From Bernoulli to Laplace and Beyond

Abstract. Reviewing Laplace's equation of gravitation from the perspective of D. Bernoulli, known as Poisson-equation, it will be shown that Laplace's equation tacitly assumes the temperature T of the mass system to be approximately $0^\circ K$. For temperatures greater zero, the gravitational field will have to be given an additive correctional field. Now, temperature is intimately related to the heat, and heat is known to be radiated as an electromagnetic field. It is shown to take two things in order to get at the gravitational field in the low temperature limit: the total square energy density of the source in space-time and a (massless) field, which defines interaction as quadratic, Lorentz-invariant, and $U(4)$-symmetric form, that restates the equivalence of inert and gravitational energy/mass in terms of absolute squares. This field not only necessarily must include electromagnetic interaction, it also will be seen to behave like it.

1. Problem Statement

A system of N particles in spacetime in Newtonian mechanics is a system that is to be defined by $3N$ location coordinates q_k as well as a common time coordinate and their associated $3N$ momentum coodinates p_k as a function of time. Mostly these systems are stably confined to a fixed region in space over time like a drop of water or a stone. So, there will be many equations of confinement, and to simplify the mathematical model, Bernoulli changed that model by replacing the particles' position with a spatial mass density $\rho(t) : \mathbb{R}^3 \ni \vec{x} \mapsto \rho(\vec{x}(t)) \geq 0$. Laplace then took over that model and showed that the gravitational force of a mass density ρ could be expressed as Poisson equation $\Delta\Phi = 4\pi G\rho$ of a potential function Φ, the gravitational field and the gravitational constant G, $\Delta := \partial_1^2 + \partial_2^2 + \partial_3^2$ being the Laplace operator. That marked the introduction of field as a concept into physics. What made it both bold and dubious, was that it said that the field was to be the sheer equivalent of the mass distribution. It was soon found out that the field was to be an harmonic function of the space coordinates, which led to the famous Laplace demon problem, and another problem then showed to be the lack of Lorentz covariance, giving evidence that the Laplace field of gravitation cannot be correct.

However, there is much more to it: Both, Bernoulli and Laplace took it as

evident that a (smooth) mass distribution $\rho(x)$ of N particles, which is confined to a bounded region $K \in \mathbb{R}^3$ (for all times t), could be resolved at each given time t into N disjoint bounded regions $K_1, \ldots, K_N$, containing a unique particle, if only the particles would stay apart from eachother. With that, it should be possible to replace ρ with the sum $\sum_k \rho_k$ of smooth, non-negative functions ρ_k of disjoint support and compact support, each (which means, they all vanish outside a bounded set, e.g. K, and if one is greater zero at some point x, then all the others must vanish at this point x). If so, the above Poisson equation could be rewritten as a sum $\sum_k \Delta\Phi_k = \sum_k 4\pi G\rho_k$ of N independent gravitational equations for each and every particle.
And indeed, mathematics proved this to be possible, now known as the partion of unity (see e.g. [2, Ch.16]). That, on one side, means that even if all particles are pointwise in nature, we can approximate these particles through Bernoulli's ingenious replacement of mass position by smooth mass densities. On the downside, that shows that Laplace's theory of gravitation must lack generality, because in it, all the particles of a body are independent from eachother: they just add up individually!
And this is incorrect, because it disregards the body's kinetic energy:

The mass m of a body B at rest is to be defined to be equal to the total energy of B. Now, if B was simply the sum of N individual oscillating particles, then the total energy E is to be the square root of $\sum_{1\leq k\leq N} m_k^2 c^4 + (cm_k v_k)^2$, where c is the speed of light, m_k are the individual masses, and the v_k are the mean speeds of these masses, so kinetic energy, a.k.a. "heat", always will add to the the total mass of B!

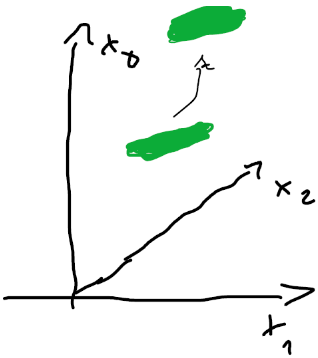
So, ρ has to become a 4-vector $j = (j_0, \ldots, j_3)$ of functions j_k in time and space, such that

$$< j(t), j(t) >:= \|j(t)\| := \int_{\mathbb{R}^3} |j_0|^2 (t, \vec{x}) + \cdots + |j_3|^2 (t, \vec{x}) d^3x$$

equates (locally) to the square of energy, which then becomes the square of the total energy of B, i.e. up to c^2 is equal to the square of the inert mass m of B. (So, j can be conceived as the macroscopically composed superposition of local quantum states, which approximates the system's particles.)

