A collection of attractors

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abstract
This note presents a collection of attractors

The Cubic Equation

$$f(z)=27z^3-27z^2-4=0 \quad , z\in\mathbb{C} \tag{1}$$

Roots

$$f(z)=0 \Rightarrow \begin{cases} z_1 = r\in\mathbb{R} \\ z_2 = w\in\mathbb{C} \\ z_3 = \overline{w}\in\mathbb{C} \end{cases} \tag{2}$$

Remark : $\overline{w}$, complex conjugate of w .

$$r=\frac{1}{3}+\frac{1}{3}\left(3+2\sqrt{2}\right)^{1/3}+\frac{1}{3}\left(3+2\sqrt{2}\right)^{-1/3} \tag{3}$$

$$w=\frac{1}{3}-\frac{1}{6}\left(3+2\sqrt{2}\right)^{1/3}-\frac{1}{6}\left(3+2\sqrt{2}\right)^{-1/3}+\frac{i\sqrt{3}}{6}\left(\left(3+2\sqrt{2}\right)^{1/3}-\left(3+2\sqrt{2}\right)^{-1/3}\right) \tag{4}$$

Some representations of r

$$r=\frac{1}{3}+\frac{1}{3}\left(\sqrt{2}+1\right)^{2/3}+\frac{1}{3}\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)^{1/3} \tag{5}$$

$$r=\sqrt[3]{\frac{4}{27}+\sqrt[3/2]{\frac{4}{27}+\sqrt[3/2]{\frac{4}{27}+\ldots}}} \tag{6}$$

$$r=\frac{1}{3}+\sqrt[3]{\frac{6}{27}+\sqrt[3]{\frac{6}{27}+\sqrt[3]{\frac{6}{27}+\ldots}}} \tag{7}$$

$$r=\frac{1}{3}+\frac{1}{3}\sqrt[3]{6+3\sqrt[3]{6+3\sqrt[3]{6+\ldots}}} \tag{8}$$

$$r=1+\frac{1}{3}\left\{\frac{2}{3}-\frac{1}{3}\left(\frac{2}{3}-\frac{1}{3}\left(\frac{2}{3}-\ldots\right)^3\right)^3\right\}^2 \tag{9}$$

$$r=\frac{1}{3}\left(\sinh\left(\frac{1}{3}\sinh^{-1}1\right)\right)^{-1} \tag{10}$$

Pi formula

$$\pi=3+\sqrt{3}\int_1^r\left(\sqrt{\frac{1}{x}-\frac{2}{x}\cos\left(\frac{2\pi+\cos^{-1}g(x)}{3}\right)}-\sqrt{\frac{1}{x}-\frac{2}{x}\cos\left(\frac{4\pi+\cos^{-1}g(x)}{3}\right)}\right)dx \tag{11}$$

$$g(x)=\frac{27x^3-27x^2-2}{2} \tag{12}$$

$$\pi=3+\sqrt{3}\int_1^r\left(\sqrt{1+\cos\left(\frac{1}{3}\cos^{-1}g(x)\right)+\sqrt{3}\sin\left(\frac{1}{3}\cos^{-1}g(x)\right)}-\sqrt{1+\cos\left(\frac{1}{3}\cos^{-1}g(x)\right)-\sqrt{3}\sin\left(\frac{1}{3}\cos^{-1}g(x)\right)}\right)\frac{dx}{\sqrt{x}} \tag{13}$$

Newton fractals for $f(z)$

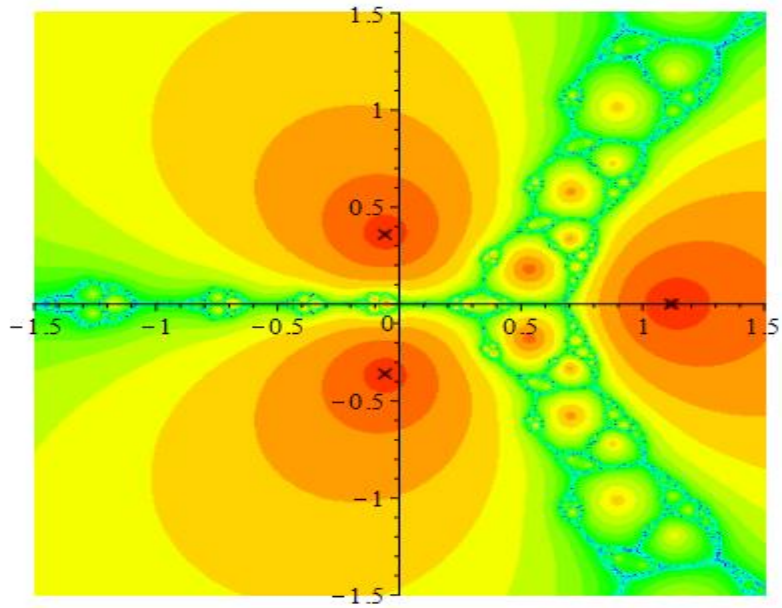


Fig. 1

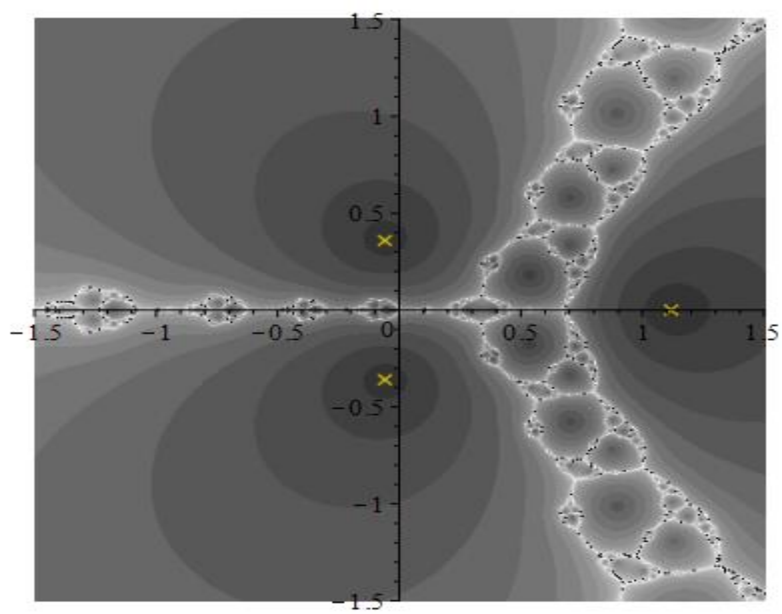


Fig. 2

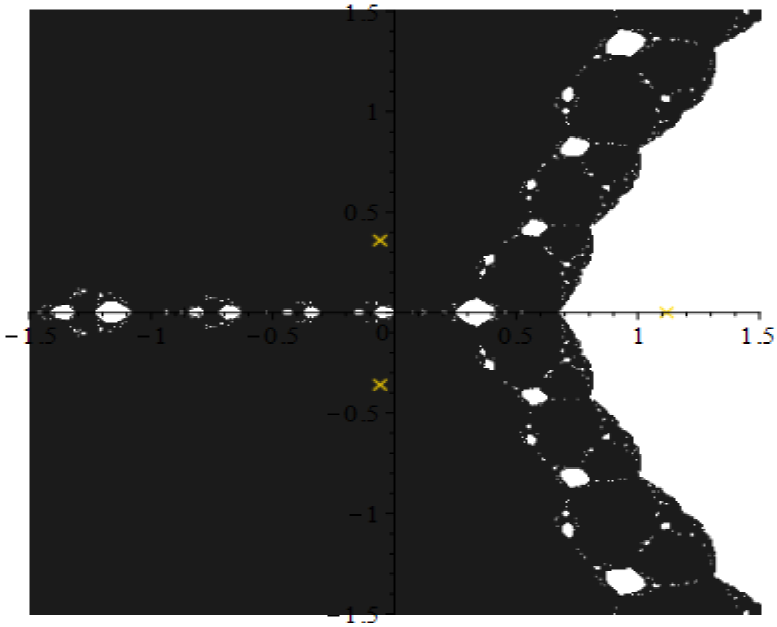


Fig. 3

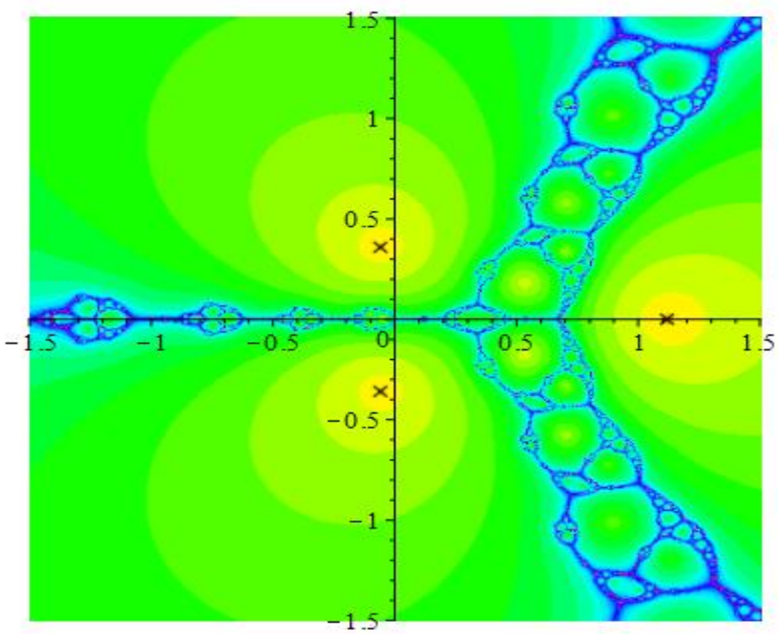


Fig. 4

$(r, w, \bar{w})$ - Attractors

Empirical functions:

$$y_r(t) = R(a,b,c,d,r,t),\ y_w(t) = W(a,b,c,d,w,t),\ y_{\bar{w}}(t) = W(a,b,c,d,\bar{w},t)$$

$$y_r(t) \to r, t \to \infty;\ y_w(t) \to w, t \to \infty;\ y_{\bar{w}}(t) \to \bar{w}, t \to \infty$$

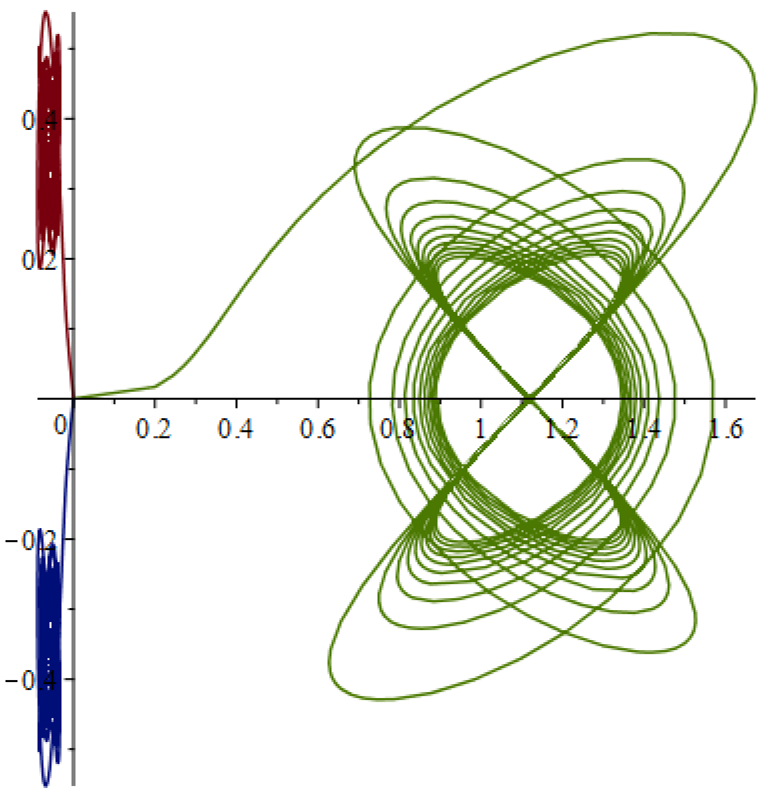


Fig. 5 : • $y_r(t) = R(3,3,2,3,r,t)$, • $y_w(t) = W(3,3,5,3,w,t)$, • $y_{\bar{w}}(t) = W(3,3,5,3,\bar{w},t)$

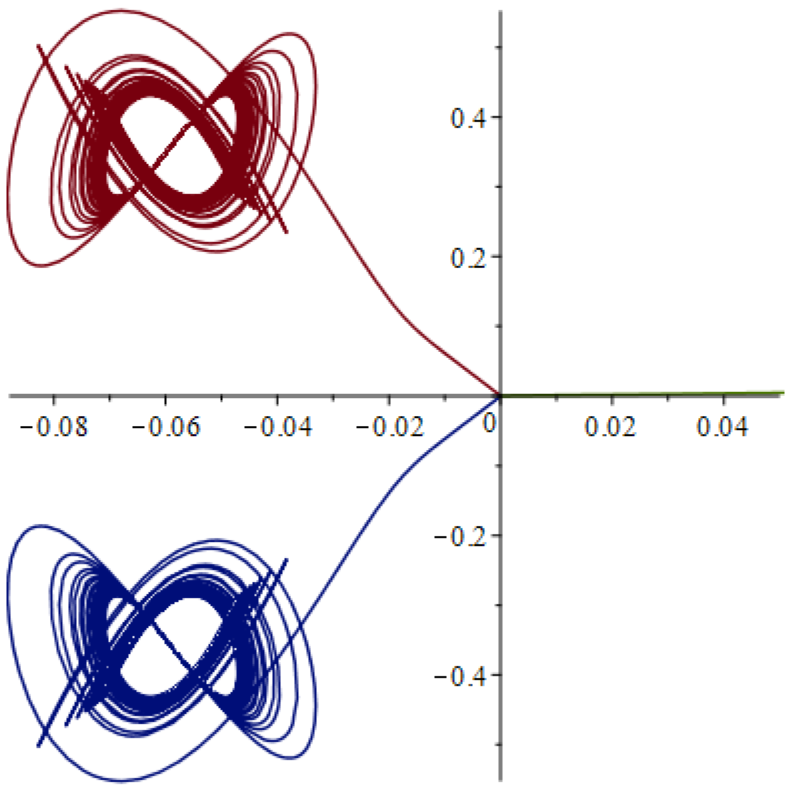


Fig. 6 : • $y_w(t) = W(3,3,5,3,w,t)$, • $y_{\bar{w}}(t) = W(3,3,5,3,\bar{w},t)$

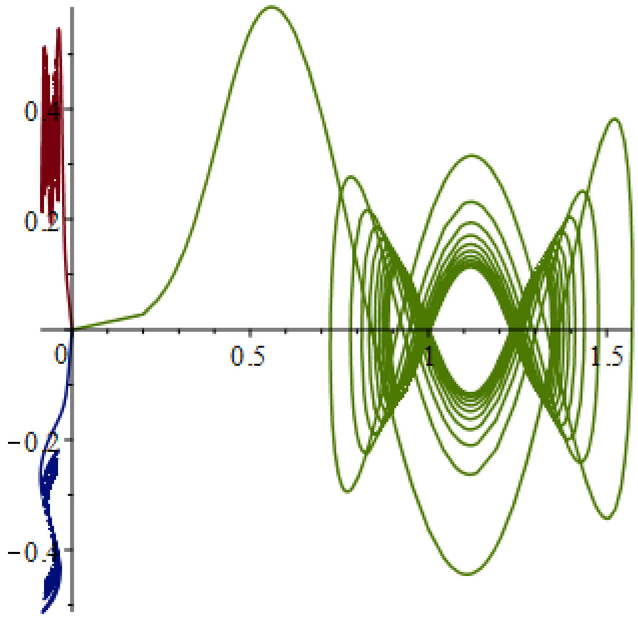


Fig. 7 : • $y_r(t) = R(1,3,3,2,r,t)$, • $y_w(t) = W(1,3,4,3,w,t)$, • $y_{\bar{w}}(t) = W(3,3,1,4,\bar{w},t)$

