The Simplest Elementary Mathematics Proving Method of Fermat's Last Theorem

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Abstract: In this paper the author gives a simplest elementary mathematics method to solve the famous ***Fermat's Last Theorem*** (FLT), in which let this equation become a one unknown number equation, in order to solve this equation the author invented a method called “Order reducing method for equations” where the second order root compares to one order root and with some necessary techniques the author successfully proved ***Fermat's Last Theorem***.

1. Some Relevant Theorems

There are some theorems for proving or need to be known. ***All symbols in this paper represent positive integers unless they are stated to be not.***

Theorem 1.1. In the equation of

$$\begin{cases} x^n + y^n = z^n \\ \gcd(x,y,z) = 1 \\ n > 2 \end{cases} \tag{1-1}$$

x, y, z meet

$x \neq y$;

$x + y > z$;

if

$x > y$

then

$z > x > y$.

Proof: Let

$x = y$,

we have

$$2x^n = z^n$$

and

$$\sqrt[n]{2}x = z$$

where $\sqrt[n]{2}$ is not an integer and x, z are all positive integers, so $x \neq y$. Since

$$(x+y)^n = x^n + C_n^1 x^{n-1} y + ... + C_n^{n-1} x y^{n-1} + y^n > z^n,$$

so we get

$x + y > z.$

Since

$$x^n + y^n = z^n ,$$

so we have

$$z^n > x^n, z^n > y^n$$

and get

$$z > x > y$$

when

$$x > y .$$

Theorem 1.2. In the equation of (1-1), x, y, z meet

$$\gcd(x, y) = \gcd(y, z) = \gcd(x, z) = 1.$$

Proof: Since $x^n + y^n = z^n$, if $\gcd(x, y) > 1$ then we have $\left(x_1^n + y^n{}_1\right) \times [\gcd(x, y)]^n = z^n$ which causes $\gcd(x, y, z) > 1$ since the left side contains the factor of $[\gcd(x, y)]^n$ then the right side must also contains this factor but contradicts against (1-1) in which $\gcd(x, y, z) = 1$, so we have $\gcd(x, y) = 1.$ Using the same way we have $\gcd(x, z) = \gcd(y, z) = 1$.

Theorem 1.3. Function $f(x) = A^X$ and $g(x) = A^X + B^X$ are all monotonically increasing "Convex functions", where A, B are all positive real numbers and X is a real number.

Proof: Since monotonically increasing "Convex functions" meets

$$f'(x) = \frac{df(x)}{dx} > 0,$$

$$f''(x) = \frac{d^2 f(x)}{dx^2} > 0,$$

for $f(x) = A^X$ and $g(x) = A^X + B^X$, we have

$$f'(x) = A^X \ln A > 0,$$
$$f''(x) = A^X \ln^2 A > 0,$$

$$g'(x) = A^X \ln A + B^X \ln B > 0,$$
$$g''(x) = A^X \ln^2 A + B^X \ln^2 B > 0,$$

so $f(x) = A^X$ and $g(x) = A^X + B^X$ are all monotonically increasing "Convex functions". This theorem means that functions $g(n) = x^n + y^n$, $f(n) = z^n$ are all monotonically increasing

“Convex functions” when n is a real number.

Theorem 1.4. In the equation of (1-1), x, y, z meet

$$x^{n-i} + y^{n-i} > z^{n-i},$$

$$x^{n+i} + y^{n+i} > z^{n+i},$$

where

$$n > i \geq 1.$$

Proof: From equation (1-1), since

$$x^n + y^n = z^n,$$

from **Theorem 1.1**, since $z > x > y$, we have

$$x^{n-i} + y^{n-i} > \left[\left(\frac{x}{z}\right)^i x^{n-i} + \left(\frac{y}{z}\right)^i y^{n-i} = z^{n-i} \right],$$

$$x^{n+i} + y^{n+i} < \left(z^i x^{n-i} + z^i y^{n-i} = z^{n+i} \right),$$

so we have

$$x^{n-i} + y^{n-i} > z^{n-i}.$$

$$x^{n+i} + y^{n+i} < z^{n+i}.$$

This theorem means given x, y, z if equation (1-1) has one positive integer solution then this solution is the only one.

Theorem 1.5. In **Figure 1-1**, x, y, z of equation (1-1) meet

$$\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1.$$

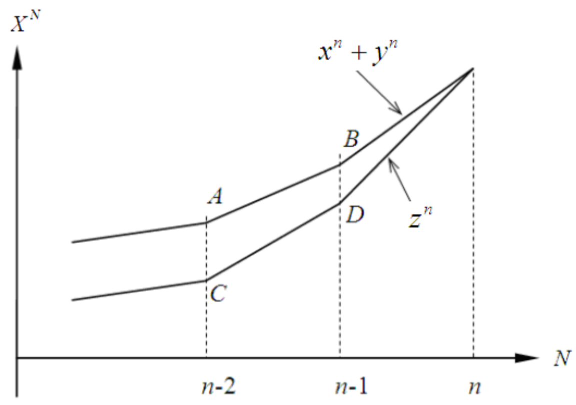

Figure 1-1 *Graph for* $x^n + y^n = z^n$

Proof: Obviously the meaning of $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} \le 1$ is the slope of AB is not greater than that of CD and if $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} = 1$ then the slope of AB equals to that of CD.

It is necessary to point out that there is a positive real number R that meets equation

$$\frac{dx^N}{dN}+\frac{dy^N}{dN}=\frac{dz^N}{dN},$$

where

$$x^R \ln x + y^R \ln y = z^R \ln z,$$

Obviously the "Slope" of $x^N + y^N$ equals to that of z^N when $N = R$. There are three cases for R in **Figure 1-1** when $R \le n-2, n-2 < R \le n-1$ and $R > n-1$. If $R \le n-2$ then it is very clear that $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} < 1$; If $n-2 < R \le n-1$ then $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} \le 1$ is possible and $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$ is also possible; If $R > n-1$ then $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$.

When $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$, there are three cases have to be considered. The first case (**Case I**) is there is a positive real number $0 < r < 1$ for $n-r$ between $n-1$ and n whose slope equals to that of AB which means

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\frac{z^{n-r}-z^{n-1}}{1-r}=\frac{\left(z^{1-r}-1\right)z^{n-1}}{1-r}$$

that can be explained by **Figure 1-2** where $AB // DF$.