(Because under relativistic conditions there is no single eigentime for an object of diameter $d > 0$, one should include include integration over a time interval t, though, but, as this is the conventional and simpler way, we'll follow that line.)

In all, the appropriate model for discussing gravity of particle systems is that of time curves $\Omega : \mathbb{R} \ni t \mapsto \Omega(t) := j_t := (j_{0,t}, \ldots, j_{3,t})$, where the $j_{\mu,t}$ are to be smooth functions with compact support in space $\mathbb{R}^3$ for each μ and t, such that their absolute squares, $\left|j_{\mu,t}\right|^2$, are the intensities of smooth, local energy-momentum packages of the particles in space and time, as sketched below:



Having $\Omega : t \mapsto j_t \in L^2(\mathbb{R}^3)^4$ in place, we can state:

Proposition 1.1. *The total energy square of a system $\Omega : t \mapsto j_t$ at time t_0, which is at rest at t_0, is given by $E^2 =< j_{t_0}, j_{t_0} >= \sum_\mu \int_{\mathbb{R}^4} \left|j_{\mu,t_0}(x)\right|^2 d^3x$.*

2. Deriving gravity

It now shows up that there is nothing else than this notion of Ω needed to discuss gravity:
If instead of inert masses m_k, the system was made of electric charges, or even hadronic baryons, or whatever could be idealistically thought of to result in massy particles, the energy-momentum distribution is already put as a quadruple j_t of complex-valued states, the absolute squares being their intensities. (We'll shortly see, why this is the case, but for the moment you might look that up from any standard text on quantum field theory.)

The covariant Maxwell equations rewrite into:

$$< j_{t_0}, \Box A >= Const < j_{t_0}, j_{t_0} >= Const\ E^2, \tag{2.1}$$

where $\Box := \partial_0^2 - \cdots - \partial_3^2$ is the wave operator, A the electromagnetic 4-vector field, and $Const$ a constant, which in Gaussian units is identically 1 along with c.

Let's now choose that constant differently, to be $Const = -4\pi G$, where G is the positive gravitational constant, such that

$$\sum_\mu < j_\mu(x), \Box A_\mu(x) >= -4\pi G E^2. \tag{2.2}$$

Equation 2.2 then states nothing but the equivalence principle: It says that $\Omega : t \mapsto j_t$ has included into the j_t a gravitational interaction potential, which, when squared and summed up, is to be proportional to E^2 and is contracting (due to negative sign of $-4\pi G$).

Theorem 2.1 (U(4)-Invariance). *We now are in the position to explain, why $\Omega : t \mapsto j_t$ suffices to describe gravitational interaction:*
Because equation 2.2 becomes $U(4)$-invariant, with $U(4)$ being the group of unitary 4×4-matrices, just by letting the bra vector $< j_t|$ be the complex adjoint of its ket vector $|j_t >$. (This is also how we get at the non-negative

square $E^2 =< j_t, j_t >$*.) And,* $U(4)$ *is reducible decomposes into a product of subgroups* $U(4) = U(2) \times U(2) \times SU(3)$*, where in turn* $U(2) = U(1) \times SU(2)$ *is the product of the phase symmetry group* $U(1)$ *and the spin group* $SU(2)$*. So,* $U(4)$ *is a super group of Standard Model (with its 12 generators). That means that all energy coming from Standard Model should be accounted for within that* $U(4)$*-invariant model.*

As another consequence, we get:

3. Charge Inversion and Parity Invariance

Naively, we can define $\mathcal{C} : j_t := (j_t^0, \vec{j}_t) \mapsto (-j_t^0, \vec{j}_t)$ and $\mathcal{P} : (j_t^0, \vec{j}_t) \mapsto (j_t^0, -\vec{j}_t)$ as charge inversion $\mathcal{C}$ and parity $\mathcal{P}$. With it, the square $< j_t, j_t >$ becomes invariant w.r.t. $\mathcal{C}$ and $\mathcal{P}$, i.e. both are non-trivial inversions on the quadrupel of functions $j_t = (j_t^0, \vec{j}_t)$. What makes these insufficient from scratch, is that these antisymmetric operators are missing their symmetric counterparts: Given two oppositely charged particles like an electron and a proton, we may add these to an hydrogen atom, which is neutral, but not uncharged: if we invert the charges of that atom, we'd get out not zero, but another neutral object, which - as to its neutral overall charge - is the very same as the hydrogen atom before. And the analogous observation can be made as to parity. Because j_t is a quadrupel, we can represent it based on the four possible states of charge inversion and parity, which both are ± 1. With it, $\mathcal{C}$ becomes the permutation of the eigenspaces for the spectrum $\{+1, -1\}$. We may then define j to be (purely) charged, if j_t^0 anti-commutes with $\mathcal{C}$, and it is neutral, if $\mathcal{C}$ leaves j invariant. Let's concentrate on the neutral states j: Because these are $\mathcal{C}$-symmetric, we can drop the symmetric anti-charge pair, and we are left with the pair of functions of either positive and negative parity.
Now, it will be superficial to attribute quantum mechanical spin to these states that would cancel out: parity comes into play, whenever the flux $\vec{j}_t$ (and not its absolute square) contributes to the energy, which here is obviously the case: when we take its square, add it to $\left|j_t^0\right|^2$, and take the square root, whether sign of parity the state takes, its absolute value is the kinetic energy.
And now, given that for a neutral system of particles, its gravitational interaction is not depending on momentum of its constituents, we get: The kinetic energy of a neutral system of particles subjected only to its own gravitational field, must be an invariant!

We derive two statements fom this:
First, defining heat as the kinetic energy of a neutral particle system, that heat may freely move to external systems, which will leave the system with heat loss in a bounded state.
And, secondly, leaving out that invariant heat, the pair of parity-dependent equations for the neutral particle system now becomes a single one, namely:

$\Box\Phi(t, \vec{x}) = (Const)\rho(t, \vec{x})$, where ρ is the neutral rest energy density, and we'll define this rest energy as rest mass.
Note that this equation is relativistically invariant, it is the simple relativistic extension of Laplace's equation of gravity, $-\Delta\Phi = (Const)\rho$, and it relies on temperature near zero as in Laplace's model.
Let's drill into that:

4. Gravitational Interaction

Head on, we are to solve the equation $\Box\Phi(x) = -4\pi G\delta(x))$ with $x = (t_0, \vec{x})$ being the 4-vector of space-time coordinates. And the solution would be what would be called the Green's function. It would mean to integrate over $\mathcal{R}^4$, which historically is felt to be awkward.
Instead, historically, the "small solution" is taken:

$$\Box\Phi = (4\pi G)\delta(t^2 - \vec{x}^2), \tag{4.1}$$

which seemingly separates t from $r^2 = \vec{x}^2$ and basically makes it possible to express the inner product of the states in terms of $L^2(\mathbb{R}^3)$ with an internal time parameter t.
Let's go with that:
We know that $-\Delta\frac{1}{|\vec{x}|} = 4\pi\delta(\vec{x})$, and we have

$$\delta(t^2 - r^2) = \frac{\delta(t + r)}{2r} + \frac{\delta(t - r)}{2r}. \tag{4.2}$$

As is common, the convolution operator $\frac{\delta(t-r)}{4\pi r}$ is called "retarded" (wave) propagator $S_+(t)$, while $\frac{\delta(t+r)}{2r}$ is termed advanced propagator $S_-(t)$. So, $4\pi\delta(t^2 - r^2) = (1/2)S_-(t) + (1/2)S_+(t)$, (see e.g.: [1][Ch. 21-3]).
It is now commonly postulated that the advanced propagator S_- was to be neglected, as it was representing waves coming in from future to the past, which was anti-causal, so that only the retarded propagator S_- was to exist, and even Feynman himself granted the advanced propagator only a virtual role as it came to relativistic particle physics. So, $(1/2)S_-(t) + (1/2)S_+(t)$ is replaced by $S_+(t)$, which out of the sudden raises major problems and might not have been the best idea:
What about reversibility? Since the time inverse maps the retarded propagator into the the advanced propagator, this turns time inversion into an anti-causal, not permitted operation - just to name one of the problems. (Another one would be the production of radial halos for particles at the origin at retarded times, which obviously do not exist.) So, we need the advanced propagator in order to cancel the halos, and we need it in order to sustain time-reversibility. On the other hand, given time-reversibility, S_- is in symmetry with S_+, and we should be able to replace S_- with S_+. What's going wrong?