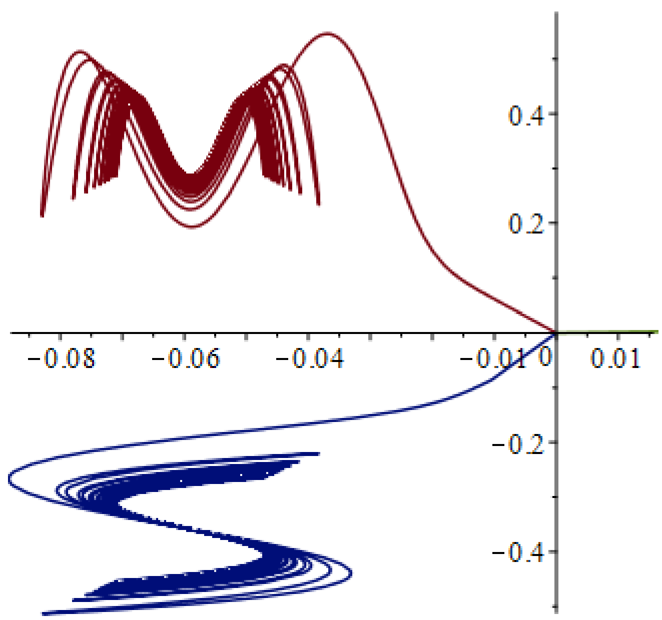


Fig. 8 : • $y_w(t) = W(1,3,4,3,w,t)$, • $y_{\bar{w}}(t) = W(3,3,1,4,\bar{w},t)$

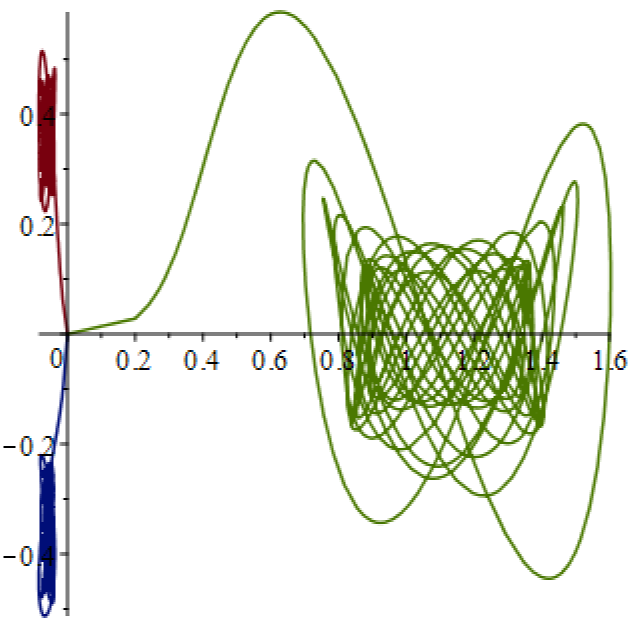


Fig. 9 :

• $y_r(t) = R(\sqrt{2},3,3,2,r,t)$, • $y_w(t) = W(1,4,\sqrt{2},3,w,t)$, • $y_{\bar{w}}(t) = W(\sqrt{2},3,1,4,\bar{w},t)$

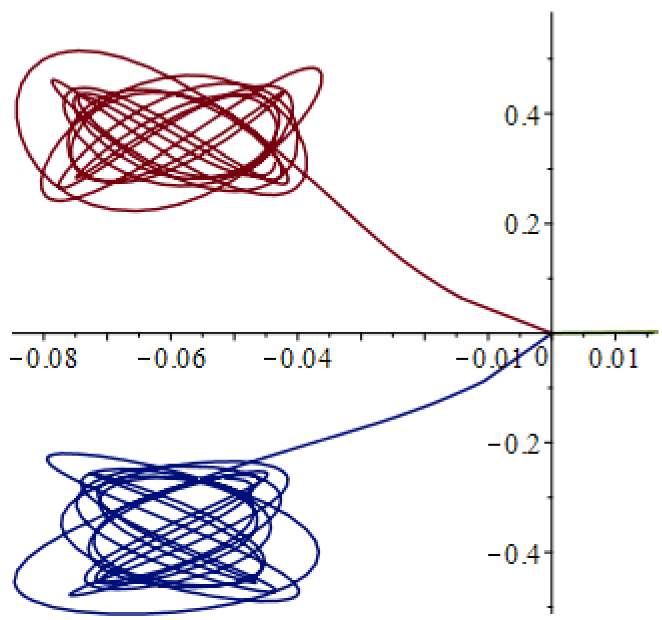


Fig. 10 : • $y_w(t) = W(1,4,\sqrt{2},3,w,t)$, • $y_{\bar{w}}(t) = W(\sqrt{2},3,1,4,\bar{w},t)$

A Collection of Attractors

Martin – Attractors (1986) , also known as Hopalongs.

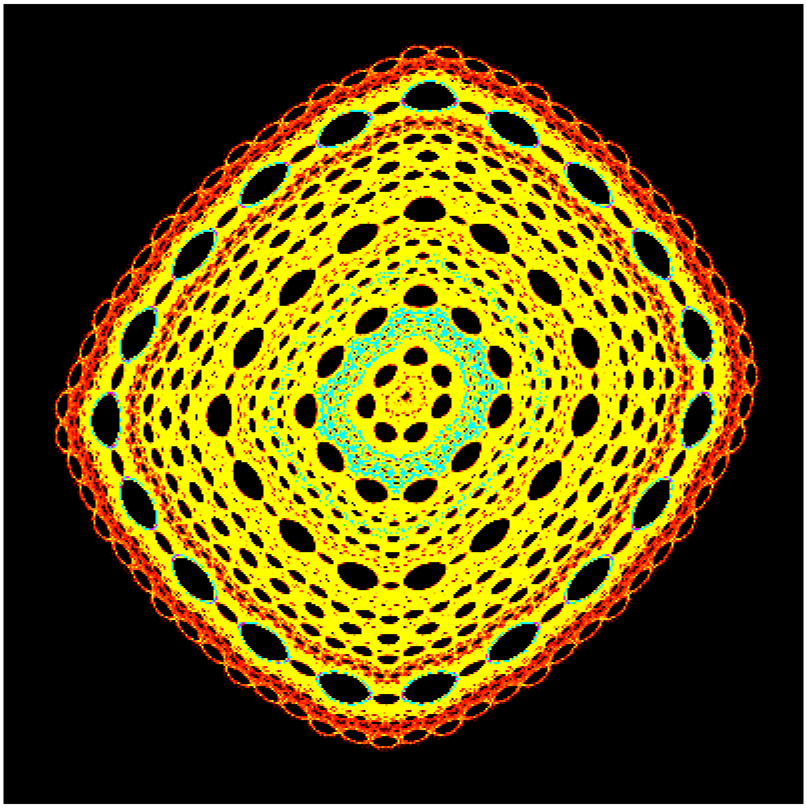


Fig. 11 (Maple example)

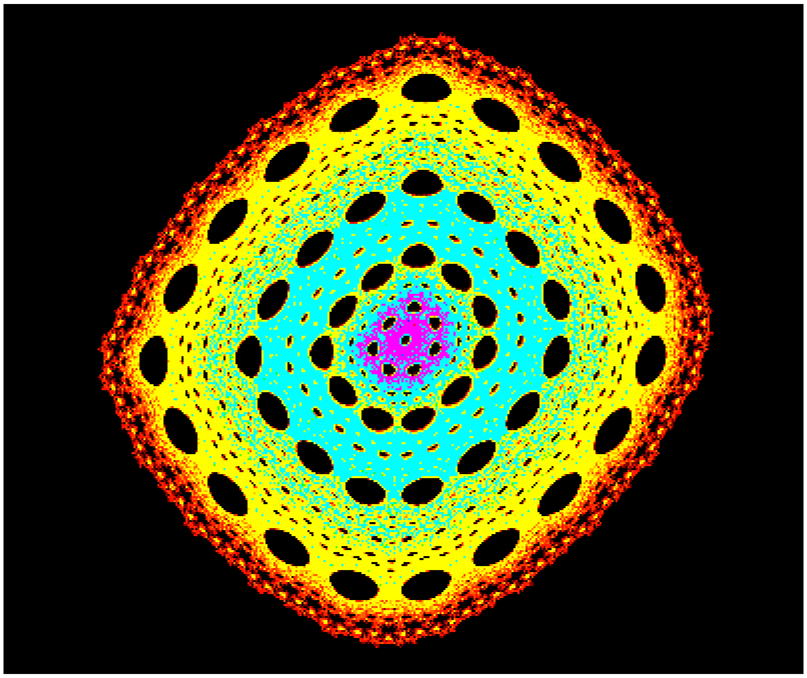
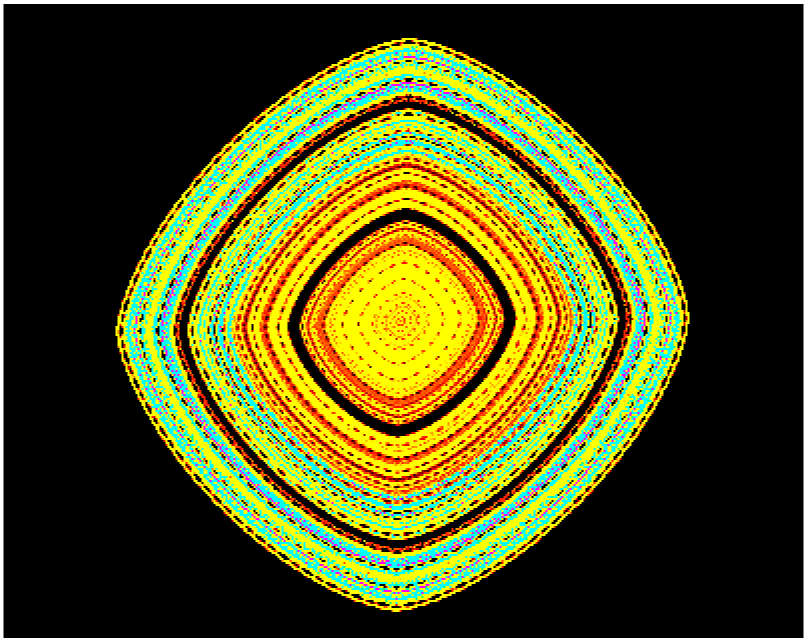


Fig. 12



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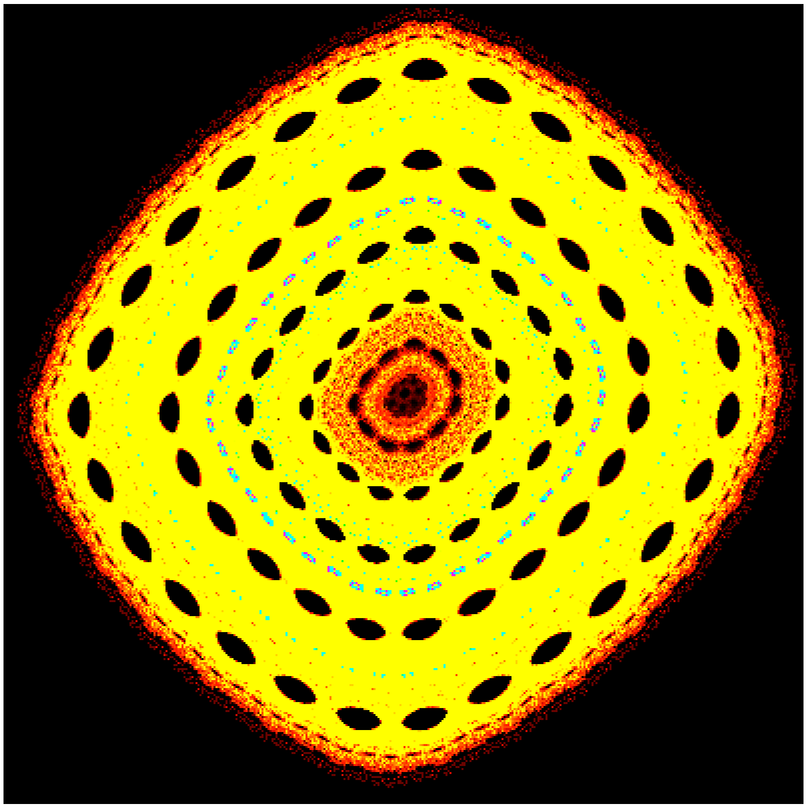
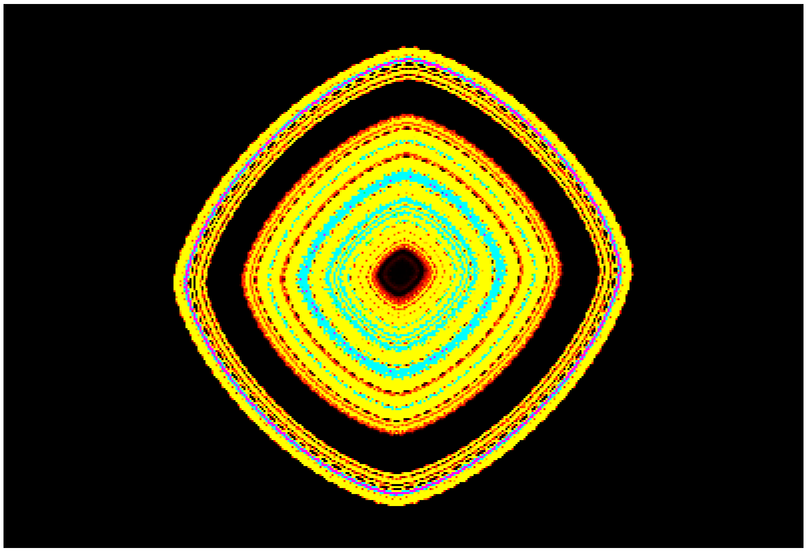


Fig. 14



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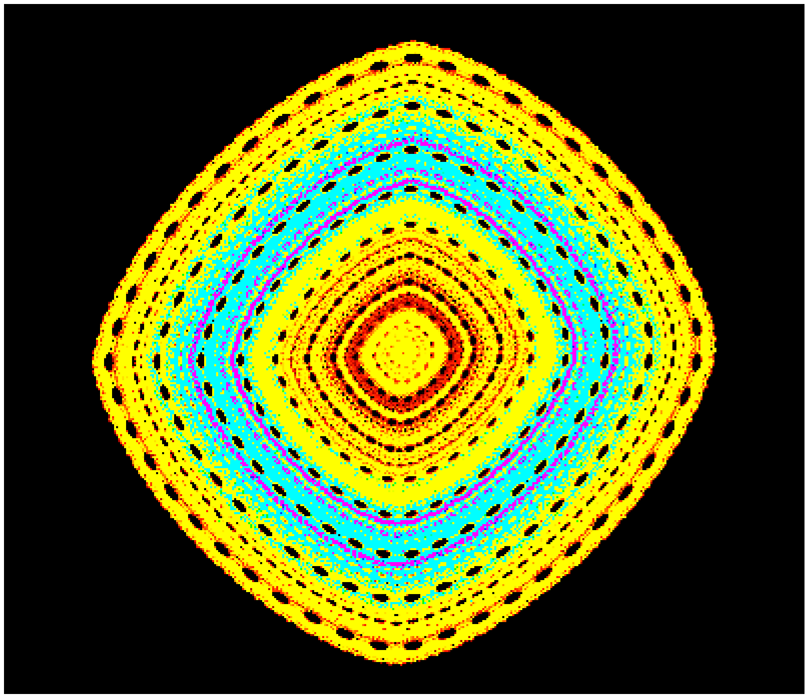