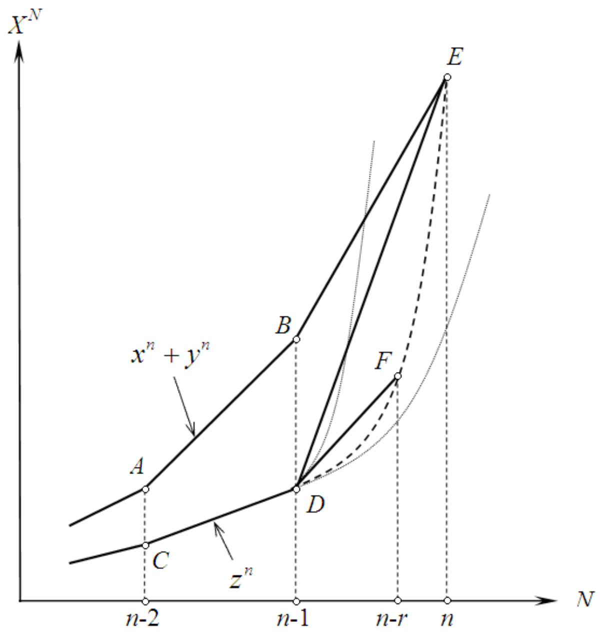

Figure 1-2 *Graph of* $x^n + y^n = z^n$ *when* $\dfrac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$

and point F *is between* n-1 *and* n *for* ***Case I***

The second case (**Case II**) is there is a positive real number $0 < r < 1$ for $n-r$ between $n-1$ and $n-2$ whose slope equals to that of AB which means

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{z^{n-1} - z^{n-r-1}}{r} = \frac{\left(1 - z^{-r}\right)z^{n-1}}{r},$$

that can be explained by **Figure 1-3** where $AB \,//\, DF \,//\, CD'$.

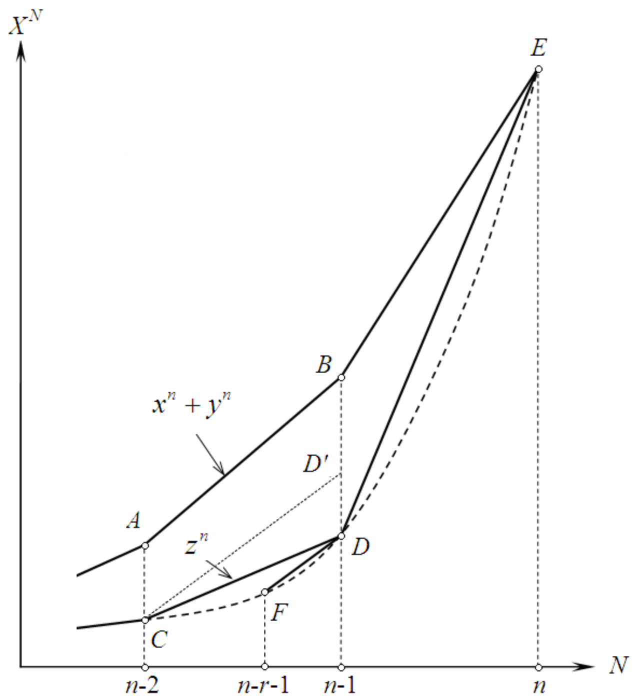

Figure 1-3 *Graph of* $x^n + y^n = z^n$ *when* $\dfrac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$

and point F *is between* n-2 *and* n-1 *for* ***Case II***

The third case (**Case III**) is there is a tangent line of curve z^n at *D* that is $D'DF$ whose slope equals to that of AB which means

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\frac{dz^N}{dN}\Big|_{N=n-1}$$

that can be explained by **Figure 1-4** where $AB\,//\,D'DF$.

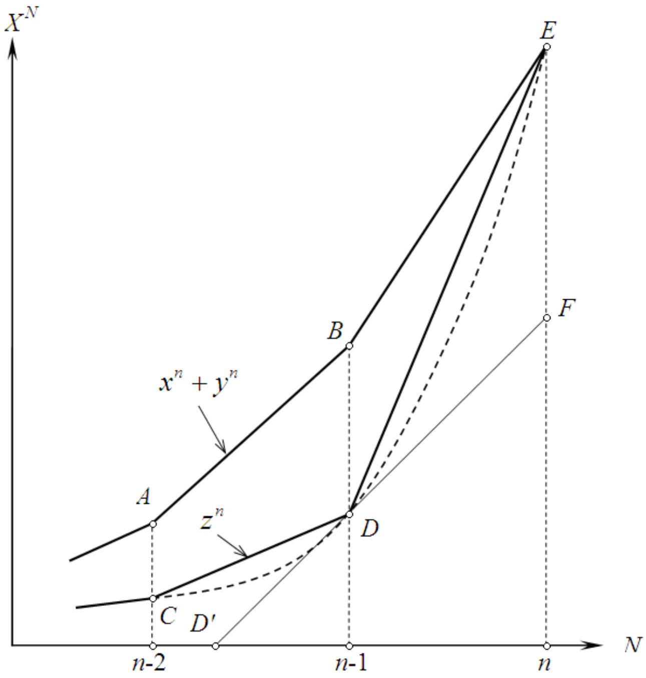

Figure 1-4 *Graph of* $x^n + y^n = z^n$ *when* $\dfrac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$

and $D'DF$ *is a tangent line of curve* z^n *for* ***Case III***

Case I : In **Figure 1-2** we have

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\left(\frac{z^{1-r}-1}{1-r}\right)z^{n-1},$$

and

$$x^{n-1}+y^{n-1}-z^{n-1}-x^{n-2}-y^{n-2}=\left(\frac{z^{1-r}-1}{1-r}\right)z^{n-1}-z^{n-1}=\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{n-1}. \qquad (1\text{-}2)$$

If we treat r as constant then $f(z)=\dfrac{z^{1-r}+r-2}{1-r}$ is a "Monotonically increasing function"; if we treat z as constant then $f(r)=\dfrac{z^{1-r}+r-2}{1-r}$ is a "Monotonically decreasing function" that can be explained by **Figure 1-5**.

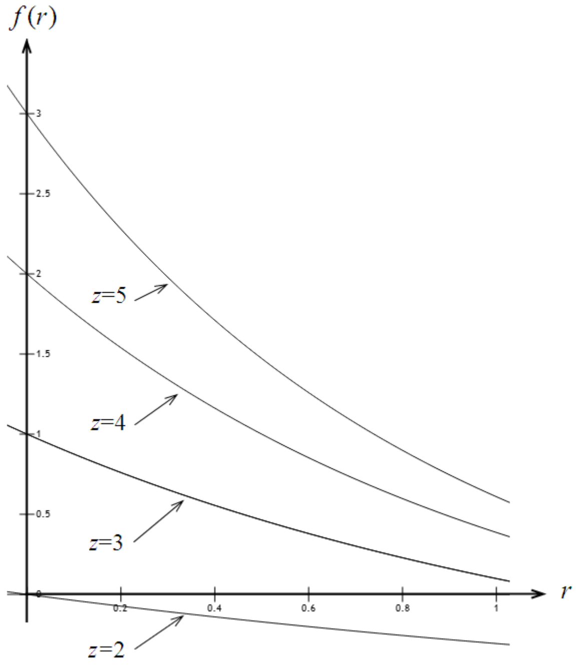

Figure 1-5 *Graph of* $f(r)=\dfrac{z^{1-r}+r-2}{1-r}$ *when* $z=2,3,4,5$

The reason why $f(r)=\dfrac{z^{1-r}+r-2}{1-r}$ is a "Monotonically decreasing function" is because:

$$f'(r)=\frac{d\left(\dfrac{z^{1-r}+r-2}{1-r}\right)}{dr}=\frac{\left(-z^{1-r}\ln z+1\right)(1-r)+z^{1-r}+r-2}{(1-r)^2}$$

$$=\frac{-z^{1-r}\ln z(1-r)+z^{1-r}-1}{(1-r)^2}=\frac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2}.$$

