Taken ABaCk by Conjecturing Out-of-Box

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ABSTRACT

Results as diverse as the ABC conjecture and the PNT could be but select areas that can be shown to have stemmed from a common domain—which additionally accounts for an inherently fuzzy line between addition and multiplication, linearity and nonlinearity of operators, and the striking simplicity underlying the broader (indeed complete) host of operations at large.

Keywords: ABC conjecture, coprimes versus prime basis, orduality and residuality, pi-sigma calculus

Making a Question into an Answer

To begin with, one should be able to appreciate just how handy the [otherwise self-serving] meta-notion of *trilinearity* will come in throughout. It will, for one, refer to a 'three-line' demonstration or schema for approaching the inexorably involved problem (which has reportedly begotten entire new [Teichmuellerian?] areas) so as to render it unbearably simple albeit allowing for multiple extensions and bridges (as ever). This manner of having the problem caught right- and left-handed (which is to say complete as well as simple in contrast to partial expositions tending to induce undue arcaneness) could be seen as a three-stroke approach, or indeed a third-strike criterion, to draw upon the legal analogy in addressing Type I and Type II errors by arguing that, too many findings concurring and pointing in largely the same direction might indeed be revealing essentially the same—credibly or cogently so. For that matter, trilinearity will, somewhat more literally, posit the special part that powers anywhere near 3 play in blurring the gap between linearity (additivity as well as products) and non-linearity (multiplicity as well as powers). Other than these implications in hindsight (yet to be unearthed), the notion may or may not have some substantive or anything-but-tenuous bearing on the way the [special] ABC conjecture is approached—the same going for the interim calculi that are only covered in passing.

Sketching the Grand Problem—or Is It but a Very Special Case?

Insofar as *coprimes* are formalized as proposed in the Appendix, it can be argued that, for any such numbers X and Y,

$$(1)\ rad(XY) = rad(X) * rad(Y)$$

By straightforward induction,

$$(1.1)\ rad(ab[a+b]) = rad(ab) * rad(a+b) = rad(a) * rad(b) * rad(a+b)$$

A [largely operational] notion of *quasi-derivative* will be introduced early on, which tends to the conventional or functional derivative for large values of ***a*** while only appearing aimed at being rendered irrelevant ultimately (in line with the *orduale* premises whereby the complete is reasonably simple, and the unknown or indefinite is of far lesser importance):

$$(A)\ rad'(a_b) \equiv \frac{rad(a+b) - rad(a)}{b}$$

In this notation, a_b refers routinely to a *b*-neighborhood of *a*. By substituting the extended [*(a+b)*-*a*] form for the *b* differential and coupling the related terms, it is straightforward to see that,

$$(A')\ (a+b) * rad' = rad(a+b)\ iff\ a * rad' = rad(a)$$

This conditional coupling, routinely abused despite its ambivalent merit, can be relaxed by applying non-unity operators *(p, q)*, as will be qualified later in text. For now, though, some of the core results can safely fare on these joint or contingent premises (which are *orduale* or *residuale* in their own right) in ways that can be acid-tested for the singular (m=1)—whether prime or *prime-basis* (as opposed to *co*-prime)—case ushering in identities in place of equivalences. Needless to say, the aforementioned operator is itself *identity-based*, which is one residuale way of inducing tentatively valid inference.

It immediately follows from the above (and will informally be invoked later on) that

$$(1.2)\ \frac{a+b}{rad(a+b)} = \frac{a}{rad(a)} = rad' = \frac{x}{rad(x)}\ \forall x$$

Alternatively, the very initial definition or identity can be solved as a functional equation, such that

$$(1.3)\ rad(x) = 1^{\frac{x}{b}} + (x-1) * rad'$$

The implied invariance can be put in terms of a parity, with the unity terms (though rigorously treated as complex decompositions) assumed (and effectively posited as a special case) just that—unity values. Alternatively, it can likewise be shown that full-fledged complexity borders on nonlinearity.

$$(1.4)\ rad' = \frac{rad(a+b)-1}{a+b-1} = \frac{rad(a)-1}{a-1} = \frac{rad(b)-1}{b-1} = \frac{rad(x)-1}{x-1}$$

