Number p-q where p and q Poulet numbers needs very few iterations of “reverse and add” to reach a palindrome

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I make the following observation: the number n = p – q, where p and q are Poulet numbers, needs very few iterations of “reverse and add” to reach a palindrome. For instance, taking q = 1729 and p = 999986341201, it can be seen that only 3 iterations are needed to reach a palindrome: n = 999986341201 – 1729 = 999986339472 and we have: 999986339472 + 274933689999 = 1274920029471; 1274920029471 + 1749200294721 = 3024120324192 and 3024120324192 + 2914230214203 = 5938350538395, a palindromic number. So, relying on this, I conjecture that there exist an infinity of n, even considering q and p successive, that need just one such iteration to reach a palindrome (see sequence A015976 in OEIS for these numbers) and I also conjecture that there is no a difference between two Poulet numbers to be a Lychrel number.

Conjecture I:

There exist an infinity of numbers n = p - q, where q and p successive Poulet numbers, that need just one iteration of “reverse and add” to reach a palindrome (see sequence A015976 in OEIS for these numbers).

The sequence of palindromes obtained from these numbers n:

: 242 (561 – 341 = 220 and 220 + 22 = 242);
: 585 (1729 - 1387 = 342 and 342 + 243 = 585);
: 383 (2047 – 1905 = 142 and 142 + 241 = 383);
: 868 (2701 – 2465 = 236 and 236 + 632 = 868);
: 141 (2821 – 2701 = 120 and 120 + 21 = 141);
: 969 (4369 – 4033 = 336 and 336 + 633 = 969);
: 4 (4371 – 4369 = 2 and 2 + 2 = 4);
: 323 (4681 – 4371 = 310 and 310 + 13 = 323);
: 1551 (6601 – 5461 = 1140 and 1140 + 411 = 1551);
: 7887 (7957 – 6601 = 1356 and 1356 + 6531 = 7887);
: 464 (8911 – 8481 = 430 and 430 + 34 = 464);
: 1881 (10261 – 8911 = 1350 and 1350 + 531 = 1881);
: 747 (10585 – 10261 = 324 and 324 + 423 = 747);
: 747 (11305 – 10585 = 720 and 720 + 27 = 747);
(...)

Note the interesting fact that for two distinct differences (324 and 720) was obtained the same palindrome, 747.

Conjecture II:

There is no a difference between two Poulet numbers to be a Lychrel number.

Two large differences which lead in three iterations to a palindrome:

: for n = 999986341201 - 561 = 999986340640 we have:
999986340640 + 46043689999 = 1046030030639;
1046030030639 + 9360300306401 = 10406330337040;
10406330337040 + 4073303360401 = 14479633697441;

: for n = 999986341201 - 1729 = 999986339472 we have:
999986339472 + 274933689999 = 1274920029471;
1274920029471 + 1749200294721 = 3024120324192;
3024120324192 + 2914230214203 = 5938350538395.

Note that frustrating but interesting results are also obtained: for instance, starting with the difference 999863018281 - 1729, is obtained, after seven iterations, the number 229408797803922, an "almost palindrome" having just two figures unfit! Or, starting with 999828475651 - 1729, is obtained, after one iteration, the number 1229203302921, again an "almost palindrome" having a digit too much and, after three iterations, the number 4933572653394, again!