For function

$$g(z)=\frac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2},$$

it is a "Monotonically decreasing function" since

$$g'(z)=\frac{d\left\{\dfrac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2}\right\}}{dz}=\frac{\dfrac{(r-1)}{z}z^{1-r}+(1-r)z^{-r}[(r-1)\ln z+1]}{(1-r)^2}$$

$$=-z^{-r}\ln z<0.$$

For function

$$g(r)=\frac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2},$$

we give the plot of it in **Figure 1-6**, in which it shows that $g(r)\neq 0, g(r)<0$ that is because

$$\lim_{r\to\infty}\left\{g(r)=\frac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2}\right\}=\lim_{r\to\infty}\frac{[(r-1)\ln z+1]z}{(1-r)^2 z^r}$$

where

$$\lim_{r\to\infty}(1-r)^2 z^r=\infty$$

$$\lim_{r\to\infty}[(r-1)\ln z+1]z=\infty,$$

and

$$\lim_{r\to\infty}\frac{[(r-1)\ln z+1]z}{(1-r)^2 z^r}=\lim_{r\to\infty}\frac{\dfrac{d[(r-1)\ln z+1]z}{dr}}{\dfrac{(1-r)^2 z^r}{dr}}=\lim_{r\to\infty}\frac{z\ln z}{[(1-r)\ln z-2](1-r)z^r}=0,$$

which means $g(r)$ has no finite value to intersect axis r and $g(r)\neq 0, g(r)<0$, since when $0<r<1$ the value of $g(r)$ is less than 0 and $g(z)$ is a "Monotonically decreasing function" , so $f(r)$ is a "Monotonically decreasing function" when $0<r<1$(*we have to say because we can not solve "Exponent equation" where the "Exponent" is the unknown number, so the solutions have to be found in numerical way, which is just "Function plot" does*).

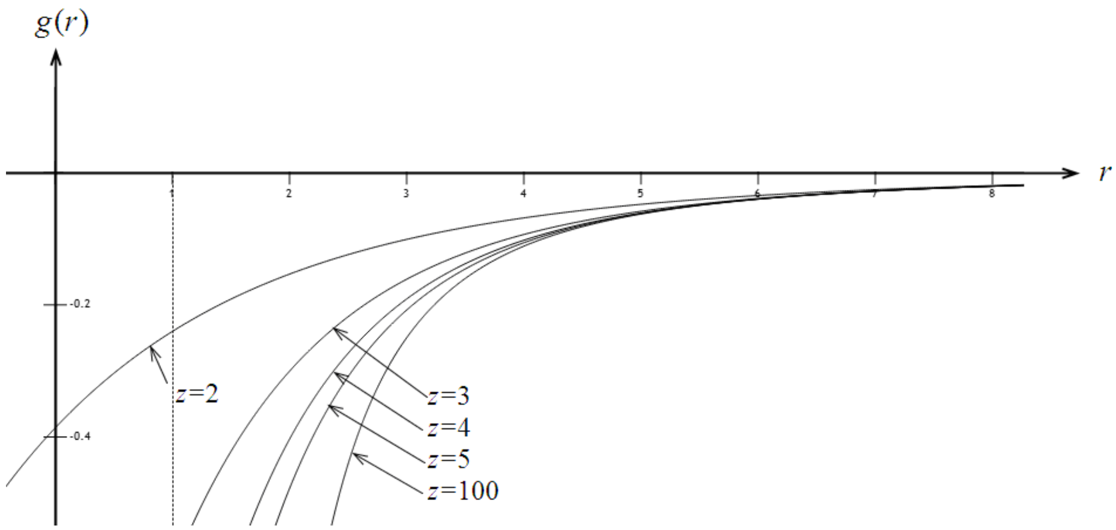

Figure 1-6 *Graph of* $g(r)=\frac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2}$ *when* $z=2,3,4,5,100$

From (1-2) we know if z (a positive real number) increases then the left side decreases and the right side also decreases. The minimum value for the right side is

$$\lim_{r\to1}\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{k-1}=\lim_{r\to1}\left[\frac{\frac{d\left(z^{1-r}+r-2\right)}{dr}}{\frac{d(1-r)}{dr}}\right]z^{k-1}=\lim_{r\to1}\left(\frac{-z^{1-r}\ln z+1}{-1}\right)z^{k-1},$$

$$=\lim_{r\to1}\left(z^{1-r}\ln z-1\right)z^{k-1}=(\ln z-1)z^{k-1}$$

since

$$\begin{cases}\lim_{r\to1}\left(z^{1-r}+r-2\right)=0\\ \lim_{r\to1}(1-r)=0\end{cases}.$$

From **Theorem 1.8** we know $z\geq4$, so we get

$$\left[\lim_{r\to1}\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{n-1}=(\ln z-1)z^{n-1}\right]>(\ln4-1)\times4^2>9.$$

From (1-2) we have

$$\left(x^{n-1}+y^{n-1}-z^{n-1}\right)-\left(x^{n-2}+y^{n-2}-z^{n-2}\right)=\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{n-1}+z^{n-2},$$

where both sides plus z^{n-2}, in **Figure 1-2** we know

$$x^{n-1}+y^{n-1}-z^{n-1}=BD,$$

$$x^{n-2}+y^{n-2}-z^{n-2}=AC,$$

there must exist a situation in **Figure 1-2** when we increase z (a positive real number) that causes

$$BD\to AC, BD>AC, r<1,$$

so the left side is almost 0 but the right side is bigger than $9+z^{n-2}\geq(9+4=13)$, that is a contradiction which means there are no positive integer solutions of equation (1-1) at **Case I**.

Case II : In **Figure 1-3** we have

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\frac{\left(1-z^{-r}\right)z^{n-1}}{r}<z^{n-1}\ln z,$$

and

$$x^{n-1}+y^{n-1}-z^{n-1}-x^{n-2}-y^{n-2}=\left(\frac{1-z^{-r}}{r}\right)z^{n-1}-z^{n-1}$$
$$=\left(\frac{1-z^{-r}-r}{r}\right)z^{n-1}<z^{n-1}(\ln z-1). \qquad (1\text{-}3)$$

If we treat r as constant then $f(z)=\dfrac{1-z^{-r}-r}{r}$ is a "Monotonically increasing function"; if we treat z as constant then $f(r)=\dfrac{1-z^{-r}-r}{r}$ is a "Monotonically decreasing function" that can be explained by **Figure 1-7**.