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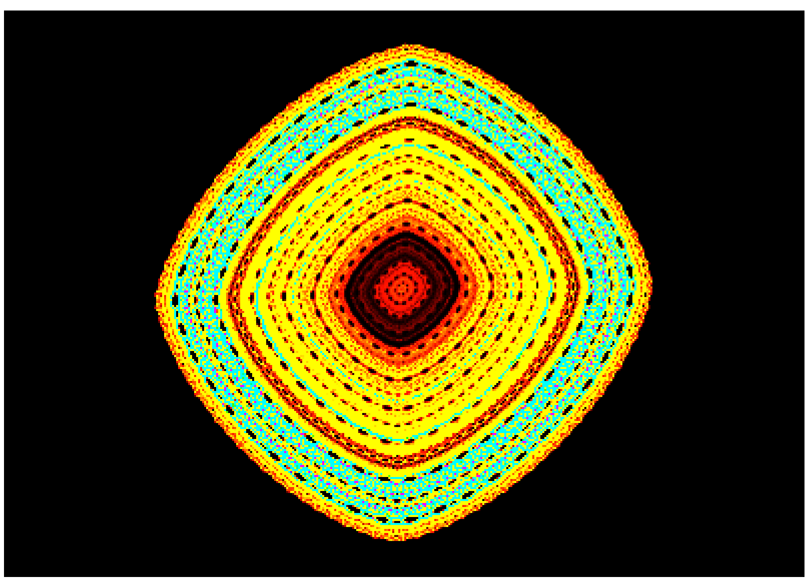

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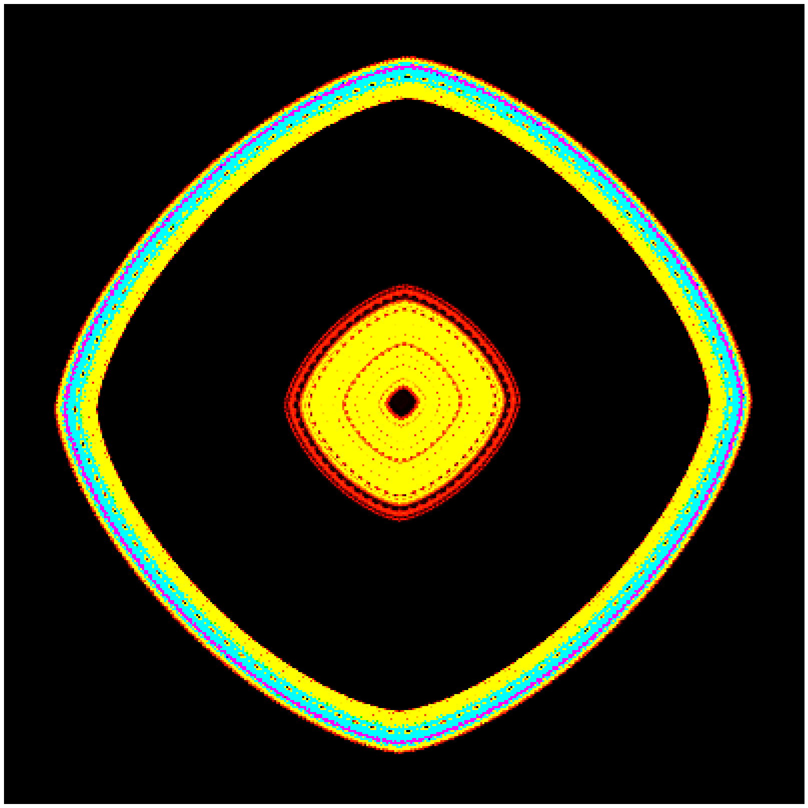

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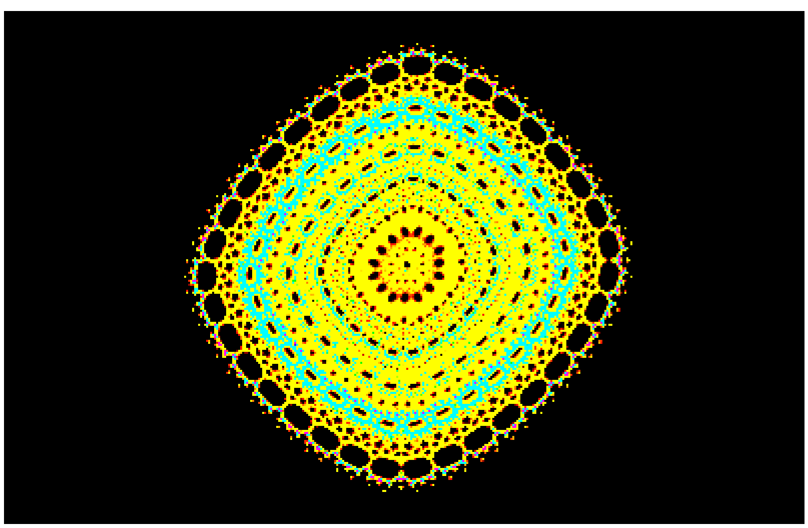

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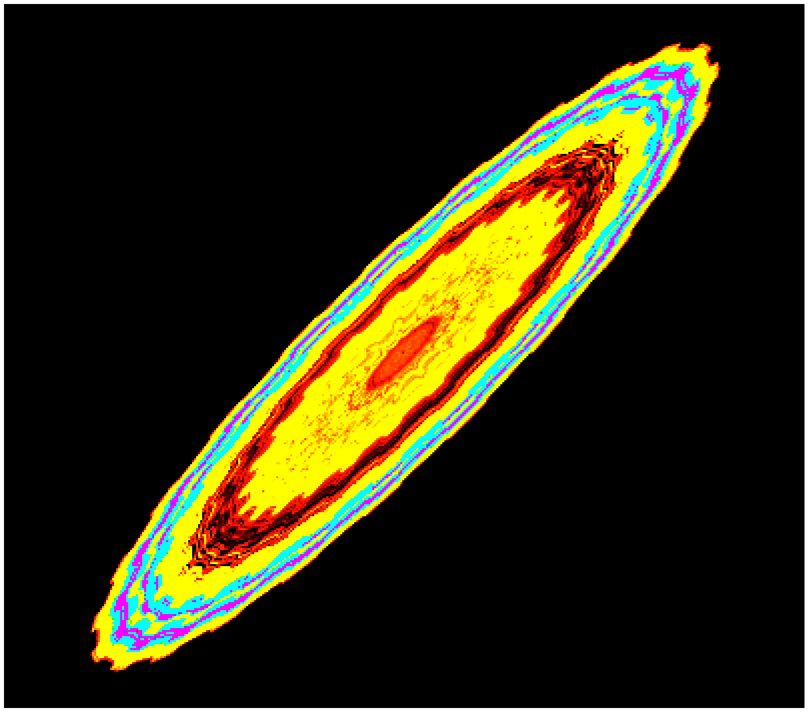

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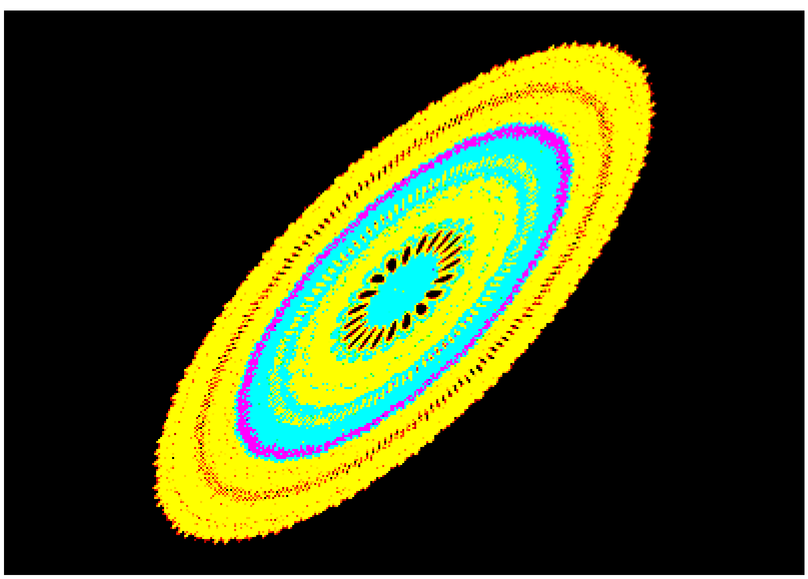

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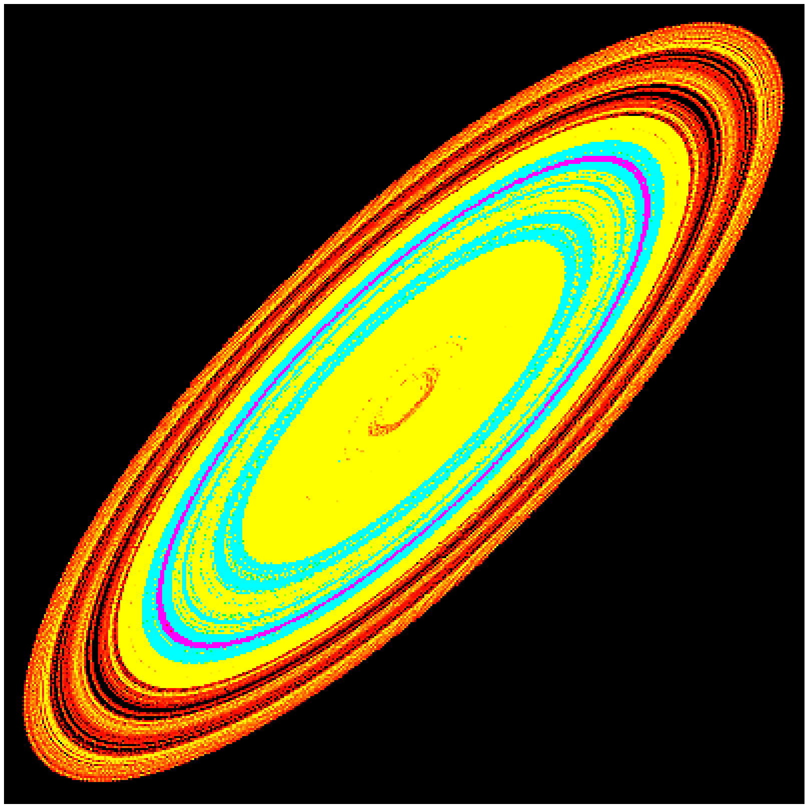

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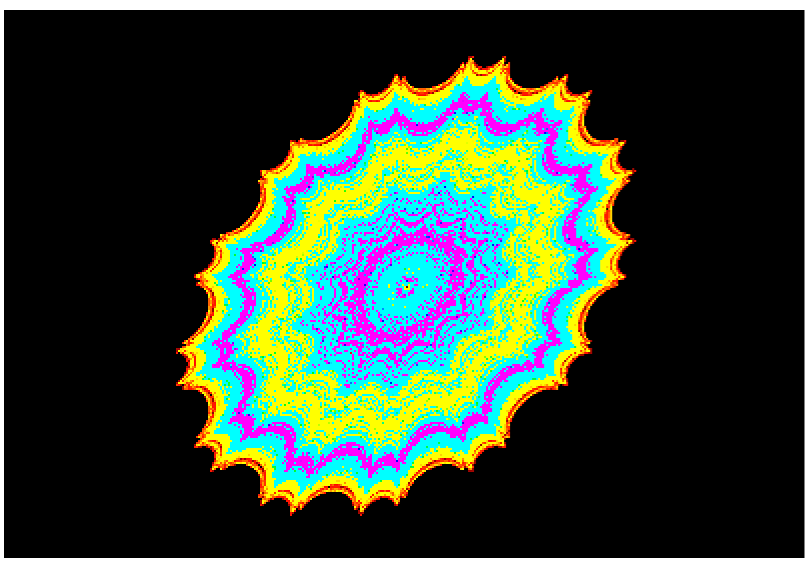

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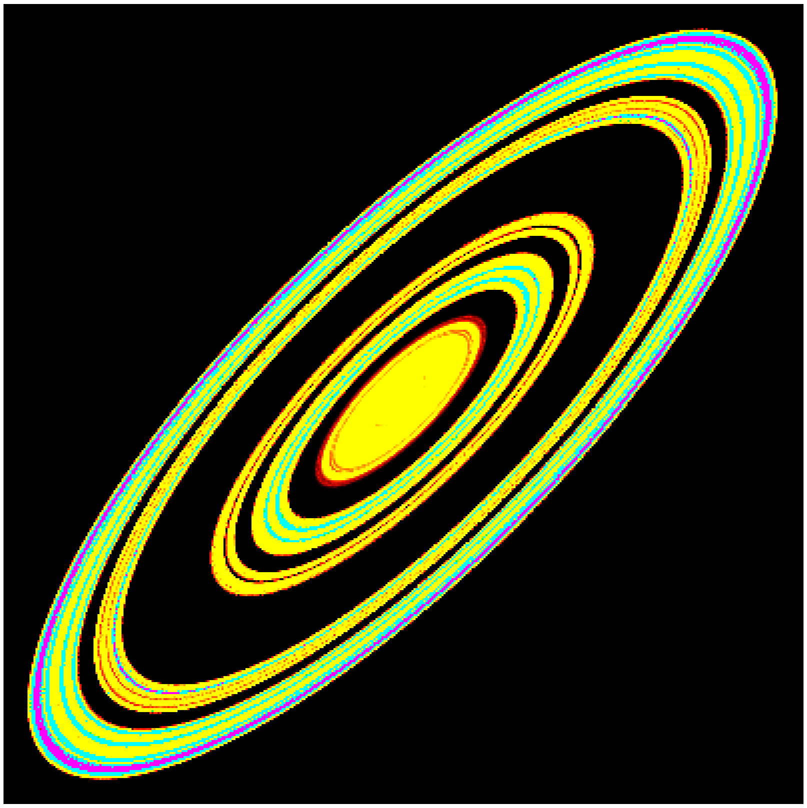

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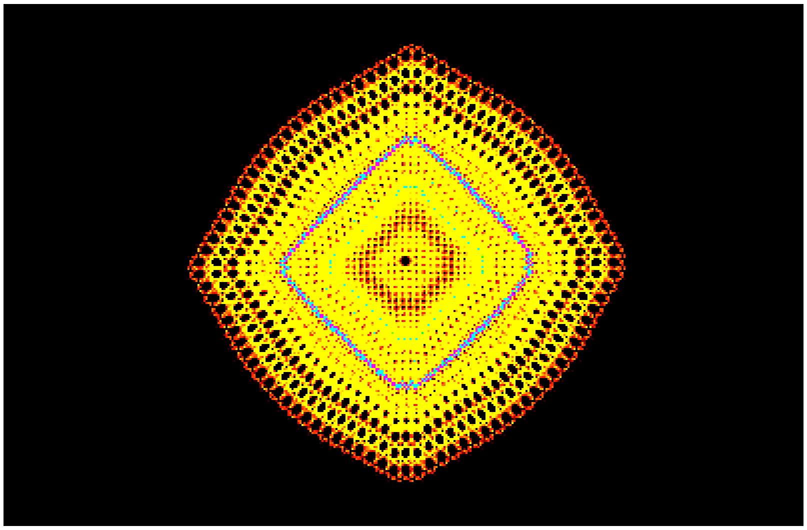

