

# **The Exact Solution of the Pioneer Anomaly and Flyby Anomaly and the Interpretation of Inertia from an asymmetric Casimir effect**

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## **Abstract**

In this paper, I'll propose an exact solution for the Pioneer anomaly and the flyby anomaly. My solution interprets inertia from an asymmetric Casimir effect, and it interprets why inertia is related to Unruh effect and a Hubble-scale Casimir effect. My solution of the Pioneer anomaly and the flyby anomaly is depending on my new transformation and my equivalence principle. According to my transformation, the reality is observer dependent as a result of translating the retardation according to the invariance by the entanglement which is leading to the wave-particle duality and the uncertainty principle by the vacuum fluctuation. In this case by translating the retardation according to my transformation, it is resulted the relativistic ether as observer dependent, and thus in this case the reality in my transformation is observer dependent. This relativistic ether is not classical, but it is depending on the energy of the vacuum by translating the retardation according to my transformation. My transformation expresses about field by the vacuum fluctuation, and that explains also why field is observer dependent, and that explains also why Unruh effect is observer dependent.

## **Introduction**

Radio metric data from Pioneer indicate an apparent anomalous, constant, acceleration acting on the spacecraft with a magnitude  $\sim 8.0 \times 10^{-10} \text{m/s}^2$ , directed towards the Sun [11,12]. Turyshev [13] examined the constancy and direction of the Pioneer anomaly, and concluded that the data a temporally decaying anomalous acceleration  $-2 \times 10^{-11} \text{m/s}^2 \cdot \text{yr}$  with an over 10% improvement in the residuals compared to a constant acceleration model. Anderson, who is retired from NASA's Jet Propulsion Laboratory (JPL), is that study's first author. He finds, so "it's either new physics or old physics we haven't discovered yet." New physics could be a variation on Newton's laws, whereas an example of as-yet-to-be-discovered old physics would be a cloud of dark matter trapped around the sun. Now I introduce an exact solution for the Pioneer anomaly depending on my transformation and my equivalence principle. and the Hubble's law. According to my solution, there are two terms of decelerations that controls the Pioneer anomaly. The first is produced by moving the Pioneer spacecraft through the gravitational field of the Sun, and this deceleration is responsible for varying behaviour of the Pioneer anomaly in Turyshev [13]. And according to the principle of quantum superposition and Heisenberg uncertainty principle by the vacuum fluctuation [1-5], we find that the second term is depending on the Hubble's law which is equal to the Hubble's constant multiplied by the speed of light in vacuum.

This solution of the Pioneer anomaly and the flyby anomaly according to my transformation and my equivalence principle will give us the origin of the problem of dark matter and dark energy and thus the cosmological constant problem.

Sonnleitner [10] showed that how a simple calculation leads to the surprising result that an excited two-level atom moving through a vacuum sees a tiny friction force of first order in  $v/c$ . That is explained in my transformation as a result of translating the retardation in my transformation according to the invariance by entanglement which is leading to the wave-particle duality and the uncertainty principle by the vacuum fluctuation [1]. Also Schrodinger [18] showed that the emission of a light quantum by a (flying) atom is regulated by the conservation laws of energy and linear momentum. Therefore, the Doppler effect for photons is the consequence of the energy and momentum exchange between the atom and the photon: a central role is played by the quantum energy jump  $\Delta E$  of the transition (a relativistic invariant).

That is explained completely according to my transformation specially in my solution of Sagnac effect [1].

An empirical equation for the anomalous flyby velocity change was proposed by J. D. Anderson et al.

$$\frac{dV}{V} = \frac{2\omega_E R_E (\cos\varphi_i \cos\varphi_o)}{c} \quad (1)$$

where  $\omega_E$  is the angular frequency of the Earth,  $R_E$  is the Earth radius, and  $\varphi_i$  and  $\varphi_o$  are the inbound and outbound equatorial angles of the spacecraft [7,9]. One of the proposed solutions of the flyby anomaly is proposing a dark matter halo around Earth [8], which is the same as proposed in case of the Pioneer anomaly, but dark matter according to my transformation and my equivalence principle is explained and thus no need to propose dark matter.

