-4}^2$$

$$G_{44} = R_{44} - \frac{1}{2} g_{44} R = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{44} (v_{\text{plane } 2-3}^2 + c_{\text{plane } 2-1}^2 + c_{\text{plane } 3-1}^2) = \frac{8 \pi G}{c^4} \rho_m \gamma^2 g_{44} v_{\text{cube planes } 1-2-3}^2$$

Join Metric Tensor & velocity cube planes squared

The velocity squared can be expressed in terms of the first fundamental form:

$$v_{\text{cube planes } 1-2-3}^2 = ds^2 = d\vec{s} \cdot d\vec{s} =$$

$$\vec{e}_1 \cdot \vec{e}_1 (du^1)^2 + \vec{e}_2 \cdot \vec{e}_2 (du^2)^2 + \vec{e}_3 \cdot \vec{e}_3 (du^3)^2 + 2 \vec{e}_1 \cdot \vec{e}_2 (du^1)(du^2) + 2 \vec{e}_1 \cdot \vec{e}_3 (du^1)(du^3) + 2 \vec{e}_2 \cdot \vec{e}_3 (du^2)(du^3) =$$

$$g_{11}(du^1)^2 + g_{22}(du^2)^2 + g_{33}(du^3)^2 + 2 g_{12}(du^1)(du^2) + 2 g_{13}(du^1)(du^3) + 2 g_{23}(du^2)(du^3) =$$

$$g_{cd}(du^c)(du^d)$$

$$v_{\text{cube planes } 1-2-3}^2 = g_{cd}(du^c)(du^d), \quad \text{with index } c = 1, 2, 3, \text{ with index } d = 1, 2, 3$$

$$v_{\text{cube planes } 2-3-4}^2 = g_{cd}(du^c)(du^d), \quad \text{with index } c = 2, 3, 4, \text{ with index } d = 2, 3, 4$$

$$v_{\text{cube planes } 1-3-4}^2 = g_{cd}(du^c)(du^d), \quad \text{with index } c = 1, 3, 4, \text{ with index } d = 1, 3, 4$$

$$v_{\text{cube planes } 1-2-4}^2 = g_{cd}(du^c)(du^d), \quad \text{with index } c = 1, 2, 4, \text{ with index } d = 1, 2, 4$$

Now let's multiply those metric tensors of 3 dimensions (g_{cd}) with the metric tensor of 4 dimensions (g_{ab}):

(We must bear in mind that the missing dimension in g_{cd} will always be in g_{cd} so that in the end we have 2 metric tensors of 4 dimensions, I develop a case to make it look better):

$$g_{ab} g_{cd}(du^c)(du^d) = (\vec{e}_a \cdot \vec{e}_b) (\vec{e}_c \cdot \vec{e}_d) (du^c)(du^d) = (\vec{e}_a \cdot \vec{e}_c) (\vec{e}_b \cdot \vec{e}_d) (du^c)(du^d) = g_{ad} g_{cb} (du^c)(du^d)$$

$$g_{11} v_{\text{cube planes } 2-3-4}^2 = g_{11} g_{cd}(du^c)(du^d), \quad \text{with index } c = 2, 3, 4, \text{ with index } d = 2, 3, 4 \quad =$$

$$g_{11} g_{22}(du^2)(du^2) + g_{11} g_{23}(du^2)(du^3) + g_{11} g_{24}(du^2)(du^4) + g_{11} g_{32}(du^3)(du^2) + g_{11} g_{33}(du^3)(du^3) + g_{11} g_{34}(du^3)(du^4) + g_{11} g_{42}(du^4)(du^2) + g_{11} g_{43}(du^4)(du^3) + g_{11} g_{44}(du^4)(du^4) =$$

$$g_{12} g_{12}(du^2)(du^2) + g_{12} g_{13}(du^2)(du^3) + g_{12} g_{14}(du^2)(du^4) + g_{13} g_{12}(du^3)(du^2) + g_{13} g_{13}(du^3)(du^3) + g_{13} g_{14}(du^3)(du^4) + g_{14} g_{12}(du^4)(du^2) + g_{14} g_{13}(du^4)(du^3) + g_{14} g_{14}(du^4)(du^4) =$$