Although (1.4) may appear slightly at odds with (1.2), the two converge for large underlying primes (referring to the prime *basis* as opposed to powers with respect to which the *rad* operator is invariant). First, (1.4) can be deployed to arrive at the extended equivalence likely underpinning the ABC conjecture:

$$(1.5)\ (a+b) = 1 + \frac{rad(a+b)-1}{rad(b)-1} * (b-1) \geq rad(ab[a+b])$$

To show that the right-hand side is the case, consider *large* values, such that the left-hand side can be approximated as follows:

$$(1.5')\ (a+b) \geq rad(a+b) * \frac{b}{rad(b)}$$

Equivalently, and fully in line with (1.2) as well as approximately so with (1.4),

$$(1.6)\ \frac{(a+b)}{rad(a+b)} \geq \frac{b}{rad(b)} \sim \frac{a}{rad(a)} \sim \frac{x}{rad(x)}$$

Consequently, the latter term could be captured (if only as a special case) in terms of an arbitrary gamma elasticity-based tossup of the other two (or indeed the rest), with gamma at ½ being one possibility:

$$(1.7)\ \frac{(a+b)}{rad(a+b)} \geq \frac{a^{\gamma} b^{1-\gamma}}{[rad(a)]^{\gamma}[rad(b)]^{1-\gamma}} \xrightarrow[\gamma \equiv 2]{} \frac{(ab)^{1/2}}{[rad(a) * rad(b)]^{1/2}}$$

Again, for the coprime case, this reduces to,

$$(1.7')\ \frac{(a+b)}{rad(a+b)} \geq [\frac{(ab)}{rad(ab)}]^{\frac{1}{2}}$$

However, a similar result obtains immediately by putting *x*=*ab*:

$$(1.7'')\ \frac{(a+b)}{rad(a+b)} \geq \frac{x}{rad(x)} \xrightarrow[x \equiv ab]{} \frac{(ab)}{rad(ab)}$$

One way of reconciling the two results could be by invoking either large numbers or small powers (anywhere around unity, with 2 and 3 standing out as cases of utmost importance):

$$(1.8)\ \frac{(a+b)}{rad(a+b)} \geq \frac{x}{rad(x)} = [rad(x)]^{m-1} = x^{1-\frac{1}{m}}$$

$$(1.9)\ [\frac{ab}{rad(ab)}]^{\frac{1}{2}} = [rad(ab)]^{\frac{m-1}{2}} \sim \frac{x}{rad(x)} \sim \frac{ab}{rad(ab)} = [rad(ab)]^{m-1}$$

By combining (1.8) and (1.9), it obtains that either

$$(1.10)\ (a+b) \geq rad(a+b) * [rad(ab)]^{\frac{m-1}{2}}$$

or,

$$(1.10')\ (a+b) \geq rad(a+b) * [rad(ab)]^{m-1}$$

Whilst both seem consistent with the ABC conjecture for m hovering around 3 (or 2 in the latter representation), of special importance is the *m=1* case, which reduces it to a *prime basis*, or *quasi-prime* setup (PB or QP as opposed to *co*-primes CP) ushering in self-identity for *(a+b)*. To remind:

$$rad(x+y) * rad(xy) = rad(xy[x+y])\ \ \forall a, b \in\ CP$$

$$rad(x) = x\ \forall x \in QP$$

As ever, the *large m* instance stands alone while likewise fully satisfying both. This completes the demonstration sketch.

Heuristic Support

The above result could be seconded from a number of alternative standpoints. First, the ABC conjecture appears to pass the dimensionality check:

$$(a+b) \geq rad(ab[a+b])$$

$$L \sim L^{\frac{1}{m_a}} L^{\frac{1}{m_b}} L^{\frac{1}{m_{a+b}}} \xrightarrow[m_x \to 3]{} L$$

As one alternative to this m=3 case, power-*invariance* could be invoked by directly drawing on the nature of the rad operator without imposing any restrictions on the underlying values save that the RHS *product* cannot possibly be a *prime* thus yielding no [absurd] case of, say, *ab* totaling near unity. In other words, the quasi-derivative can never take on a unity value for any products (other than those pertaining to the *prime-basis* or m=1 case without the primes products having to collapse to a prime value).

Not least, the LHS and RHS of the ABC conjecture could be assessed by presuming that *a* and *b* refer to the minimum versus maximum values respectively, as follows:

$$LHS \equiv (a+b) \in 2(a.b), \qquad RHS \equiv rad(ab[a+b]) \in 2[rad(a), rad(b)]$$

It is straightforward to appreciate that the LHS generally falls outside the RHS range, with near-full overlap accruing to the strong, prime-basis setup (featuring implied m=1).