## Theory

In my paper [1], I have reached to my new transformation

$$\begin{aligned} x &= \gamma^2(x' - vt') \\ t &= \gamma^2\left(t' - \frac{vx'}{c^2}\right) \\ y &= \gamma y' \\ z &= \gamma z' \end{aligned}$$

According to my transformation, space is invariant, and thus the speed of light of light is constant in the local classical vacuum and equals to  $c$ . Globally as a result of the retardation, the speed of light is not constant but fluctuates depending on the energy of the vacuum resulted by the retardation. Thus, globally we have the measured speed of light according to the phase given as

$$c' = \gamma^{-1}c$$

And the group velocity by the vacuum fluctuation

$$c' = \gamma^{-2}c$$

According to my equivalence principle [1]

$$\gamma^{-1} = \left(1 - \frac{GM}{c^2 r}\right)$$

which is depending on the gravitational potential. According to that, in the gravitational field we have the phase velocity globally

$$c' = \left(1 - \frac{GM}{c^2 r}\right)c$$

And the group velocity globally according to uncertainty principle by the vacuum fluctuation

$$c' = \left(1 - \frac{GM}{c^2 r}\right)^2 c$$

By my equivalence principle by translating the retardation in my transformation according to the invariance by the entanglement, I found the relativistic escape velocity of the free fall object under the gravitational field is given locally as

$$V_{escape-locally} = \sqrt{\frac{2GM}{r} - \frac{G^2 M^2}{c^2 r^2}} \quad (2)$$

Which is approximated in case of weak gravitational field to

$$V_{escape-locally} = \sqrt{\frac{2GM}{r}} \quad (3)$$

The escape velocity of the free fall object as observed globally is given according to two velocities, the phase velocity when we make a localization and in this case the phase velocity and the group velocity are equal which is equivalent to a motion in linear dispersion, and this in case we have

$$V_{escape-global-phase} = \left(1 - \frac{GM}{c^2 r}\right) \sqrt{\frac{2GM}{r} - \frac{G^2 M^2}{c^2 r^2}} \quad (4)$$

Now during the free fall, we have here a vacuum fluctuation, which is equivalent to motion in nonlinear dispersion, and in this case the uncertainty principle by the vacuum fluctuation plays the rule, where in this case even if we start with a fairly localized “particle”, it will soon loose this localization. According to that the group velocity of the free fall object under the gravitational field is not equal to the phase velocity, and in this case the group velocity is given according to

$$V_{escape-global-group} = \left(1 - \frac{GM}{c^2 r}\right)^2 \sqrt{\frac{2GM}{r} - \frac{G^2 M^2}{c^2 r^2}} \quad (5)$$

Which is approximated in case of weak gravitational field to

$$V_{escape-global-group} = \left(1 - \frac{2GM}{c^2 r}\right) \sqrt{\frac{2GM}{r}} \quad (6)$$

Which is the same equation derived from the Schwarzschild Geometry in case of weak gravitational for the free fall, but according to the Schwarzschild geometry this equation has no any physical meaning, because in reality it is in violation with the equivalence principle of Einstein, and also it is in violation with reality is observer independent according to Minkowski Geometry of space-time.

Sagnac effect can be explained according to my transformations by considering the t-term in my transformation.

$$t = \gamma^2 \left( t' - \frac{vx'}{c^2} \right)$$

If we considered  $t^- = \gamma^2 \left( t' - \frac{vx'}{c^2} \right)$  and  $t^+ = \gamma^2 \left( t' + \frac{vx'}{c^2} \right)$ , in this case we get

$$\Delta t = \gamma^2 \left( \frac{2x'v}{c^2} \right)$$

And since L is invariant and by considering  $x' = L$ , then we get

$$\Delta t = \gamma^2 \left( \frac{2Lv}{c^2} \right)$$

This result is exactly the same result which derived by Engelhardt [19] in explaining Sagnac effect in the framework of the ether theory and Galilean transformation, but instead according to my transformation, it is the relativistic ether from the point of view of quantum vacuum.