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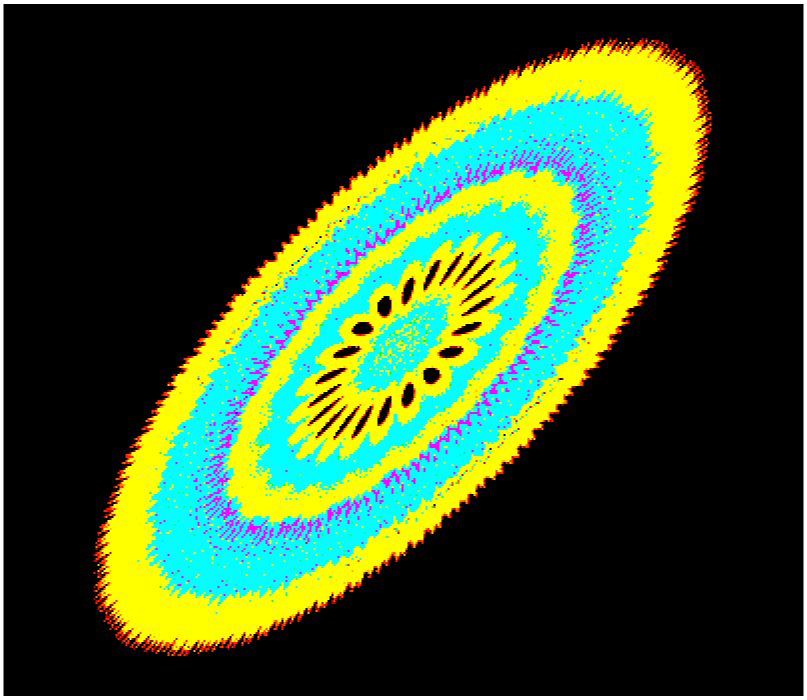

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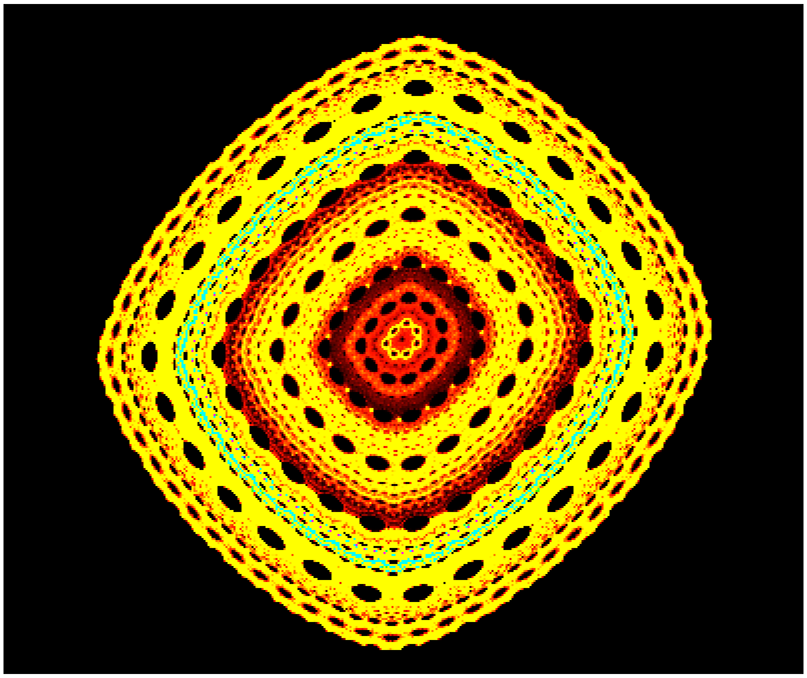

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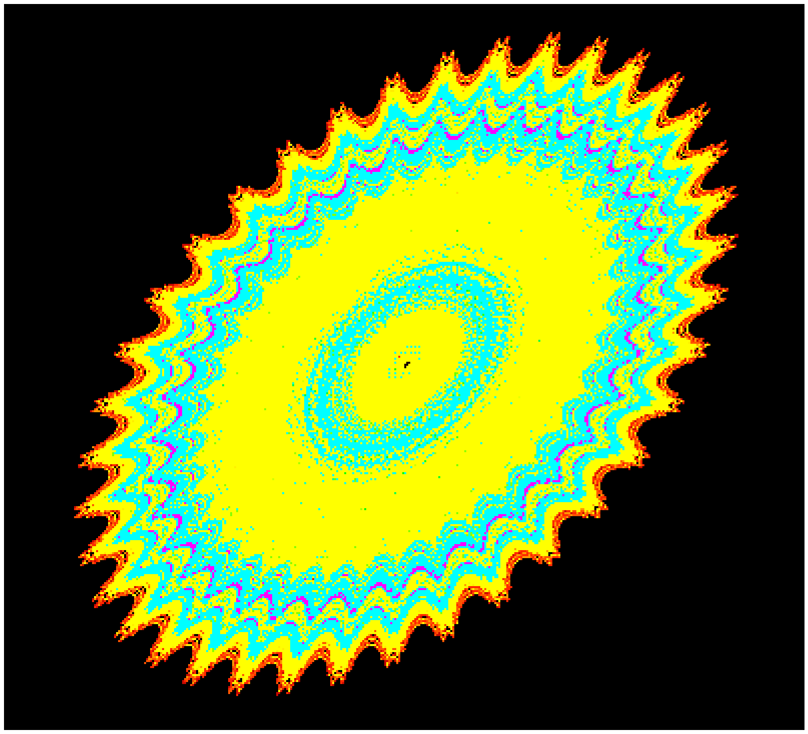

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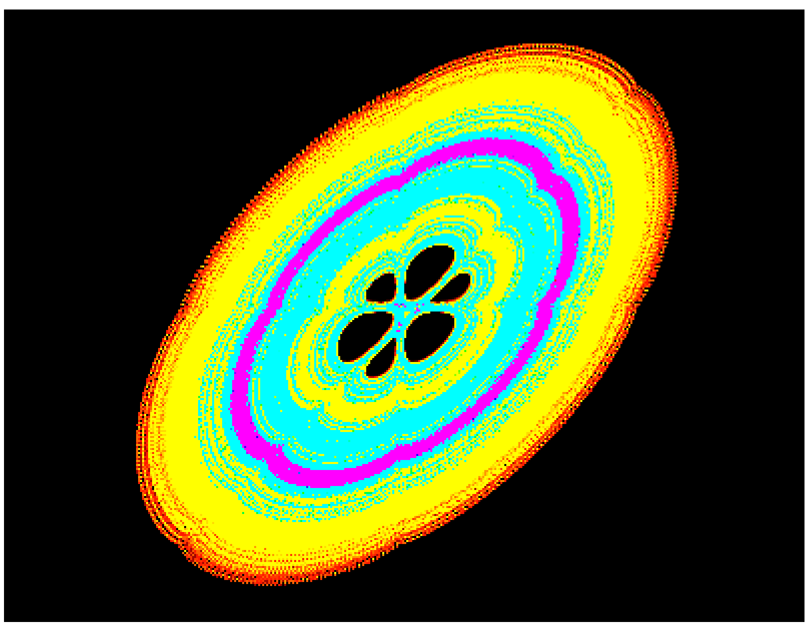

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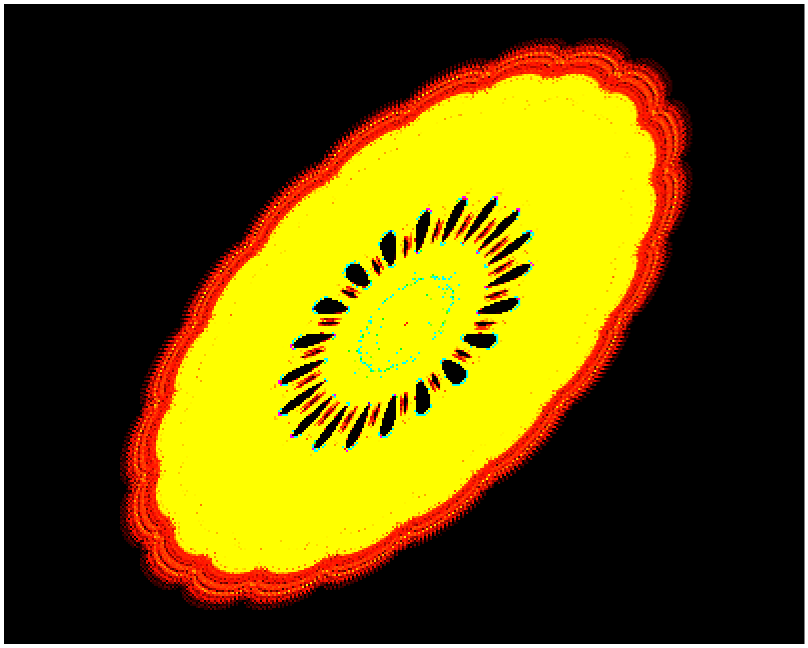

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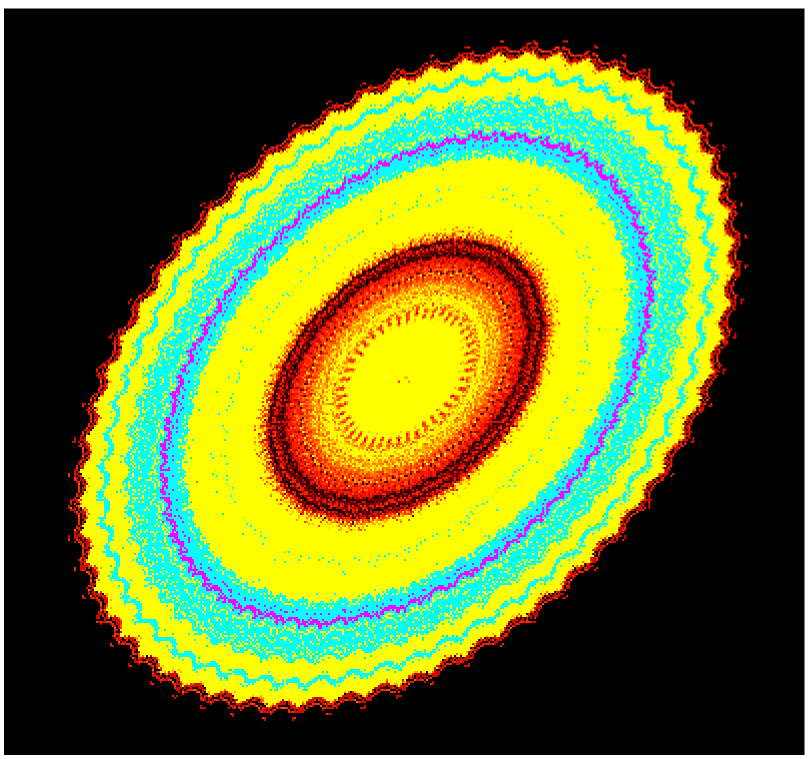

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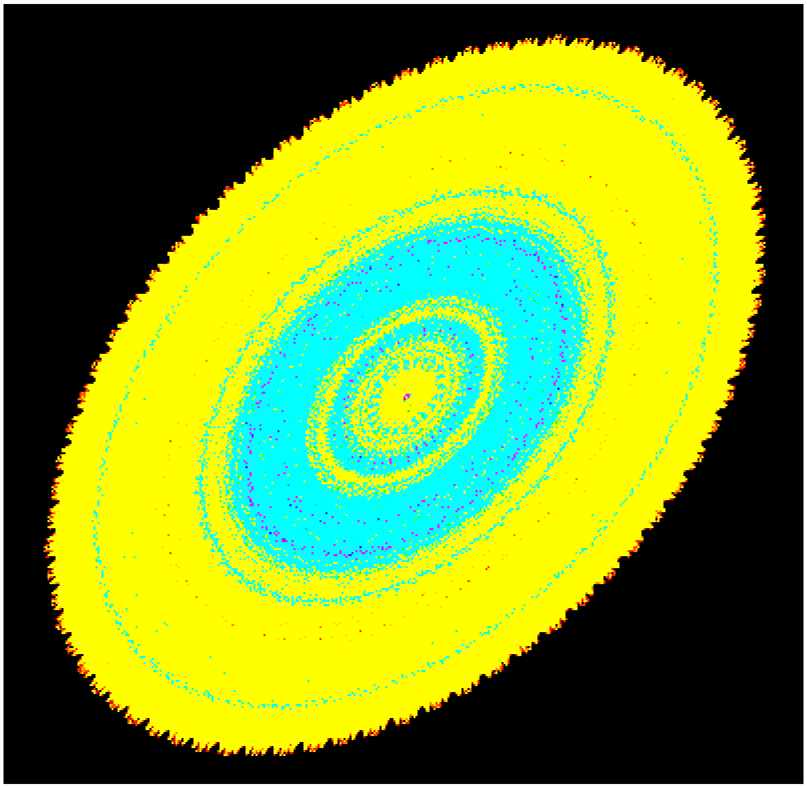

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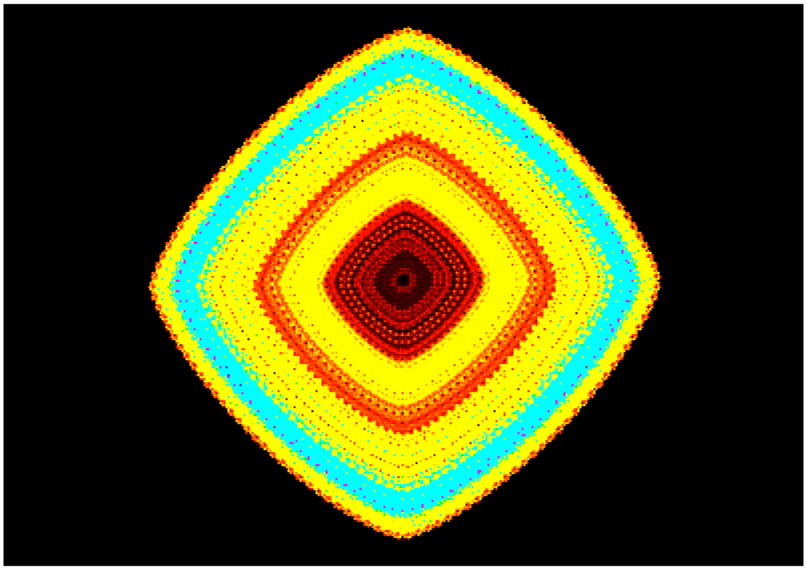