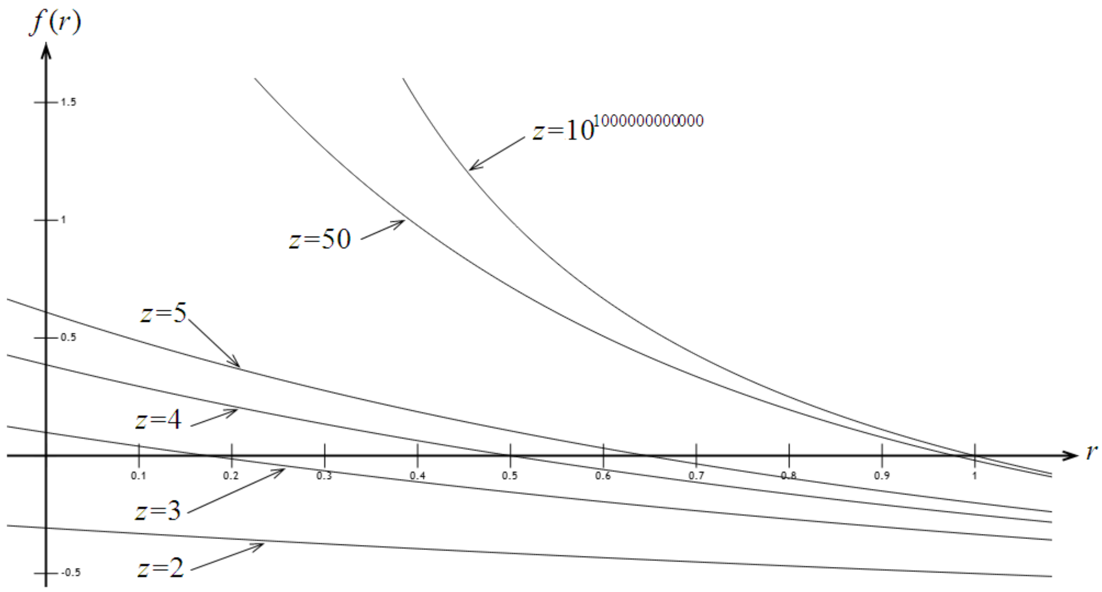

Figure 1-7 *Graph of* $f(r)=\dfrac{1-z^{-r}-r}{r}$ *when* $z=2,3,4,5,50,10^{1000000000000}$

The reason why $f(r)=\dfrac{1-z^{-r}-r}{r}$ is a "Monotonically decreasing function" is because:

$$f'(r)=\frac{d\left(\dfrac{1-z^{-r}-r}{r}\right)}{dr}=\frac{rz^{-r}\ln z-r-\left(1-z^{-r}-r\right)}{r^2}=\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2}.$$

For function

$$g(z)=\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2},$$

it is a "Monotonically decreasing function" since

$$g'(z)=\frac{d\left[\dfrac{\left(r\ln z+1\right)z^{-r}-1}{r^2}\right]}{dz}=\frac{\left[\dfrac{r}{z}-r\left(r\ln z+1\right)\right]z^{-r}}{r^2}<0,$$

in which from **Theorem 1.8** we know $z\geq 4$, so we have $\dfrac{r}{z}-r\left(r\ln z+1\right)<0$ where $\dfrac{r}{z}<r$

and $r^2 \ln z > 0$.

For function $g(r)=\dfrac{(r\ln z+1)z^{-r}-1}{r^2}$, we plot the graph of it in **Figure 1-8**, in which it shows that $g(r)\neq 0$ and $g(r)<0$ that is because:

$$\lim_{r\to\infty}\left[g(r)=\frac{(r\ln z+1)z^{-r}-1}{r^2}\right]=\lim_{r\to\infty}\frac{(r\ln z+1)}{r^2 z^r}$$

where

$$\lim_{r\to\infty}(r\ln z+1)=\infty$$
$$\lim_{r\to\infty} r^2 z^r=\infty,$$

and

$$\lim_{r\to\infty}\frac{(r\ln z+1)}{r^2 z^r}=\lim_{r\to\infty}\frac{\dfrac{d(r\ln z+1)}{dr}}{\dfrac{r^2 z^r}{dr}}=\lim_{r\to\infty}\frac{\ln z}{2rz^r+r^2 z^r\ln z}=0$$

which means $g(r)$ has no finite value to intersect axis r and $g(r)\neq 0, g(r)<0$, since when $0<r<1$ the value of $g(r)$ is less than 0 and $g(z)$ is a “Monotonically decreasing function”, so $f(r)$ is a “Monotonically decreasing function” when $0<r<1$.

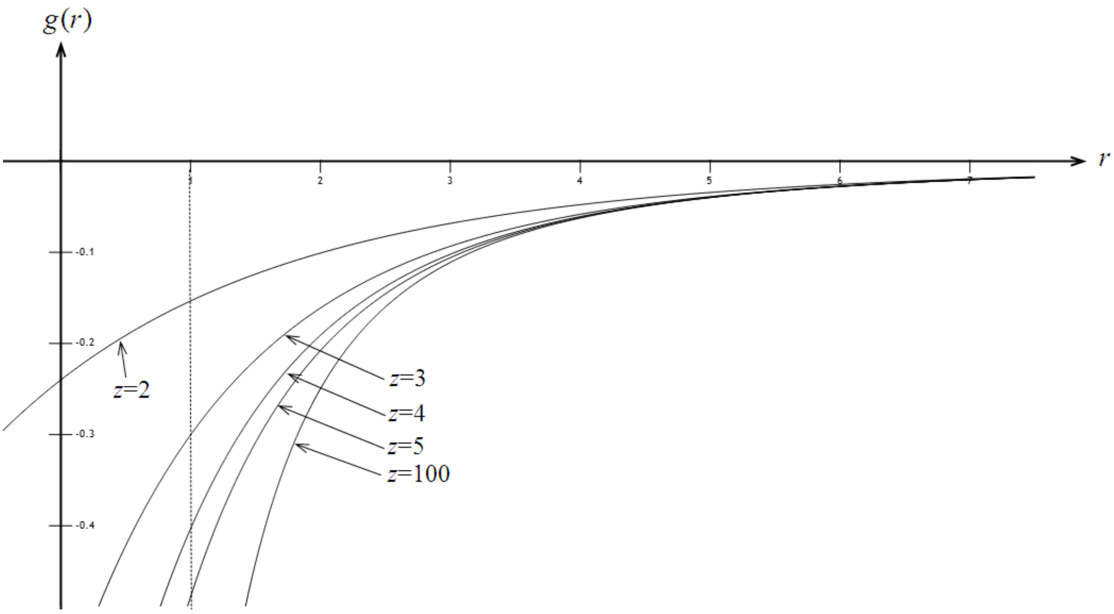

Figure 1-8 *Graph of* $g(r)=\dfrac{(r\ln z+1)z^{-r}-1}{r^2}$ *when* $z=2,3,4,5,100$