My transformation is transformation of field, quantized field, and thus the relativistic ether appears according to my transformation as observer dependent by the retardation same as in case of Unruh effect. That explains why Unruh effect is observer dependent, and thus according to my transformation and equivalence principle [1], that gives a full interpretation of inertia from an asymmetric Casimir effect.

The reference to the Doppler effect was only indirect (the experiments by Stark to the first order of  $v/c$ ) [18], and now it can be explained according to my transformation and my equivalence principle [1]. According to that for low velocities comparing to the speed of light which is equivalent to motion in weak gravitational field, the difference between the predicted frequency and the reference frequency  $\nu_0$  as the result of the red shift is  $\Delta\nu_{model}$  given as

$$\frac{\Delta v_{model}}{v_0} = \frac{V_{escape-locally}}{c} \quad (7)$$

Now by considering the observed frequency difference globally  $\Delta v_{obs}$  is depending on Eq. (6) in case of weak gravitational field according to my transformation and my equivalence principle, in this case we get

$$\frac{\Delta v_{obs}}{v_0} = \frac{\left(1 - \frac{2GM}{c^2 r}\right) V_{escape-locally}}{c} \quad (8)$$

Thus from Eqs. (7)&(8), and by substituting from Eq. (3)  $V_{escape-locally} = \sqrt{\frac{2GM}{r}}$ , we get

$$\frac{[\Delta v_{obs} - \Delta v_{model}]}{v_0} = -\left(\frac{2GM}{c^2 r}\right) \frac{\sqrt{\frac{2GM}{r}}}{c} \quad (9)$$

From eq. (9) we get the observed difference frequency is less than the predicted. That means there is a slight blue shift. According to the Pioneer team calculations, the observed, two-way anomalous effect by a DSN antenna can be expressed to first order in  $V/C$  as in [11]

$$\frac{[\Delta v_{obs} - \Delta v_{model}]_{DSN}}{v_0} = -\frac{2a'_p t}{c} \quad (10)$$

By DSN convention

$$[\Delta v_{obs} - \Delta v_{model}]_{usual} = -[\Delta v_{obs} - \Delta v_{model}]_{DSN}$$

Thus from that and from eq. (9) we get

$$-\left(\frac{2GM}{c^2 r}\right) \frac{\sqrt{\frac{2GM}{r}}}{c} = \frac{2a'_p t}{c} \quad (11)$$

By considering in Eq. (11)  $t = \frac{r}{c}$  we get

$$-\left(\frac{2GM}{c^2 r}\right) \frac{\sqrt{\frac{2GM}{r}}}{c} = \frac{2a'_p r}{c^2}$$

And from that we get

$$a'_p = -\frac{GM}{r^2} \frac{\sqrt{\frac{2GM}{r}}}{c}$$

Which is equal to

$$a'_p = -\frac{\sqrt{2}}{rc} (GM/r)^{3/2} \quad (12)$$

In Eq. (12), we find  $r$  represents the distance between the spacecraft and the Sun, and thus we find the deceleration of the spacecraft is depending on the distance of the spacecraft from the Sun according to my equivalence principle and my transformation by translating the retardation according to the invariance by the entanglement which is leading to the wave-particle duality and the uncertainty principle by the vacuum fluctuation. That's why Eq. (6) which can be derived from Schwarzschild Geometry in case of weak gravitational field has no any physical meaning in general relativity of Einstein GR, where in this case it is in violation with the equivalence principle of Einstein. Also GR can't even describe the gravitational field in case of strong gravitational field because in case of strong gravitational the escape velocity must be defined as relativistic as defined in my equivalence principle and my transformation, not classical as defined in case of Lorentz transformation and the equivalence principle of Einstein in GR.