One point of mention is the innocently appearing usage of space and time coordinates: Given a mass distribution $\rho(t, \vec{x})$ at time $t = 0$, both S_+ and S_- convolute over the space coordinates, and these space coordinates are taken at Euclidean right angles of the local Euclidean time axis t at $\vec{x} = 0$: i.e.: the local observer A at $(t = 0, \vec{x} = 0)$, say, declares his own local time as universal time, although not even being able to observe another $(t = 0, \vec{y})$ at the very same time! That's not a failure, though: Given that A sees only objects on the retarded cone, as is commonly postulated, A record his observations for some time span and replay them backwards in time. But, when playing back in time, the objects are now positioned on the advanced light cone, and because of time-reversibility, A will see the same objects for each $\vec{x} \in \mathbb{R}^3$ on either cone. So, the orthogonal projections $(\pm|\vec{x}|/c, \vec{x}) \mapsto (0, \vec{x})$ deliver the same result. And that projection is smart, because invariant w.r.t. time inversion, and above that, A needs a hyperplane of sources, not just a single point, in order to capture the wave front entirely.

That said, given a mass distribution $\rho(t = 0, \vec{x})$ for $\vec{x} \in \mathbb{R}^3$, the gravitational field is emitted independently at the speed of light at each $(0, \vec{x})$, and they simply superimpose (i.e. add) at later "local world time" $t > 0$. We can therefore restrict consideration to the origin $(t = 0, \vec{x} = \vec{0})$, while the the generalization to all $\vec{x} \in \mathbb{R}^3$ follows by integration over the location coordinates $\vec{x} \in \mathbb{R}^3$.

$\frac{\delta(t \pm r)}{2r}$ themselves are no functions, but distributions, and we get functions of these by integrating over the time t: Let' put

$$W(t) := \int_{-\infty}^{t} \frac{\delta(t' - r)}{2r} dt' = \frac{1}{2r}|_{r=t}, \tag{4.3}$$

which is defined for all $t \neq 0$, where integration from the negative t through 0 to the positive t is defined as boundary value from negative t to $-0 := lim_{-\epsilon \to 0}$ and from $+0 := \lim_{\epsilon \to 0}$ to the positive endpoint, and the boundary values cancel. So, $W(t) = \int \frac{\delta(t-r)}{2r} dt$ is the time integral of $\pm \frac{\delta(\pm t' - r)}{2r} dt$, known as principal function: in particular, we have $dW(t)/dt = \frac{\delta(t-r)}{2r} dt$ for $t \neq 0$. $W\rho(0, \vec{0})$ describes the spreading of the field for the source ρ at the origin $(0, \vec{0})$ in terms of the Euclidean local (world) time (at least for increasing $t > 0$), and to get it for the spatially neighboured locations $\vec{y}$ at the very same Euclidean time, where $\rho(0, \vec{y})$ is the source, we would need to displace $W(t)$ spatially, which will give a function $W_{\vec{y}}(t)$ to be superimposed with $W(t)$. We'll do that later. For now, it is important to see what happens, when we square $W\rho(0, \vec{0})$ up to $W^2(t)\rho^2(0, \vec{0}) = \frac{\rho^2(0,\vec{0})}{4r^2}|_{r=t}$:
Its integral over the 2-dimensional sphere $r = t$ for $t \neq 0$ is $\pi\rho^2(0, \vec{0})$ which is constant in time, and we conclude that this must hold for $t = 0$, either. And now, given that $\rho(0, \vec{x})$ vanishes outside a bounded region of space and is sufficiently smooth to be integrated, we can integrate over space the result

of all displaced sources at Euclidean world time to give:

$$< W(t,\cdot)\rho(0,\cdot), W(t,\cdot)\rho(0,\cdot) >= \pi < \rho(0,\cdot), \rho(0,\cdot) > .$$

In other words, the operator

$$\tilde{W}(t) : \rho(0,\cdot) \mapsto \pi^{-1/2} W(t,\cdot)\rho(0,\cdot) \tag{4.4}$$

is a unitary representation of the 1-dimensional group of time-translation. Consequently, $\tilde{W}(0)^*\tilde{W}(0) = \tilde{W}(t)^{-1}\tilde{W}(t)$ is the identity 1, and therefore $\tilde{W}(0) = e^{i\lambda}1$ for some arbitrary phase factor $e^{i\lambda}$.
Also, $\frac{d\tilde{W}(t)}{dt}$ equals $S_+(t)$ for $t > 0$ and it is equal to $-S_-(t) = S_+(-t)$ for $t < 0$. (The reason for the sign twist is the unfortunate twist of direction of t in S_- along $|t|$ instead of t for $t < 0$.) That said, by extending S_+ to the negative time axis by $S_+(t) := -S_+(-t)$, we can replace all occurrences of $S_-(t)$ with $S_+(-t)$, and the advanced wave propagator again is seen to exist for time-inversion symmetry and not for any unknown reasons of complementarity. With that $S_+ = \pi^{1/2}\frac{d\tilde{W}(t)}{dt}$ - up to the constant factor $\pi^{1/2}$ - is dynamically the generator of time translation, where the unitarity of time-translation expresses the homogeneity of time in the energy conserving dynamical system.