From **Figure 1-3** we know if z (a positive real number) increases then r also increases. From (1-3) we have

$$\left(x^{n-1}+y^{n-1}-z^{n-1}\right)-\left(x^{n-2}+y^{n-2}-z^{n-2}\right)=\left(\frac{1-z^{-r}-r}{r}\right)z^{n-1}+z^{n-2},$$

where both sides plus z^{n-2}, in **Figure 1-3** we know

$$x^{n-1}+y^{n-1}-z^{n-1}=BD,$$

$$x^{n-2}+y^{n-2}-z^{n-2}=AC,$$

there must exist a situation when we increase z (a positive real number) that causes

$$BD\to AC, BD>AC, r\to 1, r<1,$$

so the left side is

$$\left(x^{n-1}+y^{n-1}-z^{n-1}\right)-\left(x^{n-2}+y^{n-2}-z^{n-2}\right)=0_{+}>0,$$

when $r=1$ the right side is

$$\left[\left(\frac{1-z^{-r}-r}{r}\right)z^{n-1}+z^{n-2}\right]=\left(-z^{n-1-r}+z^{n-2}\right)=0,$$

since $f(r)=\dfrac{1-z^{-r}-r}{r}$ is a "Monotonically decreasing function", so when $r<1$, the right side is greater than 0, we do not have contradiction as **Case I** does. But **Case II** is still impossible, since there are some ways to explain why it is impossible, and at last we will give a proof.

Explanation 1. In **Figure 1-3**. It is obvious that

$$\angle CDE=360^0-\arctan\left(\frac{z^n-z^{n-1}}{1}\right)-\arctan\left(\frac{1}{z^{n-1}-z^{n-2}}\right)-90^0,$$

$$\angle CDE<\angle ABE,$$

from **Theorem 1.9** we know if $z\le 100$ then there are no positive integer solutions for equation (1-1), when $n=3$ (*which is the worst case*) we have

$$\angle CDE=270^0-\arctan\left(\frac{z^n-z^{n-1}}{1}\right)-\arctan\left(\frac{1}{z^{n-1}-z^{n-2}}\right)$$
$$=270^0-\arctan\left(100^3-100^2\right)-\arctan\left(\frac{1}{100^2-100}\right)>179.99^0,$$

and

$$\angle ABE>\angle CDE>179.99^0,$$

which means $\angle ABE, \angle CDE\to 180^0$ with $z>100, n\ge 3$, so ABE, CDE are almost lines.

Explanation 2. For function

$$f(z)=\angle CDE=270^0-\arctan\left(\frac{z^n-z^{n-1}}{1}\right)-\arctan\left(\frac{1}{z^{n-1}-z^{n-2}}\right)$$
$$=\frac{3}{2}\pi-\arctan\left(\frac{z^n-z^{n-1}}{1}\right)-\arctan\left(\frac{1}{z^{n-1}-z^{n-2}}\right),$$

we give the function plot for it in **Figure 1-9**.

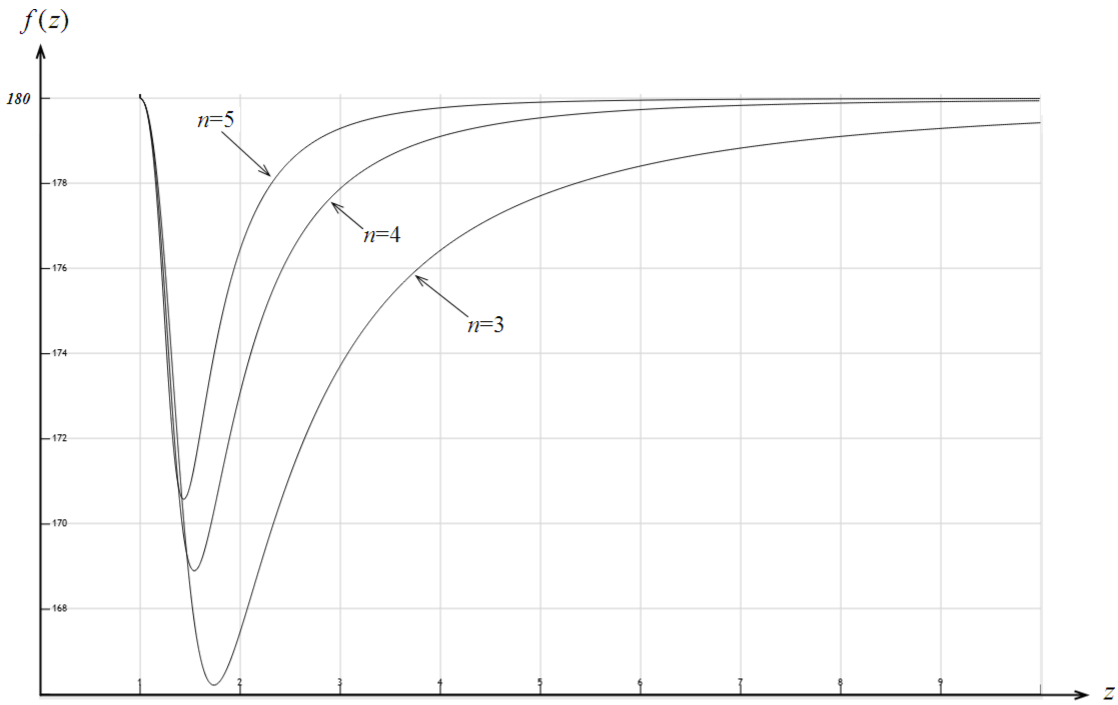

Figure 1-9 *Graph of* $f(z)=\angle CDE=270^0-\arctan\left(\frac{z^n-z^{n-1}}{1}\right)-\arctan\left(\frac{1}{z^{n-1}-z^{n-2}}\right)$

where we take $\pi=3.1415926535897932$

Obviously $f(z)=\angle CDE$ is a "Monotonically increasing function" when $z\geq 3$, and with the increasing of z the value of $f(z)=\angle CDE$ is close to 180^0.

It is very clear that $\angle ABE-\angle CDE$ is decreasing with the increasing of z, since

$$(\angle ABE-\angle CDE=\angle D'CD+\angle BED)<180^0-\angle CDE$$

where $\angle CDE$ is increasing. When $n=3$ since $\angle CDE>179.99^0$, so we have

$$(\angle D'CD+\angle BED)<180^0-\angle CDE<180^0-179.99^0<0.01^0,$$

which means

$$\angle BED, \angle D'CD < 0.01^0,$$

and when z or n is big enough, we have

$$\angle ABE - \angle CDE = (\angle BED + \angle D'CD) \to 0,$$

which means $BD < AC$.