Now by considering  $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ ,  $M = 1.99 \times 10^{30} \text{ kg}$  are respectively the gravitational constant and the mass of the Sun. Nasa data [13] show that in the very middle part (1983-1990) of the whole observation period of Pioneer 10, its radial distance from the Sun changes from  $r \cong 28.8 \text{ AU} = 4.31 \times 10^{12} \text{ m}$  to  $r \cong 48.1 \text{ AU} = 7.2 \times 10^{12} \text{ m}$ . Thus by computing  $a'_p$  from Eq. (12), we get  $a'_{10} = -1.8 \times 10^{-10} \text{ m/s}^2$  and  $a'_{10} = -0.52 \times 10^{-10} \text{ m/s}^2$ .

We have seen that the deceleration of the pioneer 10 anomalies is decreased depending on the distance from the Sun as from Eq. (12) according to my equivalence principle and my transformation [1], and that what is causing the varying behavior of the Pioneer anomalies according to Turyshev [13]. According to the period of observation 7.5 years from (1983-1990) as noted by Anderson [11], we find for the Pioneer 10  $\dot{a}'_{10}$  is given as

$$\dot{a}'_{10} = \frac{0.52 \times 10^{-10} - 1.8 \times 10^{-10}}{7.5} = -1.8 \times 10^{-11} \text{ m/s}^2 \cdot \text{yr}$$

Markwardt [14] obtained an improved fit of Pioneer 10 data when estimating a jerk of  $\dot{a}'_{p-10} = -1.8 \times 10^{-11} \text{ m/s}^2 \cdot \text{yr}$  which is exactly same as in my calculations. Also Toth [15] obtained  $\dot{a}'_{p-10} = -2.1 \times 10^{-11} \text{ m/s}^2 \cdot \text{yr}$  which is in full agreement with my calculations.

We find Eq. (6) which can be derived also from Schwarzschild geometry in case of weak gravitational field in GR can account exactly for varying behavior of the Pioneer anomaly depending on the distance of the spacecraft from the Sun depending on the gravitational potential of the Sun, but this equation has no any physical meaning in GR, because in reality if we consider that in GR, then that will be in violation with the equivalence principle of Einstein and the independent reality resulted from Minkowski geometry of the space-time in relativity of Einstein.

Now there is another term must be added to the Pioneer anomaly in Eq. (12) according to the principle of the quantum superposition in my equivalence principle and my transformation by translating the retardation depending on the Heisenberg uncertainty

principle by the vacuum fluctuation. This term is related to the Hubble's law. We have from Hubble's law this acceleration is given according to the equation

$$a_H = Hc \quad (13)$$

Where  $a_H$  is the deceleration is caused by the Hubble, where is this case since the spacecraft is going far away from the Sun, in this case it is observed for an observer on ground, there is a slight blue-shift given according to the Eqs. (12)&(13). If the spacecraft is in a free fall toward the Sun, in this case, it will be observed a slight red-shift which is given also according to Eqs. (12)&(13), and that in reality explains the Hubble's law which is leading to solve the cosmological constant problem by solving the problem time in physics according to my transformation by translating the retardation according to invariance by the entanglement.

According to that we get the full Pioneer anomaly is given according to

$$a_p = -Hc - \frac{\sqrt{2}}{rc} (GM/r)^{3/2} \quad (14)$$

An estimate of the Hubble constant, which used a new infrared camera on the Hubble Space Telescope (HST) to measure the distance and redshift for a collection of astronomical objects, gives a value of  $H = 73.8 \pm 2.4$  (km/s)/Mpc or about  $H = 73.8 \pm 2.4$  (km/s)/Mpc [16,17]. Thus from Eq. (40) we get for the Pioneer 10 at distance  $r = 28.8$  AU or after 11 years of lunch

$$a_{10} = -a_H - a'_{10} = -7.20 \times 10^{-10} - 1.87 \times 10^{-10} = -9.07 \times 10^{-10} \text{ m/s}^2$$