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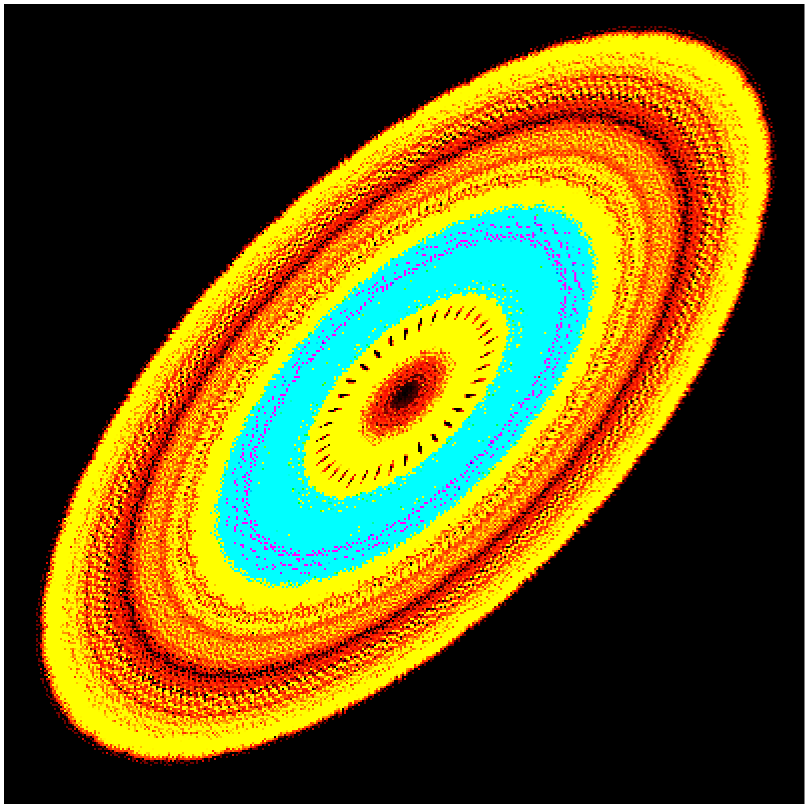

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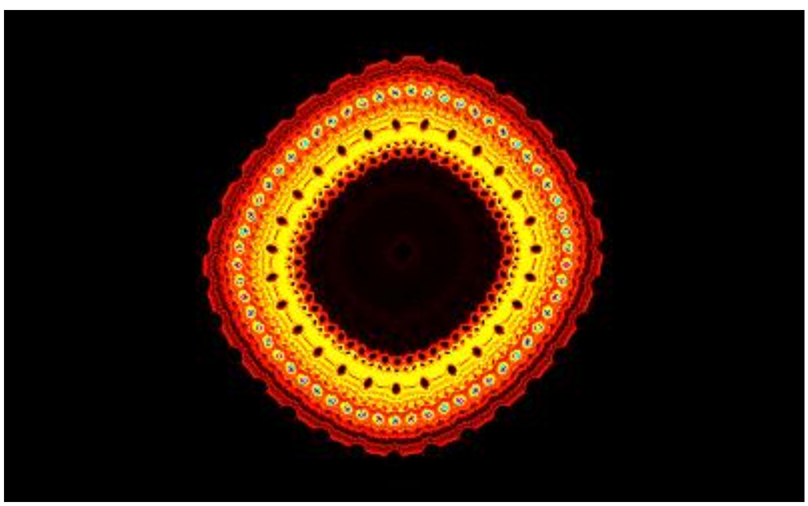

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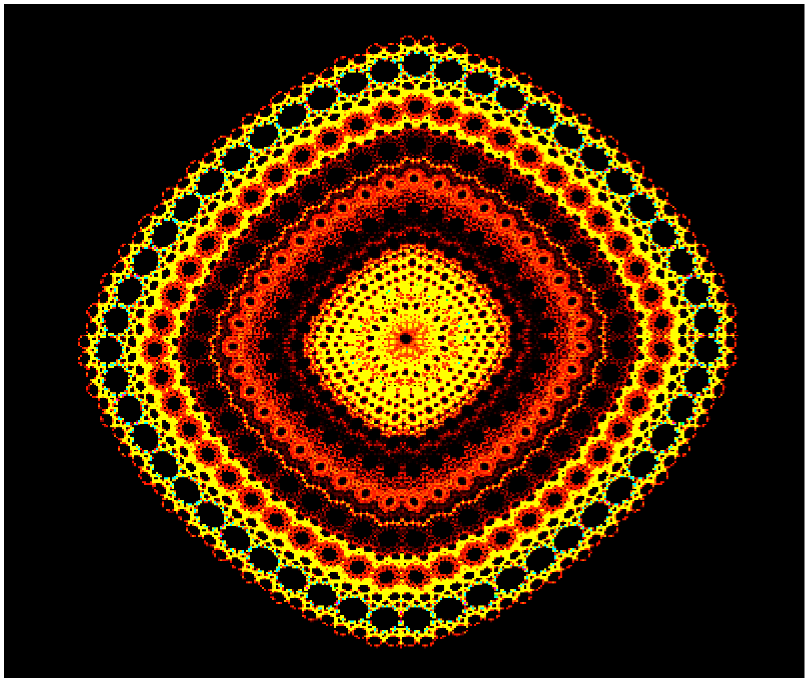

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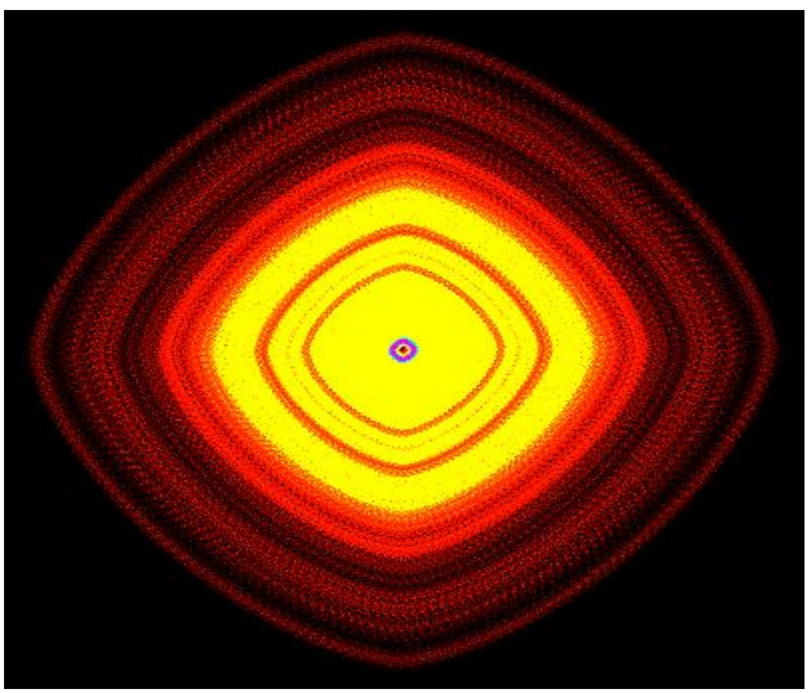

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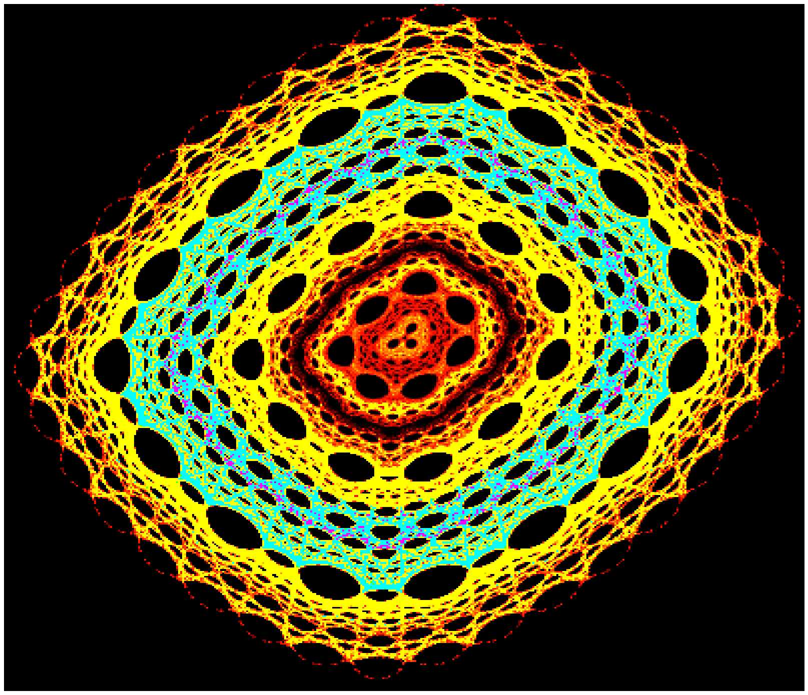

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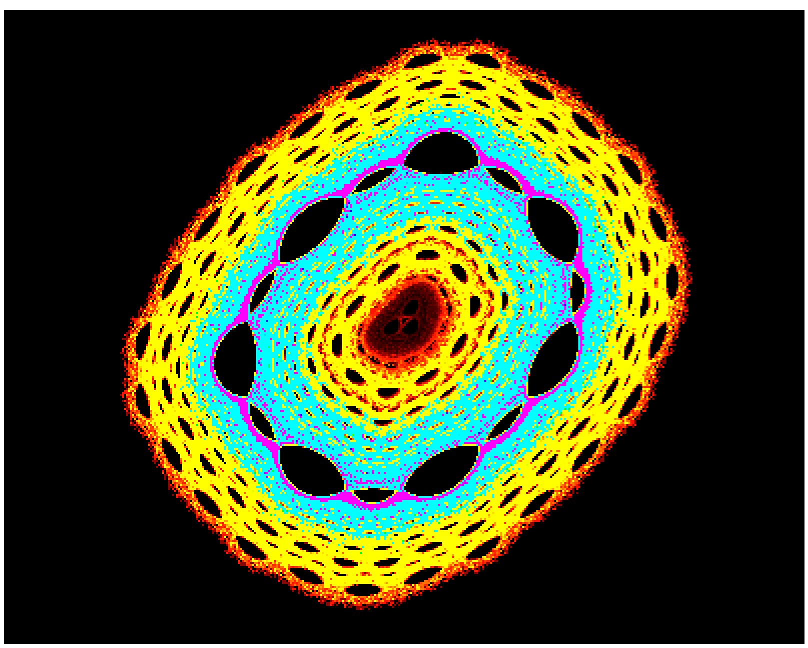

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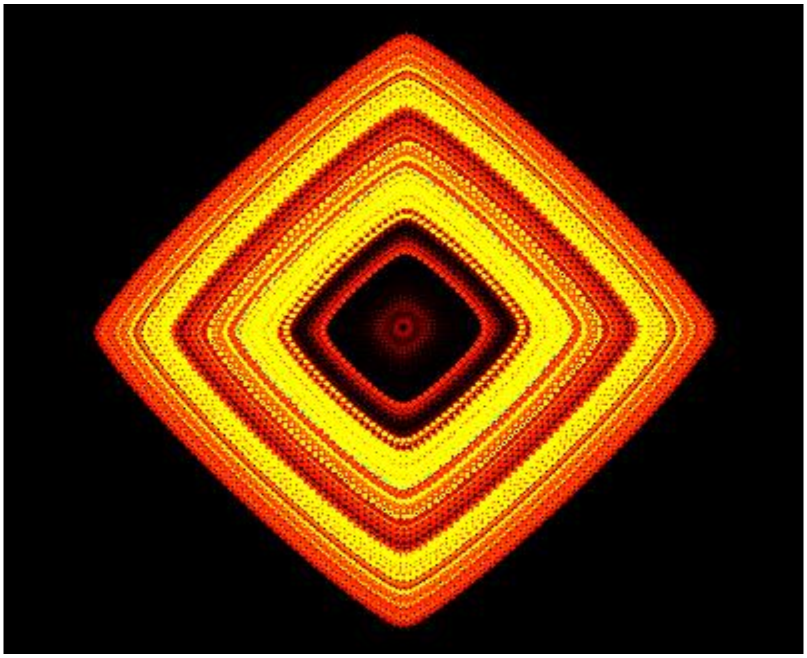

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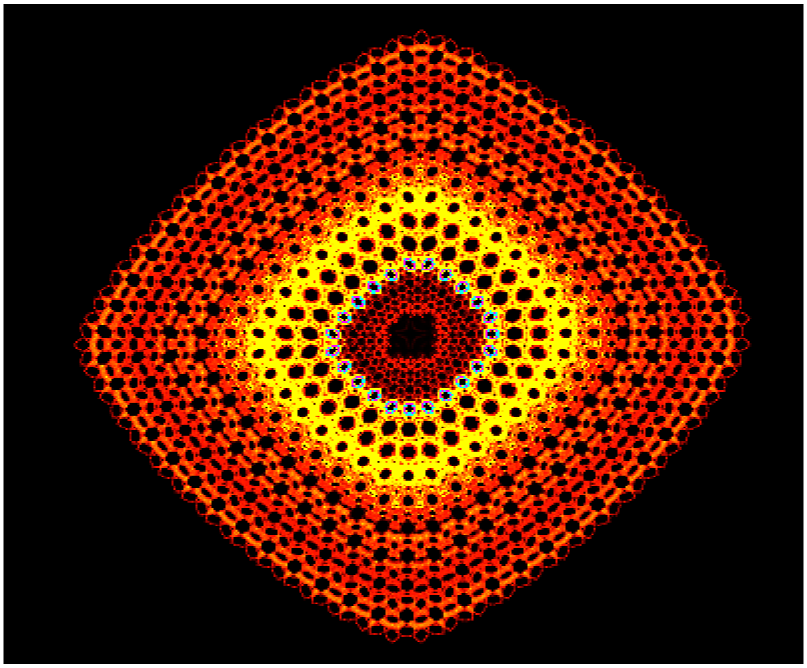

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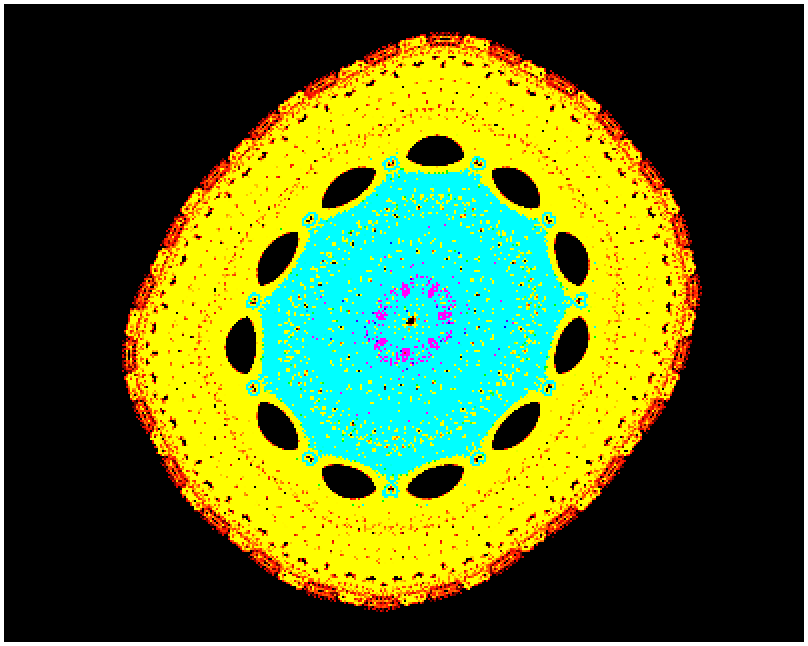