Explanation 3. In **Figure 1-3** we have

$$\begin{aligned}
&\angle ABE \\
&= \frac{3}{2}\pi - \arctan\left(\frac{x^n + y^n - x^{n-1} - y^{n-1}}{1}\right) - \arctan\left(\frac{1}{x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2}}\right) \\
&= \frac{3}{2}\pi - \arctan\left(\frac{(x-1)x^{n-1} + (y-1)y^{n-1}}{1}\right) - \arctan\left(\frac{1}{(x-1)x^{n-2} + (y-1)y^{n-2}}\right),
\end{aligned}$$

from **Theorem 1.9** we know $x >> 1$, so we have

$$\begin{aligned}
&(x-1)x^{n-1} \approx x^n >> 1, \\
&(x-1)x^{n-2} \approx x^{n-1} >> 1,
\end{aligned}$$

and

$$\angle ABE \approx \frac{3}{2}\pi - \arctan\left(\frac{x^n + (y-1)y^{n-1}}{1}\right) - \arctan\left(\frac{1}{x^{n-1} + (y-1)y^{n-2}}\right)$$

$$\approx \left[\frac{3}{2}\pi - \arctan\left(\frac{z^n - y^n + (y-1)y^{n-1}}{1}\right) = \frac{3}{2}\pi - \arctan\left(\frac{z^n - y^{n-1}}{1}\right)\right],$$

since $\angle ABE > \angle CDE$, so we get

$$\frac{3}{2}\pi - \arctan\left(\frac{z^n - y^{n-1}}{1}\right) > \frac{3}{2}\pi - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right),$$

since $z > x >> 1$, so we have

$$\arctan\left(\frac{z^n - y^{n-1}}{1}\right) < \arctan\left(\frac{z^n - z^{n-1}}{1}\right) \Rightarrow$$

$$(z^n - y^{n-1} < z^n - z^{n-1}) \Rightarrow$$

$$(y^{n-1} > z^{n-1}) \Rightarrow$$

$$y > z,$$

that is impossible. So we have the conclusion of there are no positive integer solutions of equation (1-1) at **Case II** when $x >> 1$, which is true in order to have positive integer solutions for equation (1-1) in which $x >> 1$ must be met.

The proof for **Case II** to have no positive integer solutions is to draw the function plot for functions

$$f(z) = \angle CDE = 270^0 - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right),$$

$$g(x) = \angle ABE$$
$$= 270^0 - \arctan\left(\frac{(x-1)x^{n-1} + (y-1)y^{n-1}}{1}\right) - \arctan\left(\frac{1}{(x-1)x^{n-2} + (y-1)y^{n-2}}\right),$$

$$h(x) = \angle ABE - \angle CDE = g(x) - f(z).$$

For $h(x) = \angle ABE - \angle CDE = g(x) - f(z)$, we "Imagine" its plot as showed in **Figure 1-10**, where there are two "Intersections" with axial x, z, but one of them is at $x, z \to \infty$.

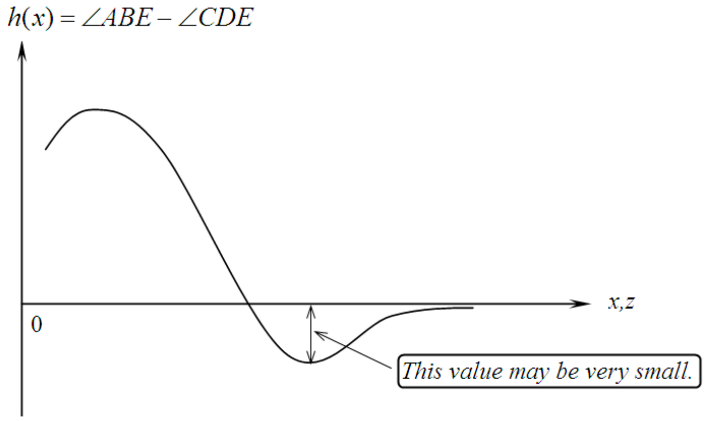

Figure 1-10 *Graph of* $h(x) = \angle ABE - \angle CDE = g(x) - f(z)$

For $g(x) = \angle ABE$, we take $y = 4$ and $y = x - 1$, the plot is showed in **Figure 1-11**.

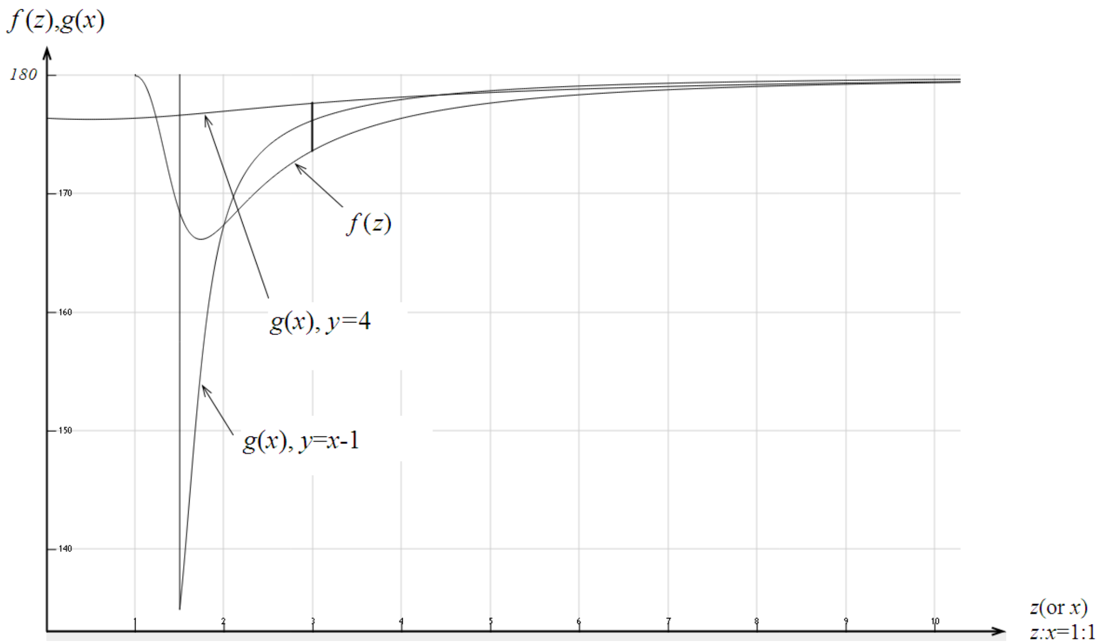

Figure 1-11 *Graph of* $f(z) = \angle CDE = 270^0 - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right)$,