Let's now put it together:
Instead of trying to find a linear solution of the gravitational field problem, let's go for a quadratical one: Given a free mass density $\rho_{free}(t) : \mathbb{R}^{\nvDash} \ni \vec{y} \mapsto \rho_{free}(t,\vec{y}) \in \mathbb{C}$, which is smooth in $\vec{y}$ and of compact support for each t, to solve is:

$$< \rho(t), \rho(t) >=< \rho_{free}(t), \rho_{free}(t) > - < \rho_{free}(t), V^2\rho_{free}(t) >, \tag{4.5}$$

where $-V^2$ is a negative (possibly linear) operator, representing the gravitational interaction, and because gravitation is to be independent of the motion of the masses, we can drop all motion from the right hand side; the above equation then becomes

$$< \rho(t), \rho(t) >=< \rho_{free}(0), \rho_{free}(0) > - < \rho_{free}(0), V^2\rho_{free}(0) > . \tag{4.6}$$

We now put $\rho(t) := (1+S_+(t))\rho_{free}(0,\cdot)$. Then $< \rho(t), \rho(t) >=< \rho_{free}(0,\cdot), (1+S_+(t))^*(1+S_+(t))\rho_{free}(0,\cdot)$. But $S_+(t)^* = -S_+(t)$, so $(1+S_+(t))^*(1+S_+(t)) = (1+S_+(t))(1+S_+(t)) = 1 - S_+^2(t)$. Therefore, $\rho(t) := (1+S_+(t))\rho_{free}(0,\cdot)$ solves the equation 4.6 with $V^2 = S_+^2$; moreover, $S_+(t)$ goes to the classical gravitational potential in the non-relativistic limit (i.e. letting the velocity of light become infinite).

So, dropping the bra side, we get $\rho = (1+S_+)\rho_{free}$, which converges to the classical gravitational problem in the non-relativistic limit.
Several notes are to be made:

Remark 4.1 (Boundedness and Temperature Independence of Gravity). It should be remarked that the replacement $S_+ \to S_-$ is an invariant of the steps above, in other words: the equations are time-inversion invariant: if particles attract gravitationally, then their time inverses attract either. So, how can gravitational attraction happen?
Summarizing the basic steps from above, it rests on 4 pilars:

(1) Independence of gravity from the speed/momentum of the particles
(2) Boundedness of the gravitationally interacting system
In free system of n elementary particles, each of these maintain their energy, such that their absolute square, and therefore the sum of these is maintained over time. That system of particles may stay together at whole, but any addition of kinetical energy makes it unstable. When that free system starts an attracting interaction, i.e. some of these particles begin sticking together, then the kinetic energy of that system, which always is positive, increases. So, the square of the kinetic energy increases, but then the complementary rest of the system's square energy must decrease, and when that additional kinetic square energy transfers to an exterior system (which is possible because of (1)), a bounded system results, and the total energy square of it will be smaller than it was in its free state. Conversely, a bounded system needs a positive kinetic energy to be transformed into a free system again.
(3) The interaction evolves radially from the particle sources itself at the speed of light,
and finally
(4) The strength of the gravitational field is to be proportional to what is called the gravitational masses of the particle sources.
So, it is irrelevant whether the potential energy is positive or negative, as long as the square of the potential energy is negative in the sum of energy squares of the system: what counts is that the particles in the bounded system stick together.