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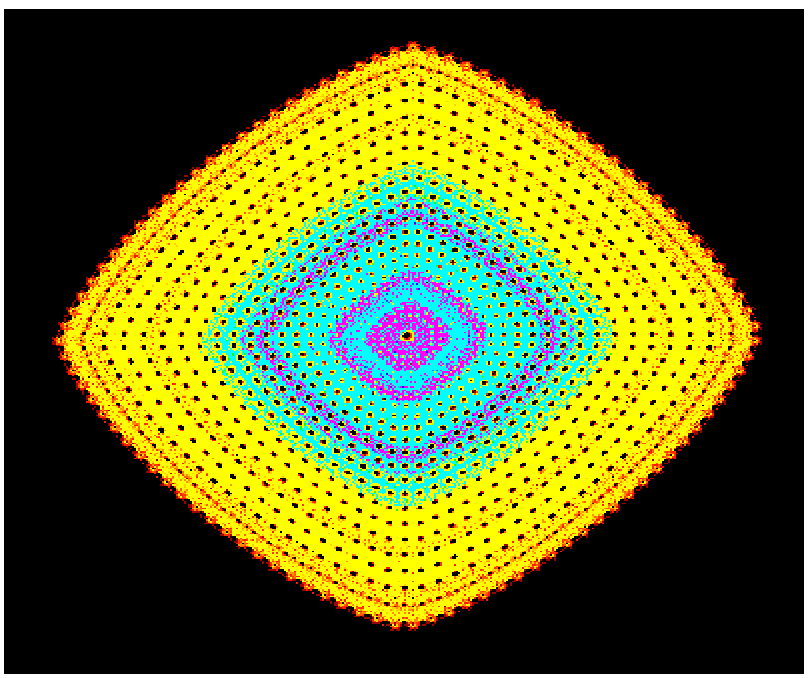

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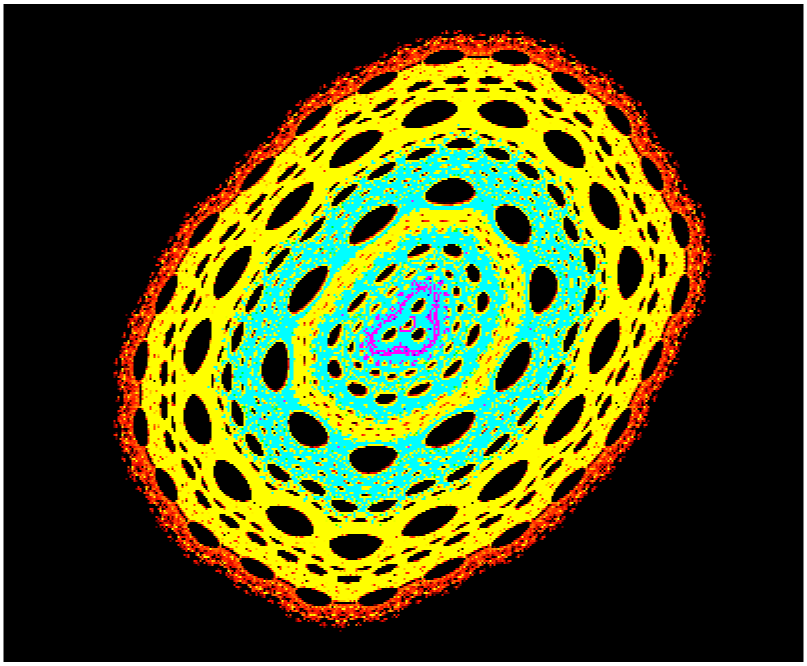

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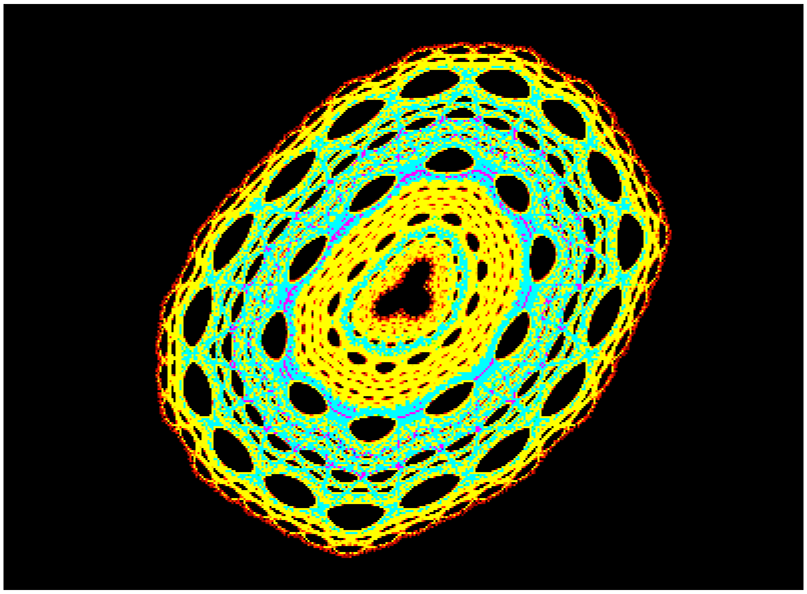

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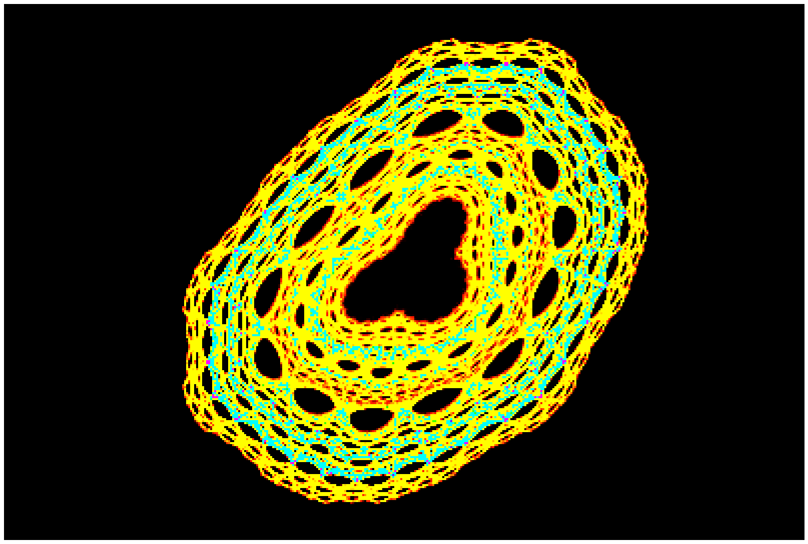

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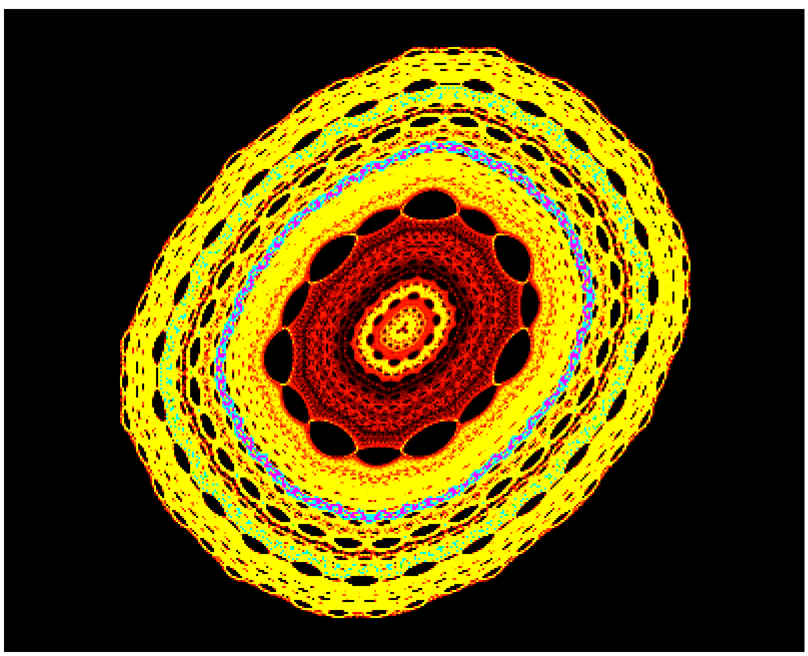

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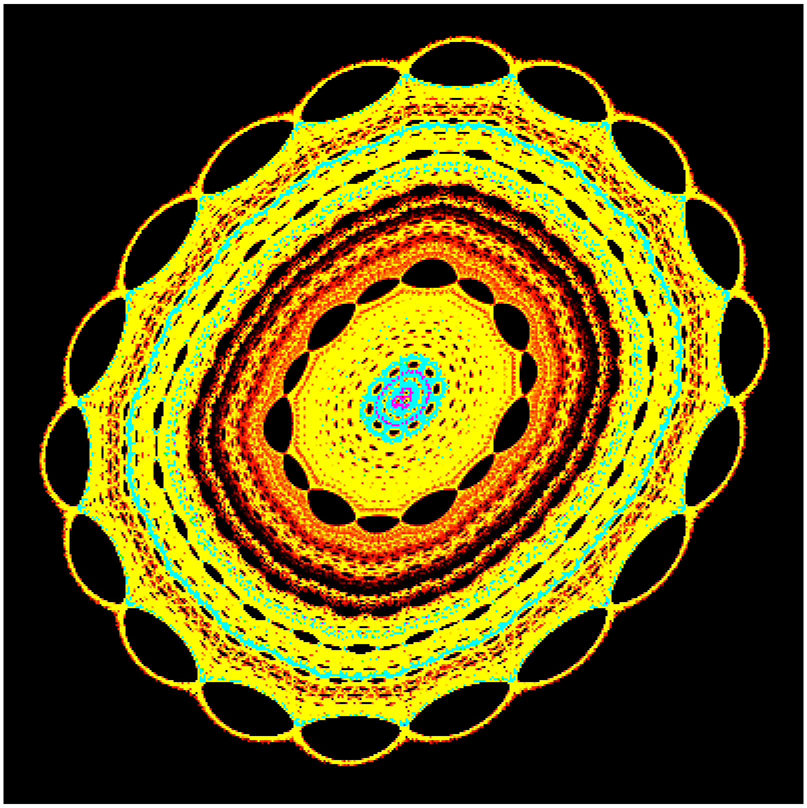

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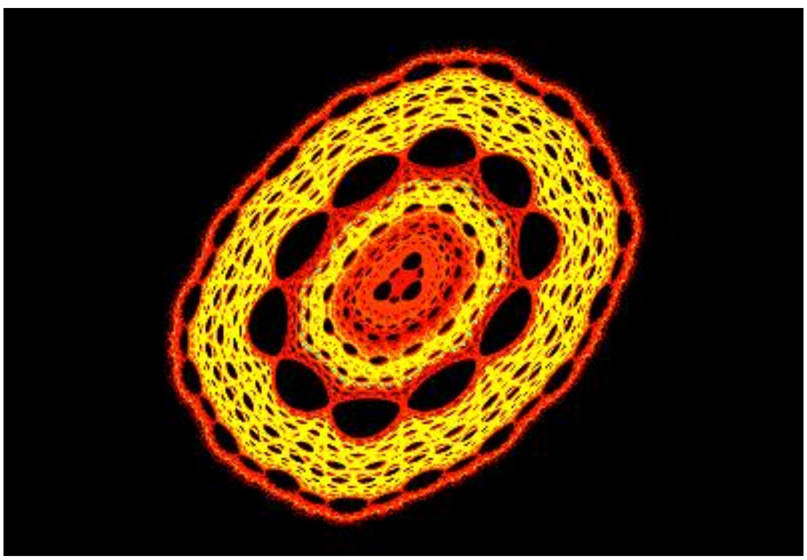

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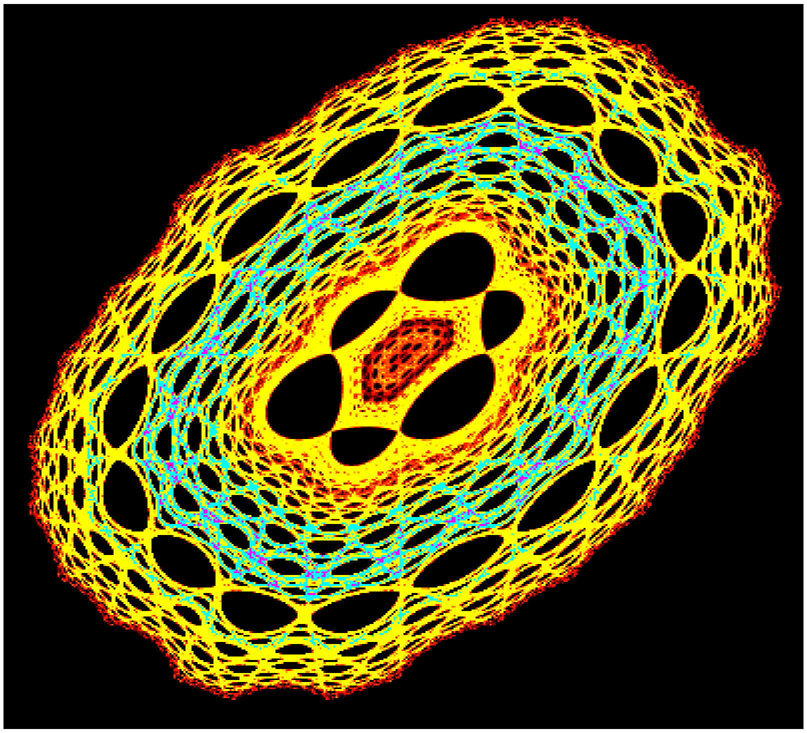

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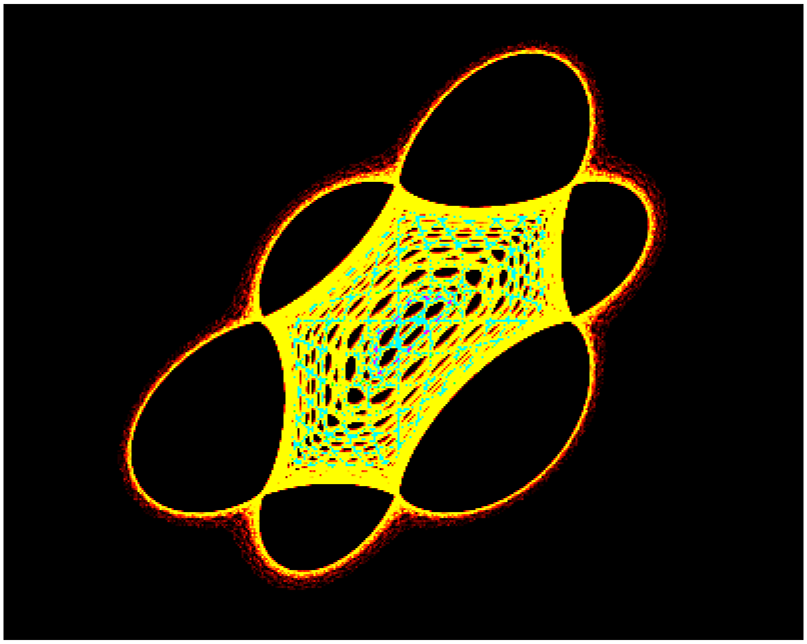