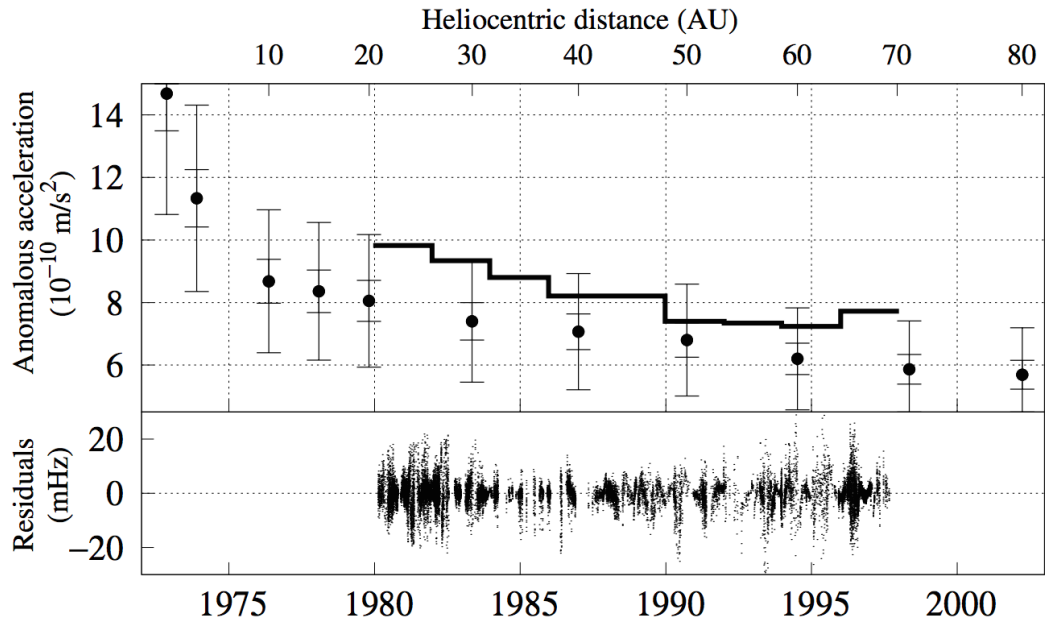
This quantity is in complete agreement with the observed Pioneer 10 acceleration ( at  $t=11$  years of lunch), in Fig. (1) taken from Turyshev [13].

At a distance  $r = 48.1$  AU at  $t = 18$  years of lunch, we get

$$a_{10} = -7.20 \times 10^{-10} - 0.52 \times 10^{-10} = -7.72 \times 10^{-10} \text{ m/s}^2$$

This quantity is in complete agreement with the observed Pioneer 10 acceleration ( at  $t=18$  years of lunch), in Fig. (1) taken from Turyshev [13].

We find from my transformation by translating the retardation according to the invariance by the entanglement which is leading to the wave-particle duality and the uncertainty principle by the vacuum fluctuation, the gravitational field is expressed according to the energy fluctuation, the vacuum energy fluctuation effectively gives a correct explanation of dark energy and dark matter, where in this case dark matter and dark energy are explained , and that will provide a solution to the cosmological constant problem. Figure (2) illustrates the predicted Pioneer 10 anomaly according to Eq. (14) which gives an exact solution of the Pioneer anomaly.