$g(x) = \angle ABE = 270^0 - \arctan\left(\frac{(x-1)x^{n-1} + (y-1)y^{n-1}}{1}\right) - \arctan\left(\frac{1}{(x-1)x^{n-2} + (y-1)y^{n-2}}\right)$,

when $n = 3$

In **Figure 1-11** if $x, z \geq 3$ when $x : z = 1:1$, the value of $\angle ABE - \angle CDE = g(x) - f(z)$ is "Monotonically decreasing", since $z < \sqrt[n]{2}x$, so the actual value is around 3. From **Figure 1-10**, we know there are two intersection of $f(z), g(x)$, but one of them is at $x, z \to \infty$, so the "First point" we find $g(x) - f(z) \to 0$ is the "First intersection", since when $x = 100$,

$$\angle ABE - \angle CDE = g(x) - f(z) \to 0,$$

so we treat $x = 100$ as the "First intersection", in this case we have $\frac{BD}{AC} < 1$, and with the increasing of x (*which means* $x > 100$), $\frac{BD}{AC} < 1$ will be more certain to be satisfied but it contradicts against $BD > AC$. From **Section 2** we will know in order to have positive integer solutions for equation (1-1), $\frac{BD}{AC}$ must satisfy $\frac{BD}{AC} > 40$, so this is a contradiction which means when $x > 100$ there are no positive integer solutions for equation (1-1) (*Or we can say in order to have positive integer solutions for equation (1-1), we have to increase* x, z, *that causes* $\frac{BD}{AC}$ *decrease and the contradiction is that* $\frac{BD}{AC}$ *must increase to have positive integer solutions for equation (1-1)*). Using the same way we can prove when $n > 3$, the value of x meets $x < 100$, so there are no positive integer solutions of equation (1-1) at **Case II**.

Case III : In **Figure 1-4** we have

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{dz^N}{dN}\Big|_{N=n-1} = z^{n-1} \ln z,$$

and

$$x^{n-1} + y^{n-1} - z^{n-1} = z^{n-1} \ln z - z^{n-1} + x^{n-2} + y^{n-2} = (\ln z - 1)z^{n-1} + x^{n-2} + y^{n-2},$$

that is impossible since for any positive integer solutions of equation (1-1) when z increases then the left side is becoming smaller but the right side is becoming bigger(*since from* ***Theorem 1.8*** *we know* $z \geq 4$, *so* $(\ln z - 1) > 0$) which is a contradiction, so there are no positive integer solutions of equation (1-1) at **Case III**.

So from **Case I**, **Case II** and **Case III** we have the conclusion of $\dfrac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}>1$ is impossible and $\dfrac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}\le 1$.

Theorem 1.6. There are no positive integer solutions for

$$1^n + y^n = z^n .$$

Proof: Since

$$1 = z^n - y^n = (z-y)\left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right)$$

where

$$\begin{cases} z - y = 1 \\ \left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right) = 1 \end{cases}$$

that causes z, y to be non positive integers, so there are no positive integer solutions for

$$1^n + y^n = z^n .$$

Theorem 1.7. There are no positive integer solutions for

$$2^n + y^n = z^n .$$

Proof: Since

$$2^n = z^n - y^n = (z-y)\left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right),$$

if

$$\begin{cases} z - y = 1 \\ z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} = 2^n \end{cases}$$

then taking the least value for $y = 2, z = 3$, we have

$$3^{n-1} + 2\times 3^{n-2} + \ldots + 2^{n-1} > 2^n$$

when $n > 2$ that is impossible. If

$$\begin{cases} z - y = 2^i \\ z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} = 2^j \\ i + j = n \\ i \ge 1 \end{cases}$$

then $z > 2$ and taking the least value of $y = 2, z = 3$, we get

$$3^{n-1}+2\times 3^{n-2}+...+2^{n-1}>2^{j}$$

with $n>2$ that is also impossible, so there are no positive integer solutions for

$$2^{n}+y^{n}=z^{n}.$$

Theorem 1.8. There are no positive integer solutions for equation (1-1) when $n\to\infty$ and x, y, z in equation (1-1) meet

$$\sqrt[n]{2}y<z<\sqrt[n]{2}x,$$
$$x>2,$$
$$y>1,$$
$$z>3.$$

Proof: Since $x^{n}+y^{n}=z^{n}$, let $x>y$, we get

$$\left(\frac{z}{x}\right)^{n}-\left(\frac{y}{x}\right)^{n}=1,$$

since

$$z>x>y,$$

so we have

$$z<\sqrt[n]{2}x,$$

and

$$\lim_{n\to\infty}\left(\frac{z}{x}\right)^{n}-\left(\frac{y}{x}\right)^{n}=\infty>1$$

which means there are no positive integer solutions for equation (1-1) when $n\to\infty$. And according to **Theorem 1.1, 1.6** we have $x>2, y>1, z>3$.

If $y^{n}\geq\frac{z^{n}}{2}$ then since $x>y$, so we have

$$x^{n}+y^{n}>z^{n},$$

that is impossible so we have

$$\sqrt[n]{2}y<z.$$

Theorem 1.9. There are no positive integer solutions for equation (1-1) when $x, y, z\leq 100$.

Proof: From **Theorem1.8**, we know $\sqrt[n]{2}y<z<\sqrt[n]{2}x$, so we have

$$y < \frac{100}{\sqrt[n]{2}} < x,$$

when $n = 3$, we have the smallest values for x, so we get

$$\left(y < \frac{100}{\sqrt[3]{2}} < x \right) \Rightarrow (y < 79 < x),$$

since from **Theorem 1.10** we know x or y is not a prime number. When $n = 4$ we have

$$\left(y < \frac{100}{\sqrt[4]{2}} < x \right) \Rightarrow (y \le 84 < x).$$

From **Theorem 1.10** we only consider the not prime numbers for x, y. There are below combinations of x, y, z when $x, y, z \le 100$:

$$(80 \sim 99)^n + (4 \sim 78)^n = (81 \sim 100)^n.$$

Here we take $7^n + 9^n = 10^n$ for example to explain how to prove. We plot the graph for this equation as showed in **Figure 1-10**.

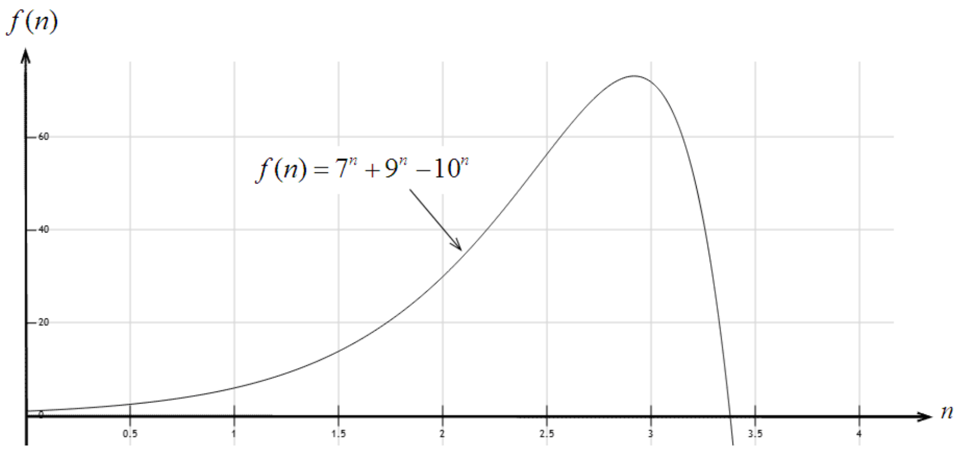

Figure 1-10 *Graph of* $f(n) = 7^n + 9^n - 10^n$

Obviously for equation $f(n) = 7^n + 9^n - 10^n$ in **Figure 1-10**, we have $3 < n < 4$ is not an integer so there are no positive integer solutions, using this method we have the conclusion of there are no positive integer solutions for equation (1-1) when $z \le 100$.