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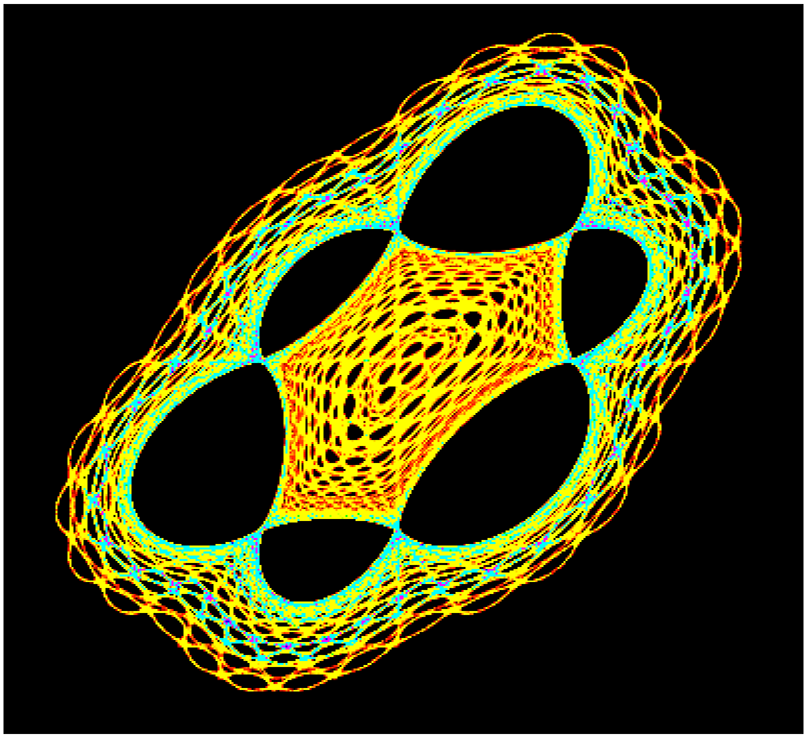

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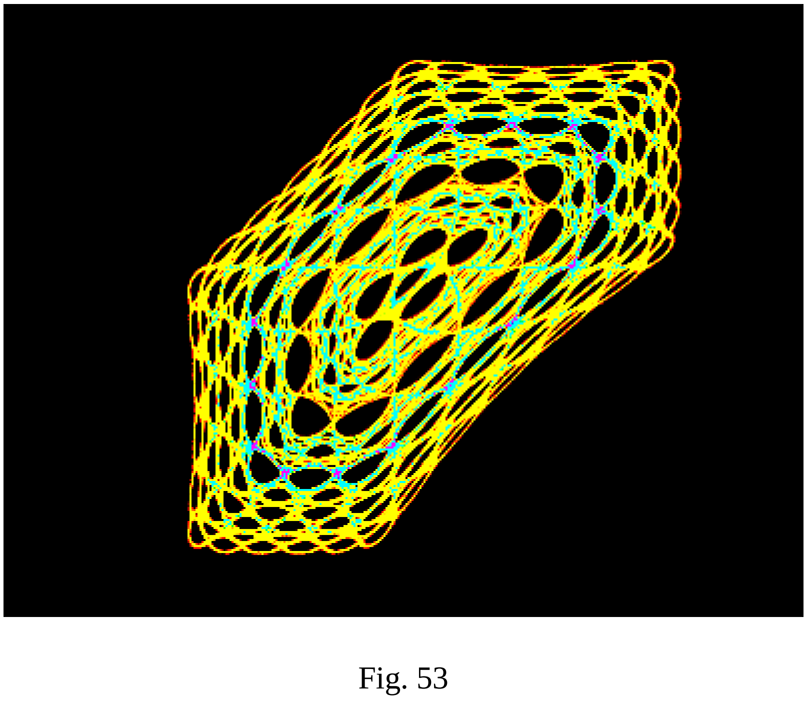

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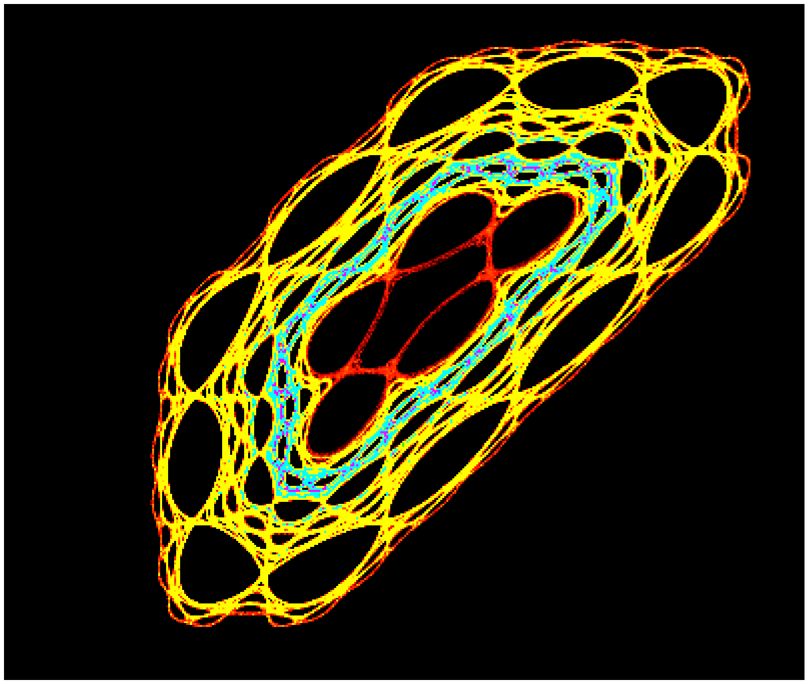

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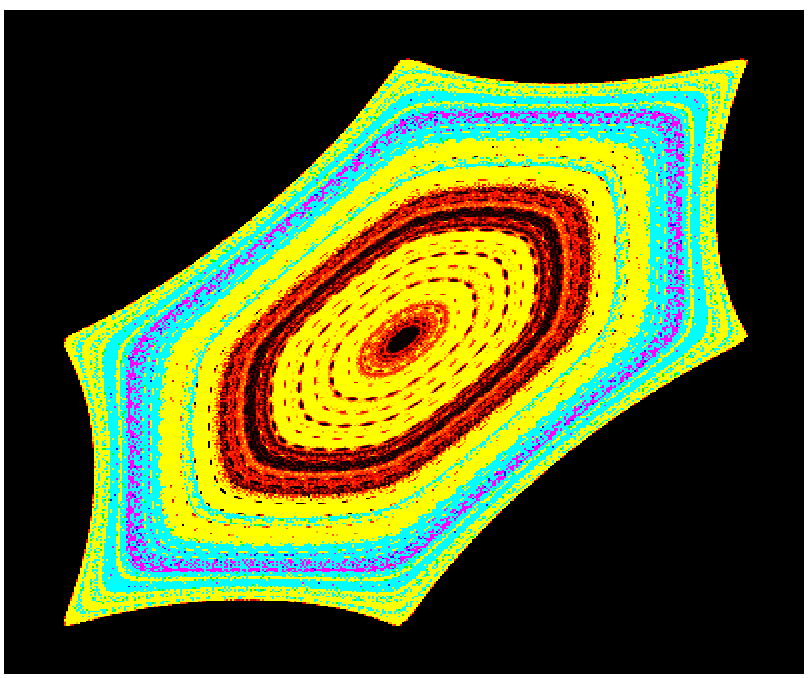

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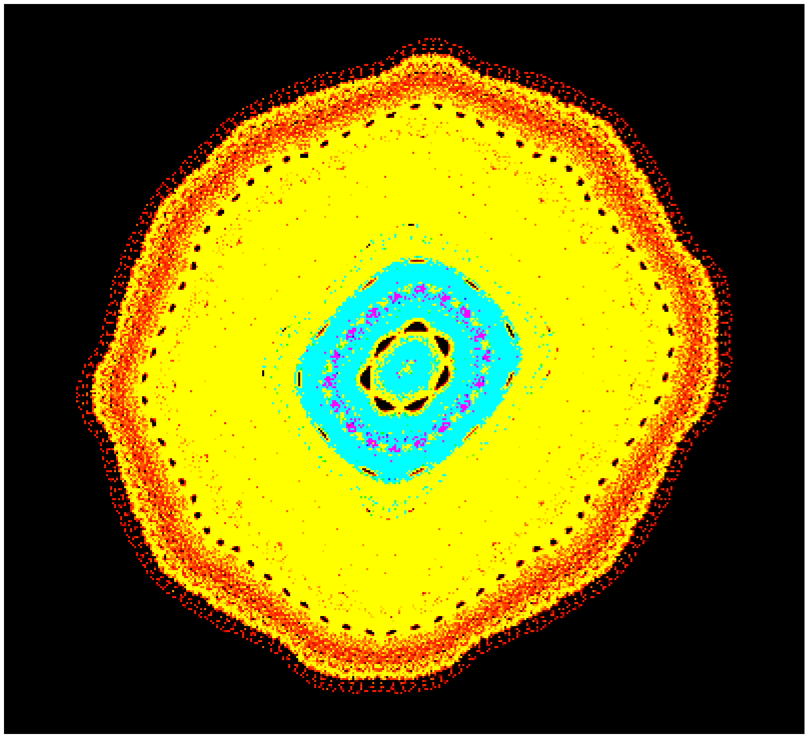

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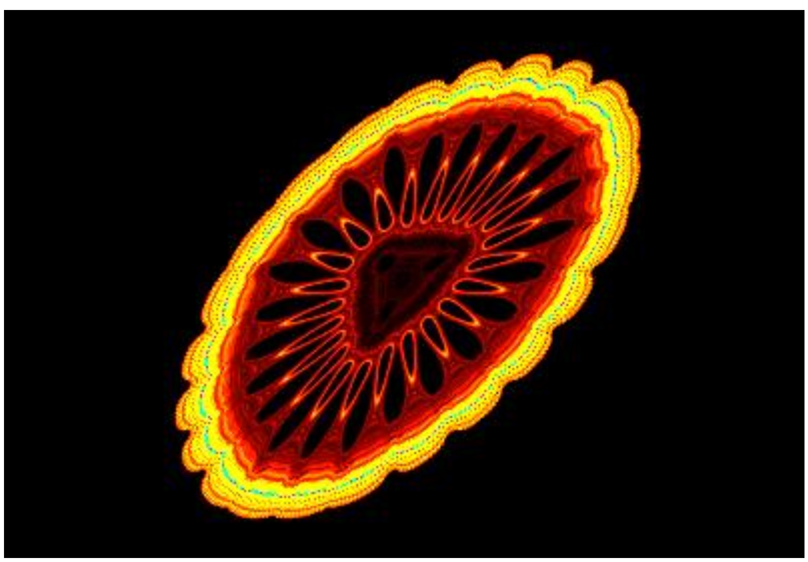

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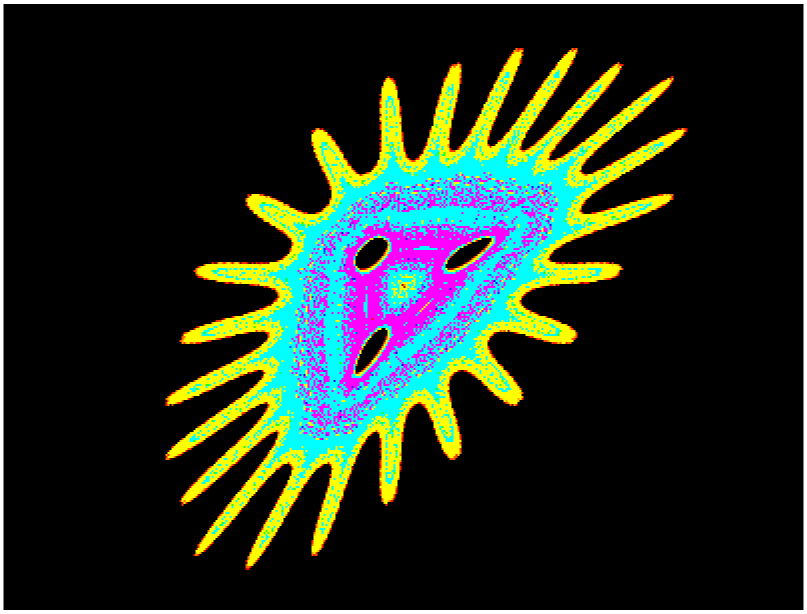

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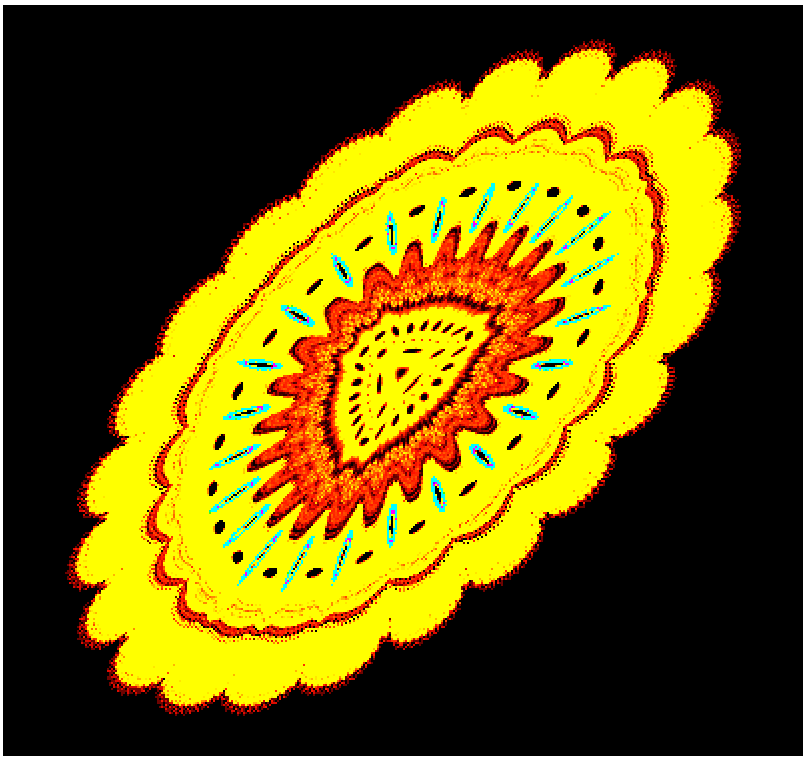

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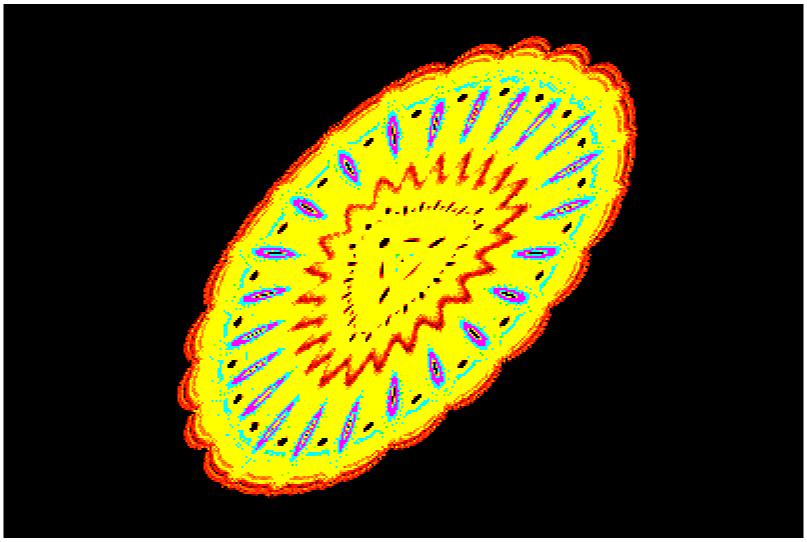