With what:

$$g_{ab} (\sum^2 \text{velocity planes}) = g_{ad} g_{cb} (du^c)(du^d), \quad \text{with } (\text{index } c \text{ and } d) \neq (\text{index } a \text{ and } b)$$

And we have:

$$R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G}{c^4} \rho_m \gamma^2 g_{ad} g_{cb} (du^c)(du^d), \quad \text{with } (\text{index } c \text{ and } d) \neq (\text{index } a \text{ and } b)$$

Finally some tensor algebra

Multiplying both sides by g^{bi} :

$$R_a^i - \frac{1}{2} \delta_a^i R = \frac{8\pi G}{c^4} \rho_m \gamma^2 g_{ad} \delta_c^i (du^c)(du^d)$$

Multiplying both sides by g^{aj} :

$$R^{ij} - \frac{1}{2} g^{ij} R = \frac{8\pi G}{c^4} \rho_m \gamma^2 \delta_d^j \delta_c^i (du^c)(du^d)$$

$$R^{ij} - \frac{1}{2} g^{ij} R = \frac{8\pi G}{c^4} \rho_m \gamma^2 (du^i)(du^j)$$

The derivative of a coordinate is the velocity component in that coordinate

(Remember that the speed in coordinate 1 (r) it is c):

$$R^{ij} - \frac{1}{2} g^{ij} R = \frac{8\pi G}{c^4} \rho_m \gamma^2 v^i v^j$$

This expression $\rho_m \gamma^2 v^i v^j$ it is the stress-energy tensor T^{ij} with what, we have the Einstein field equations:

$$R^{ij} - \frac{1}{2} g^{ij} R = \frac{8\pi G}{c^4} T^{ij}$$

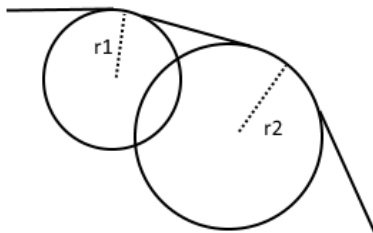
Gravity

We can see that the previous equation was obtained with spheres in one point that includes the whole universe, with all its radius and all its mass-energy density (because mass and energy are related by $E = mc^2$).

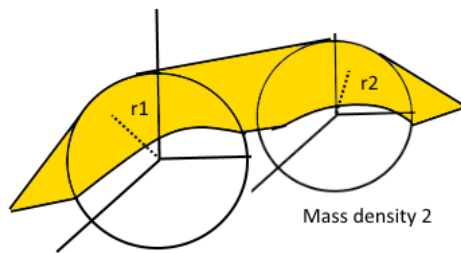
This mass-energy density It cause the all universe spherical curvature form.

Obviously if we have a smaller mass-energy, we will have a smaller sphere with a smaller radius.

And the curvature of this 2 dimension plane we can calculate it with the radius of that's smaller sphere, similar to how we calculate the curvature of a 1 dimension line using the radius of circles.



CURVATURE OF LINE CAUSES FOR CIRCLES

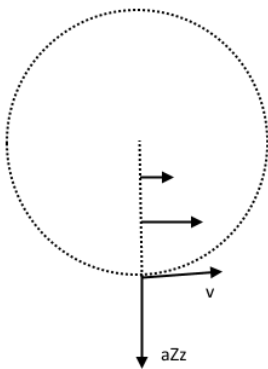


PLANE CURVATURE CAUSE FOR SPHERES

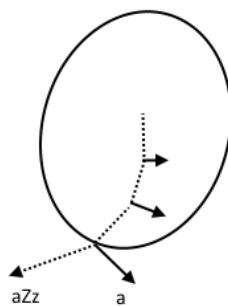
It was already known that gravity was caused by the curvature of the 4 dimensions of space-time, but with this zoom universe model and its 4 dimensions of space-zoom it may look better.