Twin & Twixt Multiplicity versus Additivity

A host of peripheral yet suggestive results could be proposed along the lines of reconciling linear and non-linear representations. For starters, based on the parities, the underlying values and their rad operators could be seen as either lacking any independent substance (which finding is consistent with orduale residuality as well as the *inherently Diophantine* nature of primes, as will be shown in further expositions).

$$(2.1)\ rad(a) = \frac{a-1}{b-1} * rad(b) + \frac{b-a}{b-1}$$

$$(2.2)\ rad(a+b) = \frac{a}{a-1} * [rad(a) - 1] + rad(b)$$

$$(2.3)\ \frac{rad(a+b) - rad(a)}{rad(a+b) - rad(b)} = \frac{b}{a}$$

From considering the fully blown unity-decomposition cases (albeit without expanding them beyond reconciliation around 1), it follows that:

$$(2.4)\ [\,]_{a+b} = [\,]_a^a * [\,]_b^{b-1}, \qquad [\,]_x \equiv rad(x) - rad' * (x-1)$$

In fact, the latter alone refers back to the initial quasi-derivative structure, to suggest:

$$(2.5)\ rad' = rad(a+1) - rad(a)$$

$$(2.6)\ rad' * x = rad(a+x) - rad(a) = x * [rad(a+1) - rad(a)]$$

$$(2.7)\ rad(y) = \frac{rad(y+x) - x * rad(y+1)}{1-x} = rad(y+1) + \frac{rad(y+x) - rad(y+1)}{1-x}$$

$$(2.8)\ \frac{y}{x} = \frac{rad(a+y) - rad(a)}{rad(a+x) - rad(a)}$$

By reconciling (2.4) against (2.5) through (2.7), the non-linear counterpart could be boiled down to,

$$(2.9)\ 2rad(a+b) - rad(2[a+b]-1) = [2rad(a) - rad(2a-1)]^a$$
$$= [2rad(b) - rad(2b-1)]^{b-1}$$

In all of the above instances, the trivial identity obtains as a prime [basis] check. Even so, the very basic special values may lack independent or '*cardinale*' nature, even as they reinforce the unity power irrelevance (i.e. essential linearity) as postulated from the outset without discarding the more general setup. This can be deduced from solving (2.6) as a dual functional equation wherein either side can be exogenized in the interim:

$$rad(a) = 1^{\frac{a-1}{x}} * rad(1) + (a-1) * [rad(a+1) - rad(a)]$$
$$= 1^{a-1} * rad(1) + \frac{a-1}{x} * [rad(a+x) - rad(a)]$$

Since direct comparison of the latter terms in the RHS's leads up to (2.6), the same should go for the former terms (being equivalent thereby). This points to unity power invariance, with *a=0* suggesting the entangled natures of the basic rad values:

$$(2.10)\ rad(0) = \frac{1}{2} * [rad(1) + rad(-1)]$$

Although the values of 0, +1 and -1 might appear plausible as special [*cardinalcy*] conventions, this is but one arbitrary way of reducing the inherently intertwined nature of the values in question.

As proposed from the outset, though, a strong-form parity as posited by (1.4) might not appear very plausible, if only because a varying *m* power would likely result in varying ratios.

While this might be less of an issue for coprimes (the Appendix showing in what ways they are complements or duals with the implied, averaged, or effective power being comparable), still one might want to consider a relaxed case other than the one building on direct, like-for-like coupling of terms. In other words, assume:

$$(B)\ (a+b) * rad' \equiv p * rad(a+b), \qquad a * rad' \equiv q * rad(a)$$

The resultant parity would generalize (1.2):

$$(1.2')\ [rad']^{-1} = \frac{(a+b)}{rad(a+b)} * \frac{1}{p} = \frac{a}{rad(a)} * \frac{1}{q} = \frac{b}{rad(a+b) - rad(a)}\ , \qquad p \neq q \neq 1$$

The latter term is invariant vis-à-vis the operator or coupling convention attempted and could be seen as the more reliable core of analysis. However, since the other two render it indirectly contingent thereon, it could be reduced to either of them, e.g.:

$$(1.3)\ [rad']^{-1} = \frac{b}{rad(a+b)[1 - \frac{p}{q} * \frac{a}{a+b}]} = \frac{b}{rad(a)[\frac{q}{p} * \frac{a+b}{a} - 1]}$$

By juxtaposing the LHS's of (1.2') and (1.3), it follows that,

$$(a+b) = \frac{pb}{1 - \frac{p}{q} * \frac{a}{a+b}}$$

The above collapses to trivial identity under the former *p=q=1* case (a similar outcome accruing on *b* being very small amidst its asymmetry being evident against *a*, and likewise for *p* versus *q*).