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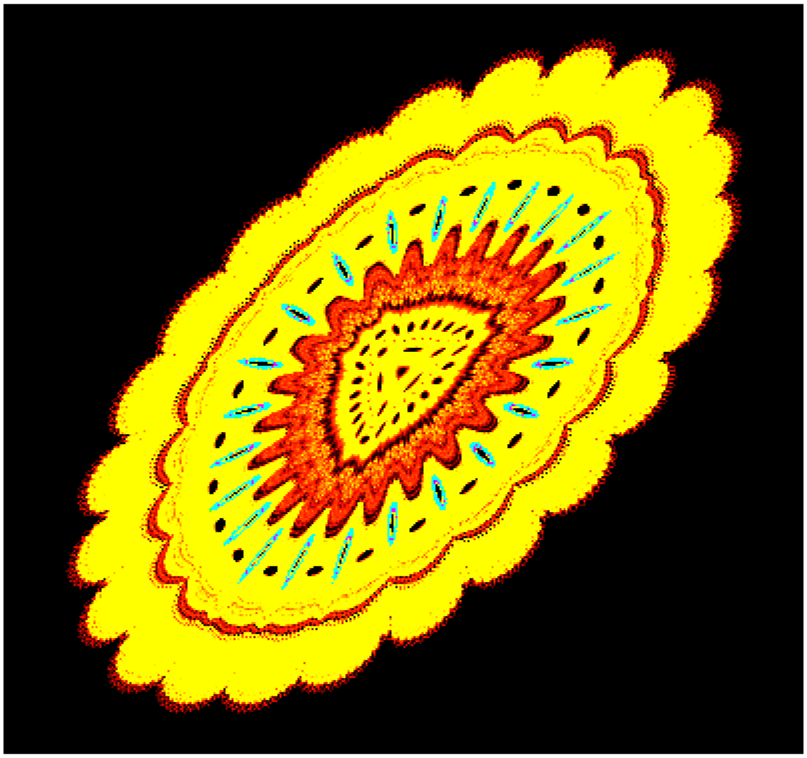

Fig. 61

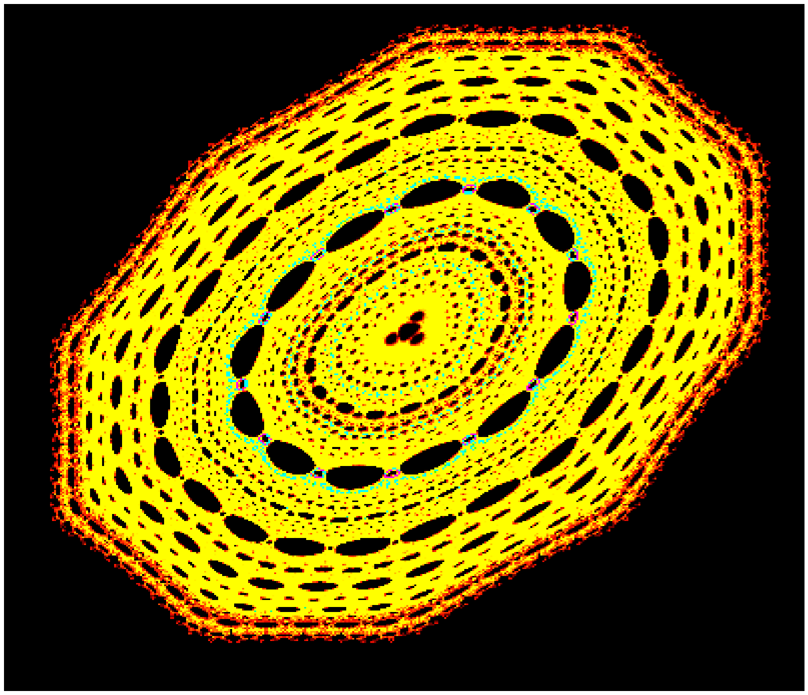

Fig. 62

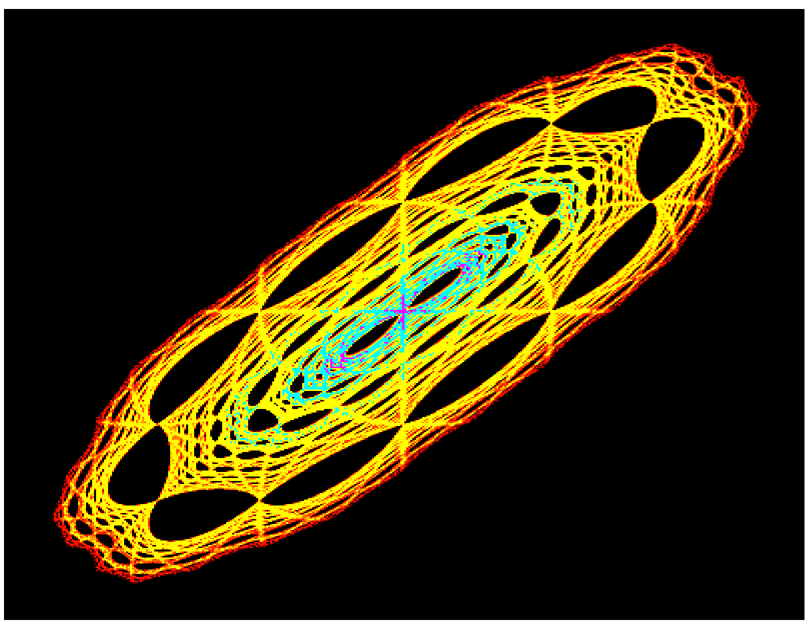

Fig. 63

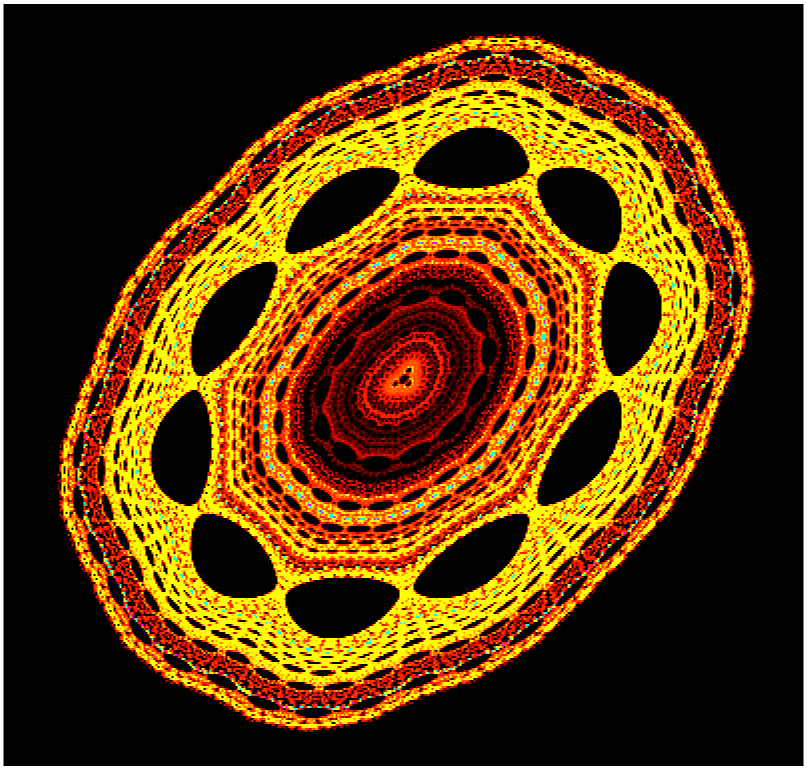

Fig. 64

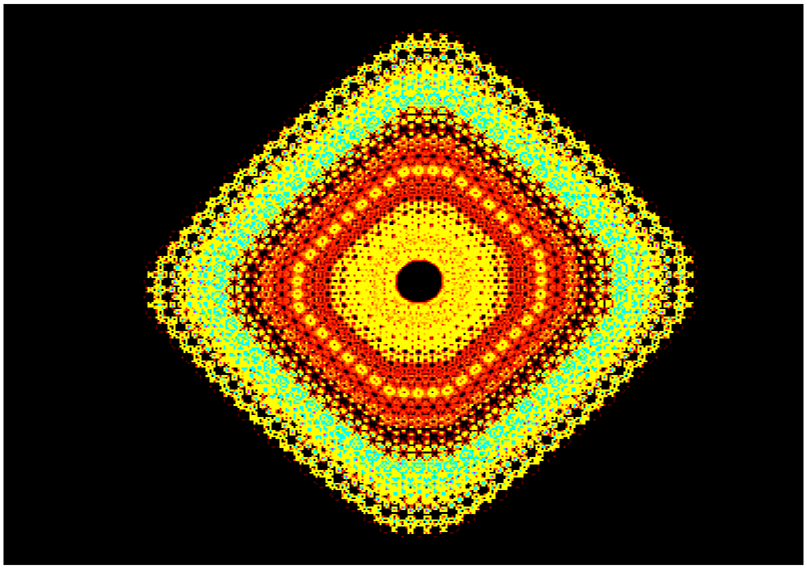

Fig. 65

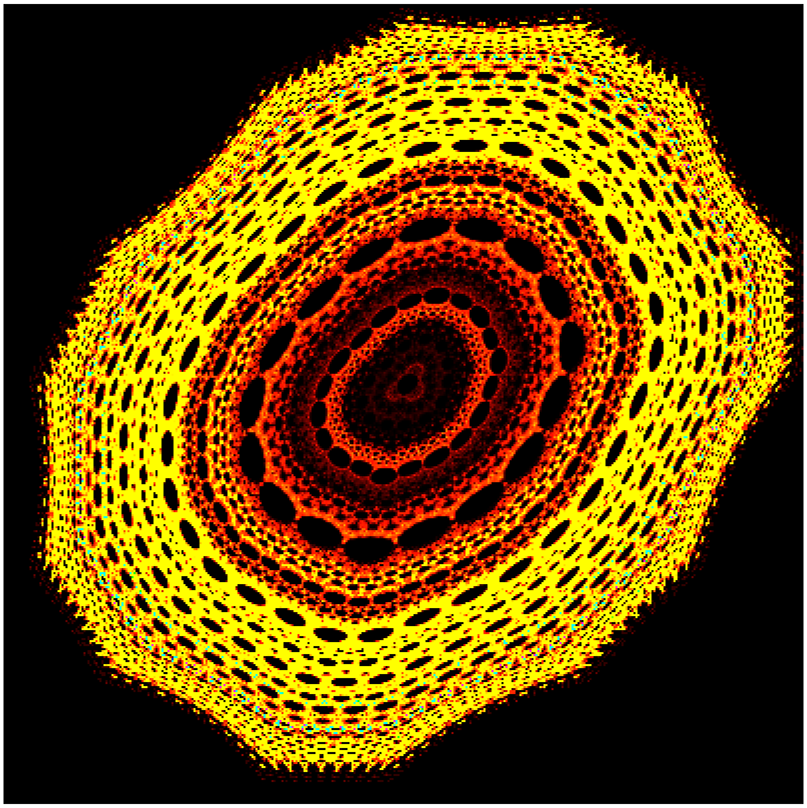

Fig. 66

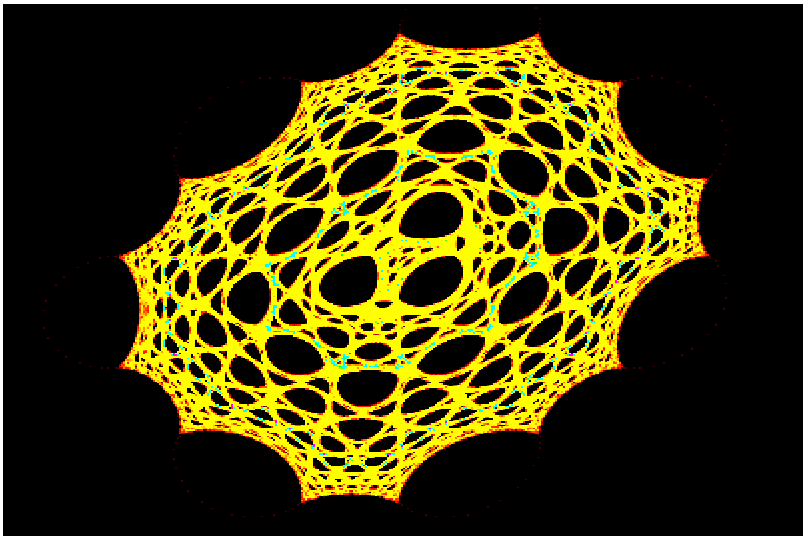

Fig. 67

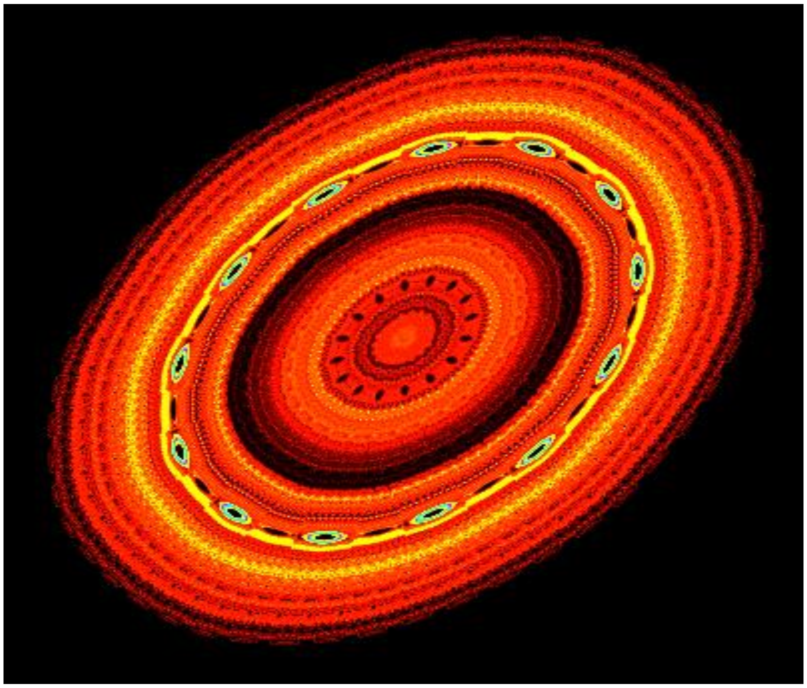

Fig. 68

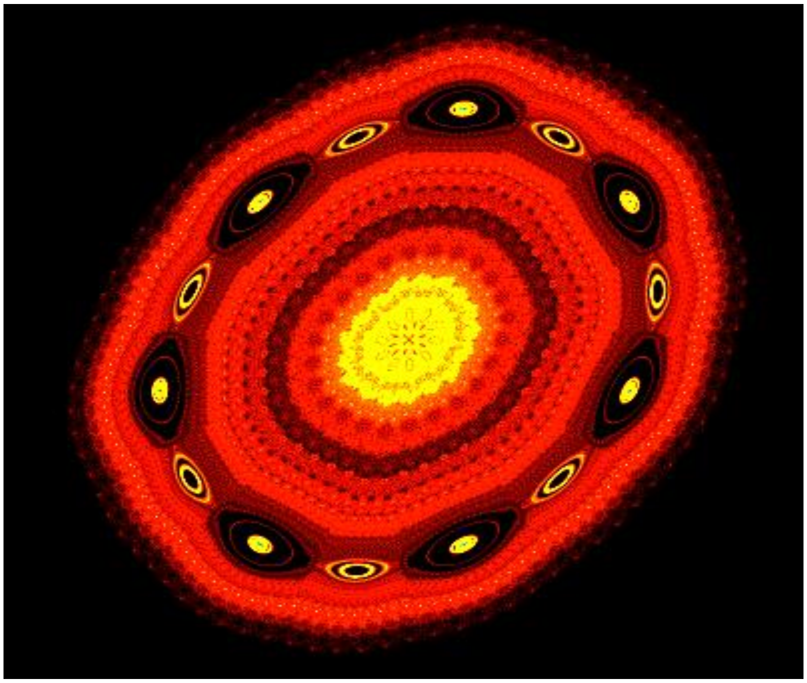

Fig. 69

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