A object with constant velocity causes a centrifugal acceleration as shown in [1] your velocity will be perpendicular and centrifugal acceleration it travels for the radius dimension (r) there is no curvature.

But if that object have a acceleration, this acceleration will be added to the centrifugal acceleration with what will bend that radius and how the velocity is perpendicular, we see how the acceleration of an object curves the space-zoom.

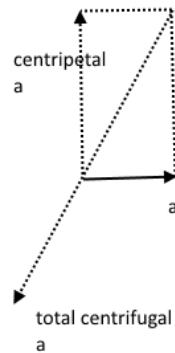


OBJECT WITH CONSTANT VELOCITY



OBJECT WITH ACCELERATION

Remember..



And if an acceleration causes a curvature, then a curvature then causes an acceleration.

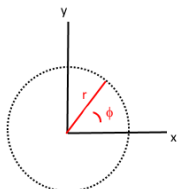
That's the gravity

Sphere example (drawn with one dimension less) (without 3d perspective)

First we draw a circle with yours polar coordinates $(x, y) \rightarrow (r, \phi)$:

$$x = r \cos \phi$$

$$y = r \sin \phi$$



Now we take the radius dimension and we replace it with another circle perpendicular to the rest of the dimensions

$$r \rightarrow z_x = r \cos \theta$$

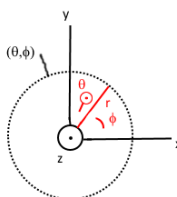
$$z_y = r \sin \theta$$

We can see that the z_y is a new dimension and the z_x matches the radius, with that:

$$z = r \sin \theta$$

$$x = r \cos \theta \cos \phi$$

$$y = r \cos \theta \sin \phi$$

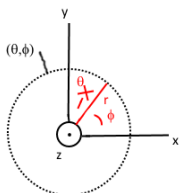


But if we want the last angle to begin with the last dimension:

$$z = r \cos \theta$$

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$



We have the spherical coordinates $(x, y, z) \rightarrow (r, \theta, \phi)$

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

$$\vec{N} = \frac{(x, y, z)}{r} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

If we make constant $r \rightarrow dr = 0$

$$g = \begin{pmatrix} g_{22} & g_{23} \\ g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} g_{\theta\theta} & g_{\theta\phi} \\ g_{\phi\theta} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} r^2 & 0 \\ 0 & r^2 \sin^2 \theta \end{pmatrix}$$

$$l = \begin{pmatrix} l_{22} & l_{23} \\ l_{32} & l_{33} \end{pmatrix} = \begin{pmatrix} l_{\theta\theta} & l_{\theta\phi} \\ l_{\phi\theta} & l_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -r & 0 \\ 0 & -r \sin^2 \theta \end{pmatrix}$$

$$K_{\text{plane } \theta-\phi} = \frac{1}{r^2}, K_{\text{plane } r-\phi} = 0, K_{\text{plane } \theta-r} = 0$$

$$\text{Ricci Tensor} = \begin{pmatrix} R_{22} & R_{23} \\ R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} R_{\theta\theta} & R_{\theta\phi} \\ R_{\phi\theta} & R_{\phi\phi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2 \theta \end{pmatrix}, \quad \text{Ricci Scalar} = \frac{2}{r^2}$$

$$\text{Einstein Tensor} = \begin{pmatrix} G_{22} & G_{23} \\ G_{32} & G_{33} \end{pmatrix} = \begin{pmatrix} G_{\theta\theta} & G_{\theta\phi} \\ G_{\phi\theta} & G_{\phi\phi} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

We observed that the flat coordinate and the curved coordinates could be chosen differently

If we make constant $\theta \rightarrow d\theta = 0$ and:

$z = \theta \cos r$
 $x = \theta \sin r \cos \phi$
 $y = \theta \sin r \sin \phi$

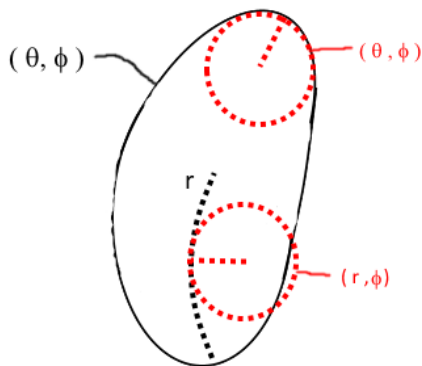
$K_{\text{plane } \theta-\phi} = 0,$ $K_{\text{plane } r-\phi} = \frac{1}{\theta^2}, K_{\text{plane } \theta-r} = 0$

If we make constant $\phi \rightarrow d\phi = 0$ and:

$z = \phi \cos \theta$
 $x = \phi \sin \theta \cos r$
 $y = \phi \sin \theta \sin r$

$K_{\text{plane } \theta-\phi} = 0,$ $K_{\text{plane } r-\phi} = 0, K_{\text{plane } \theta-r} = \frac{1}{\phi^2}$

And we can use that's spheres to determine the curvature of any pseudo-spherical object



Hypersphere example (drawn with two dimension less) (without 3d perspective) (without 4d perspective)

First we draw a sphere with ours spherical coordinates $(x, y, z) \rightarrow (r, \theta, \phi)$:

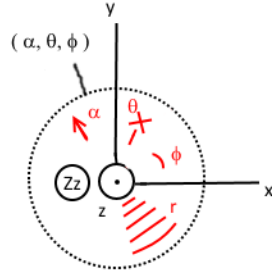
$z = r \cos \theta$
 $x = r \sin \theta \cos \phi$
 $y = r \sin \theta \sin \phi$

Now we take the radius dimension and we replace it with another circle perpendicular to the rest of the dimensions:

$$r \quad \rightarrow \quad \begin{aligned} z_y &= r \cos \alpha \\ z_z &= r \sin \alpha \end{aligned}$$

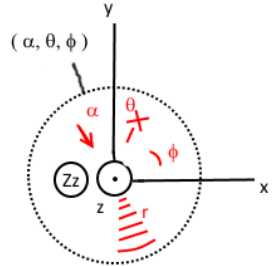
We can see that the z_z is a new dimension and the z_y matches the radius, with that:

$$\begin{aligned}
z_z &= r \sin \alpha \\
z &= r \cos \alpha \cos \theta \\
x &= r \cos \alpha \sin \theta \cos \phi \\
y &= r \cos \alpha \sin \theta \sin \phi
\end{aligned}$$



But if we want the last angle to begin with the last dimension:

$$\begin{aligned}
z_z &= r \cos \alpha \\
z &= r \sin \alpha \cos \theta \\
x &= r \sin \alpha \sin \theta \cos \phi \\
y &= r \sin \alpha \sin \theta \sin \phi
\end{aligned}$$



We have the hyperspherical coordinates $(x, y, z, z_z) \rightarrow (r, \alpha, \theta, \phi)$

$$ds^2 = dr^2 + r^2 d\alpha^2 + r^2 \sin^2 \alpha d\theta^2 + r^2 \sin^2 \alpha \sin^2 \theta d\phi^2$$

$$\vec{N} = \frac{(\alpha, y, z, z_z)}{r} = (\sin \alpha \sin \theta \cos \phi, \sin \alpha \sin \theta \sin \phi, \sin \alpha \cos \theta, \cos \alpha)$$