Using the method of which we prove **Theorem 1.6, 1.7** we can prove when $x, y \le 100$, there are no positive integer solutions for equation (1-1).

Theorem 1.10. There are no positive integer solutions for equation (1-1) when x or y is a prime number .

Proof: When x is a prime number, since

$$x^n = z^n - y^n = (z-y)\left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right),$$

so we have

$$\gcd(z-y, x) = x\ ,$$

which means

$$z - y \geq x\ ,$$

we have

$$x + y \leq z\ ,$$

that contradicts against **Theorem 1.1** in which $x + y > z$. So it is with y .

2. Proving Method

In equation (1-1), let

$$\begin{cases} a = x^{n-2} \\ b = y^{n-2} \\ c = z^{n-2} \end{cases},$$

we have

$$\begin{cases} ax^2 + by^2 = cz^2 \\ a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}y = c^{\frac{n-1}{n-2}}z \end{cases}. \qquad (2\text{-}1)$$

Since we reduce the order of equation so the method is called "Order reducing method for equations".

Let $x > y$ and

$$\begin{cases} y = x - f \\ z = x + e \end{cases}. \qquad (2\text{-}2)$$

From (2-1) and (2-2) we have

$$\begin{cases} ax^2 + b(x-f)^2 = c(x+e)^2 \\ a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}(x-f) = c^{\frac{n-1}{n-2}}(x+e) \end{cases}$$

and

$$\begin{cases} (a+b-c)x^2 - 2(bf+ce)x + \left(bf^2 - ce^2\right) = 0 \\ a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}(x-f) - c^{\frac{n-1}{n-2}}(x+e) = 0 \end{cases},$$

the roots are

$$x = \frac{(bf + ce) \pm \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}, \tag{2-3}$$

and

$$x = \frac{c^{\frac{n-1}{n-2}} e + b^{\frac{n-1}{n-2}} f}{a^{\frac{n-1}{n-2}} + b^{\frac{n-1}{n-2}} - c^{\frac{n-1}{n-2}}} = \frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}}. \tag{2-4}$$

There are two cases for bf^2, ce^2 when $bf^2 \geq ce^2$ and $bf^2 < ce^2$.

Case A: If $bf^2 \geq ce^2$, from (2-3) when

$$x = \frac{(bf + ce) + \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}},$$

From **Theorem 1.4** we know $a + b - c = x^{n-2} + y^{n-2} - z^{n-2} > 0$, so we have

$$x \leq \frac{2(bf + ce)}{x^{n-2} + y^{n-2} - z^{n-2}},$$

and also from **Theorem 1.4** we have $x^{n-1} + y^{n-1} - z^{n-1} > 0$, compare to (2-4) we get

$$\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \leq \frac{2(bf + ce)}{x^{n-2} + y^{n-2} - z^{n-2}}.$$

From **Theorem 1.5** we know $\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1$, so we have

$$bfy + cez \leq 2(bf + ce)$$

that is impossible since from **Theorem 1.8** we know $y \geq 2$ and $z > 3$.

When

$$x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}.$$

we have

$$x \leq \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}},$$

compare to (2-4) we get

$$\frac{bfy+cez}{x^{n-1}+y^{n-1}-z^{n-1}} \le \frac{bf+ce}{x^{n-2}+y^{n-2}-z^{n-2}}.$$

From **Theorem 1.5** we have

$$bfy+cez \le bf+ce$$

that is impossible since from **Theorem 1.8** we have already known $y \ge 2$ and $z > 3$.

Case B: If $bf^2 < ce^2$, from (2-3) when

$$x = \frac{(bf+ce)+\sqrt{(bf+ce)^2+(a+b-c)(ce^2-bf^2)}}{x^{n-2}+y^{n-2}-z^{n-2}},$$

we can prove $(bf+ce)^2 > (a+b-c)(ce^2-bf^2)$ since if not we have

$$(bf+ce)^2 \le (a+b-c)(ce^2-bf^2)$$

and

$$[(2b+a)-c]bf^2+2bfce+[2c-(a+b)]ce^2 \le 0$$

that is impossible since $a+b-c>0$ and $c>a, c>b, 2c-(a+b)>0.$ So we have

$$x < \frac{(bf+ce)(1+\sqrt{2})}{x^{n-2}+y^{n-2}-z^{n-2}}$$

compare to (2-4) we get

$$\frac{bfy+cez}{x^{n-1}+y^{n-1}-z^{n-1}} < \frac{(bf+ce)(1+\sqrt{2})}{x^{n-2}+y^{n-2}-z^{n-2}}.$$

From **Theorem 1.5** we have

$$bfy+cez < (bf+ce)(1+\sqrt{2}) < 2.5(bf+ce)$$

and

$$bf(x-f)+ce(x+e) < 2.5(bf+ce)$$

that leads to

$$x < \left[\frac{2.5(bf+ce)+bf^2-ce^2}{bf+ce} = 2.5-\frac{ce^2-bf^2}{bf+ce}\right] < 2.5$$

where possible values for x are 1, 2 but according to **Theorem 1.6**, **1.7** we know there are no positive integer solutions.

When

$$x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 + (a + b - c)(ce^2 - bf^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}$$

is not possible since $x \le 0$.

Obviously we have

$$bfy + cez < 2.5 \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} (bf + ce),$$

from **Theorem 1.9** we know $x, y, z \le 100$ there are no positive integer solutions for equation (1-1), so we have

$$\frac{BD}{AC} = \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 40,$$

which must be satisfied to have positive integer solutions for equation (1-1).

Now we have completely solved no positive integer solutions for equation (1-1) when $n > 2$ using “Order reducing method for equations”.

3. Conclusion

Through the above contents we can see clearly that the proving of *Fermat's Last Theorem* is just a problem of elementary mathematics. “Order reducing method for equations” that the author invented is a very effective method in the proving of *Fermat's Last Theorem* and the author’s technique in which let $y = x - f$ and $z = x + e$ is a very important step for solving.

Fermat's Last Theorem is a problem that has lasted for about 380 years. Proving methods are not important but the theorem’s correctness is very necessary because many useful inferences can be deduced that are obviously better than “conjectures”.

The author has been working on proving of *Fermat's Last Theorem* for quite some times (253 days) without any reference and many methods have been thought about, for example “Method of prime factorization” but not work. So the author has already known that there are no ways to solve except “Solving high order equations” which is also an important aspect in solving other mathematic problems.