*Fig. (1):* Comparison of the thermally-induced and anomalous accelerations for Pioneer 10. The estimated thermal acceleration is shown with error bars [13].



$$H = 73.8 \pm 2.4 \text{ (km/s)/Mpc}, \quad a_p = Hc + \frac{\sqrt{2}}{rc} (GM/r)^{3/2}$$

$a_{10} (\times 10^{-10} \text{ m/s}^2)$ ,

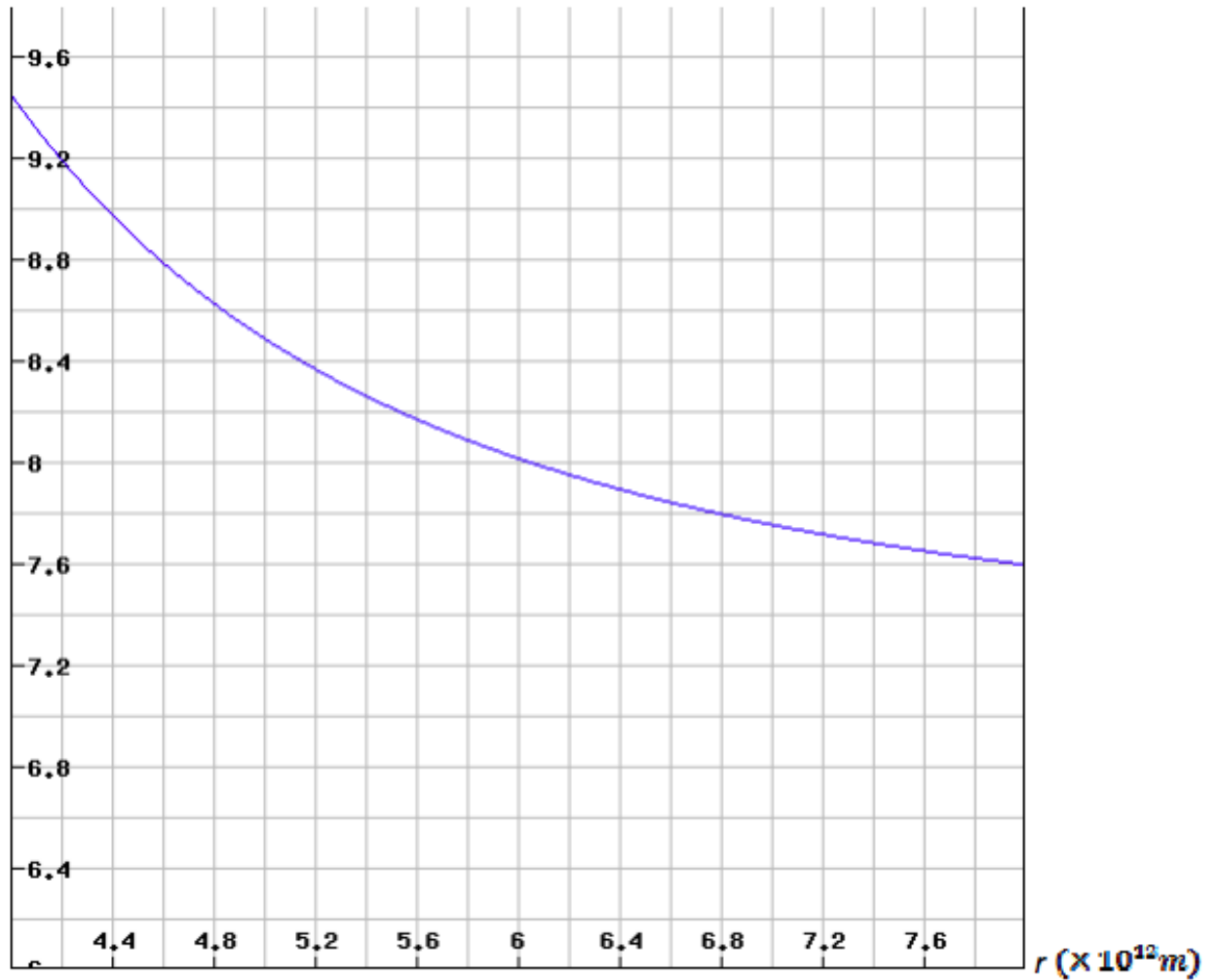


Fig. (3), the predicted Pioneer 10 anomaly versus distance from the Sun according to my solution.

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