If we make constant $r \rightarrow dr = 0$

$$g = \begin{pmatrix} g_{22} & g_{23} & g_{24} \\ g_{32} & g_{33} & g_{34} \\ g_{42} & g_{43} & g_{44} \end{pmatrix} = \begin{pmatrix} g_{\alpha\alpha} & g_{\alpha\theta} & g_{\alpha\phi} \\ g_{\theta\alpha} & g_{\theta\theta} & g_{\theta\phi} \\ g_{\phi\alpha} & g_{\phi\theta} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} r^2 & 0 & 0 \\ 0 & r^2 \sin^2 \alpha & 0 \\ 0 & 0 & r^2 \sin^2 \alpha \sin^2 \theta \end{pmatrix}$$

$$l = \begin{pmatrix} l_{22} & l_{23} & l_{24} \\ l_{32} & l_{33} & l_{34} \\ l_{42} & l_{43} & l_{44} \end{pmatrix} = \begin{pmatrix} l_{\alpha\alpha} & l_{\alpha\theta} & l_{\alpha\phi} \\ l_{\theta\alpha} & l_{\theta\theta} & l_{\theta\phi} \\ l_{\phi\alpha} & l_{\phi\theta} & l_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -r & 0 & 0 \\ 0 & -r \sin^2 \alpha & 0 \\ 0 & 0 & -r \sin^2 \alpha \sin^2 \theta \end{pmatrix}$$

$$K_{\text{plane } \alpha-\theta} = \frac{1}{r^2}, K_{\text{plane } \alpha-\phi} = \frac{1}{r^2}, \quad K_{\text{plane } \theta-\phi} = \frac{1}{r^2}$$

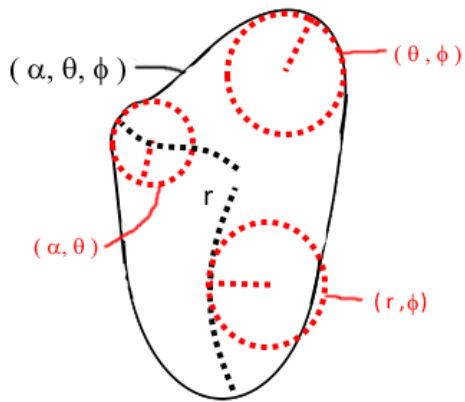
$$K_{\text{plane } r-\alpha} = 0, K_{\text{plane } r-\theta} = 0, \quad K_{\text{plane } r-\phi} = 0$$

$$\text{Ricci Tensor} = \begin{pmatrix} R_{22} & R_{23} & R_{24} \\ R_{32} & R_{33} & R_{34} \\ R_{42} & R_{43} & R_{44} \end{pmatrix} = \begin{pmatrix} R_{\alpha\alpha} & R_{\alpha\theta} & R_{\alpha\phi} \\ R_{\theta\alpha} & R_{\theta\theta} & R_{\theta\phi} \\ R_{\phi\alpha} & R_{\phi\theta} & R_{\phi\phi} \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 \sin^2 \alpha & 0 \\ 0 & 0 & 2 \sin^2 \alpha \sin^2 \theta \end{pmatrix}, \quad \text{Ricci Scalar} = \frac{6}{r^2}$$

$$\text{Einstein Tensor} = \begin{pmatrix} G_{22} & G_{23} & G_{24} \\ G_{32} & G_{33} & G_{34} \\ G_{42} & G_{43} & G_{44} \end{pmatrix} = \begin{pmatrix} G_{\alpha\alpha} & G_{\alpha\theta} & G_{\alpha\phi} \\ G_{\theta\alpha} & G_{\theta\theta} & G_{\theta\phi} \\ G_{\phi\alpha} & G_{\phi\theta} & G_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\sin^2 \alpha & 0 \\ 0 & 0 & -\sin^2 \alpha \sin^2 \theta \end{pmatrix}$$

We observed that the flat coordinate and the curved coordinates could be chosen differently, as in the example of the sphere.

And as in the example of the sphere we can use spheres to determine the curvature of any pseudo-hyperspherical object, as the zoom-universe model.



References

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