Remark 4.2 (Complex Conjugation and Phase Invariance). Either by going through the above or by Fourier transforming $\frac{\delta(t-r)}{r}$ and $\frac{\delta(t+r)}{r}$ it follows that S_+ and S_- are complex conjugates of eachother, and the interaction was written as (Hermititian or complex) square: $<\rho(0,\cdot),\rho(0,\cdot)>$. The complex conjugated bra side, $<\rho(0,\cdot)|$ can therefore be interpreted as the time-inverted ket-side $|\rho(0\cdot)>$. And indeed, given two distinct particles, any interaction at the speed of light mandates it to happen on the light cone, where one particle is advanced w.r.t. the retarded partner. As long as time-reversibility is valid, noone can tell which is the advanced an which is the retarded one, but we can deliberately make the choice: the advanced ones go to the bra-side, while the retarded one goes to the ket-side. So, the bra-particle interacts with the ket-sided one backwards in Euclidean time (in line with S_-), while the ket-particle interacts along the positive Euclidean time (according to S_+).
(In line with this, if the interaction between particles happens at the speed of light, then that means that interaction between particles on the light cone is instantaneos. And that implies that noone can tell whether that interaction happens forward or backwards: it just happens at the very instance of relativistic time.)
There is another take-away from that: if we take the (Euclidean) time-direction of light as our base, then we know what our advanced bra-particles are, but then its complex conjugate will generally differ from its non-conjugated ket-particle! Shouldn't it be the same, i.e. only represented by real functions?

Well, it is phase symmetry, that is needed to be able to identify a state with its complex conjugate, and it is just what quantum theory does! And that means mathematically that the gravitational theory is to be a $U(1)$-theory, because a circle in terms of complex numbers is the set of $z \in \mathbb{C}$, for which $\bar{z}z = Const.$
This is important, when it comes to electrodynamics:

5. Electrodynamics

Electrodynamics differs from a gravitationally interacting particle system in two important aspects: charge inversion and space inversion (a.k.a. parity), and both are no symmetries, but anti-symmetries:
In electromagnetism, the energy density function is a 4-vector $j = (j_0, \vec{j})$, where j_0 is called charge density and $\vec{j}$ the flux. With this, these inversions express into $\mathcal{C} : j \mapsto -j$ and $\mathcal{P} : j \mapsto (j_0, -\vec{j})$, and neither one has a non-trivial invariant subspace. Now, we want to come up with a Hermititian quadratic form $< j, j >$ which allows to express $< j, \mathcal{C}j >$ and $< j, \mathcal{P}j >$ as interaction of j with its charge and parity inverts. The charge inversion is not a problem, it's just a factor -1, but parity is: as was even clear to Maxwell in his time, it needs 2×2-spin matrices $\sigma_1 \ldots, \sigma_3$, then called "quaternions" $q_1 = i\sigma_1, \ldots, q_3 = i\sigma_3$, which are anticommuting and satisfy $\sigma_1^2 = \cdots = \sigma_3^2 = 1$. With it, the interchange operator $\mathcal{P} : (\lambda_1, \lambda_2) \mapsto (\lambda_2, \lambda_1)$ maps $\sigma_1 j_1 + \cdots + \sigma_3 j_3$ to $-\sigma_1 j_1 - \cdots - \sigma_3 j_3$, which allows us to map the 4-vector j bijectively to $ij_0 + \sigma_1 j_1 + \cdots + \sigma_3 j_3$, where $\mathcal{P}$ and $\mathcal{C}$ are well-defined as operators on the ket (and bra) vectors. Now look what happens, when we apply the commuting product $\mathcal{PC}$ to the ket-vectors: we get $-ij_0 + \vec{\sigma} \cdot \vec{j}$, and, assuming the j_k to be all real-valued, then $\mathcal{PC}$ turns out to be the complex conjugation, which from the section above was identified as the time-inversion $\mathcal{T}$. So, we get the TCP-theorem

$$\mathcal{T} = \mathcal{PC}, \tag{5.1}$$

and we see, what it is that needs the phase symmetry in order to ensure time-inversion invariance: these are the charges, not the neutral masses! Sofar, the electromagnetism is a pure $SU(2)$-theory built entirely on the real vector field generated by the spin-matrices, and it enforces ij_0 to be purely imaginary.
But what about the neutral particles? As to pure electrodynamics, these don't count: they mapped to zero as being "chargeless". And to include these, all it needs is to factor this $SU(2)$-theory with the $U(1)$-theory of gravitation, independent from electrodynamics, because the latter is purely symmetric w.r.t. time and charge inversion, and then it even must be symmetric w.r.t. parity. In all, that gives a $U(2) = SU(2) \times U(1)$-theory as the simplest model to unify electromagnetism and gravity into a single theory.

6. Conclusion

It was shown that gravity can be derived from the electromagnetic field. It can be equated to the maximal amount of heat which the body can stably contain.

References

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[2] François Trèves, *Topological Vector Spaces, Distributions, And Kernels*, Academic Press, 1967.