As will be reiterated in passing, the RHS of *(1.2')* fits into (1.2) amidst *a* nearing zero while maintaining *rad (0)=0*. Among other possibilities, the resultant uncertainty could be handled *a la* L'Hopitale whilst ushering in the patterns below (as one way around *unity* adjustment operators) as per the prime-basis (m near 1) setting:

$$[rad'(a_b)]^{-1} \underset{m\to 1}{\longrightarrow} rad'(a_b) = \frac{a}{rad(a)} * \frac{1}{q} \underset{a\to 0}{\longrightarrow} \frac{a'}{rad'(a_b)} * \frac{1}{q}$$

$$\lim_{a\to 0} rad'(a_b) = \frac{1}{\sqrt{q}}$$

$$\lim_{b\to 0} p = q$$

$$\lim_{b\to 0} rad'(a_b) = rad'(a_b) * \frac{q}{p}$$

$$\lim_{a,b\to 0} rad'(a_b) = \frac{1}{\sqrt{p}}$$

This may not appear to stand to scrutiny—perhaps because the latter term of the *(1.2')* parity was based on *(A')*, whereas *(B)* would suggest,

$$a * rad' \equiv q * rad(a) = p * rad(a+b) - b * rad'$$

Intriguing (and possibly valid as a kind of 'non-Cartesian' extension) as this $rad' = \frac{p*rad(a+b)-q*rad(a)}{b}$ implied generalization might promise to be, it proves at odds with the very original definition of the quasi-derivative, as if to imply that the *p* and *q* operators are identically unity.

This shortcut impossibility spares the bulk of effort. However, should one be interested in toying with the relaxed parity and the implied relationships, the gravest concern would perhaps rest with whether or not the *q/p* ratio is so finite (compressed) as to keep the entire parity terms well-behaved beyond the weak dimensionality check (with this ratio being dimensionless and hence unlikely to affect the prior test). In other words, of special interest could be a rethinking of the *ab=1* case insofar as it (*inter alia*) captures as well as expands on the conventional derivative which builds on *a/b* laden discontinuity (*b* being very small compared to *a*) thus paradoxically questioning the well-behaved or finite-ratio outcomes. Again, though, bearing in mind the nature of the values (coprime), this frontier of tradeoffs appears altogether irrelevant—as does any alternative to the special q=p=1 case as imposed at the outset.

That said, by trying to reconcile the two alternate representations of the 'characteristic slope,' general and special, one could arrive at one other parity frontier:

$$\frac{rad(a+b)}{rad(a)} = \frac{q-1}{p-1} = \frac{q}{p} * \frac{a+b}{a}$$

It may for one appear that, for the *p=q* case, the initial parity obtains. However, as per the *p=q=1* restriction, one could further surmise,

$$\frac{\partial rad(a+b)}{\partial rad(a)} = \frac{a+b}{a}$$

if and only if *b* is held constant or exogenous (e.g. the way an independent-variable differential is treated under a conventional derivative—a dual case of variations). One other way of showing the implied local linearity (or homogeneity 1) of the operator would be to check,

$$\partial rad(ka) = \frac{ka}{a} * \partial rad(a)$$

In any event, more generally,

$$\left(1+\frac{b}{a}\right)*\left(1-\frac{1}{p}\right)=1-\frac{1}{q}$$

As q tends to 1, so does p—unless b is very large compared with a. By contrast, for b small *or* p, q large (or both):

$$\frac{1}{p}-\frac{1}{q}\sim\frac{b}{a}$$

To rehash on this, consider again:

$$(1.2')\ [rad']^{-1}=\frac{(a+b)}{rad(a+b)}*\frac{1}{p}=\frac{a}{rad(a)}*\frac{1}{q}=\frac{b}{rad(a+b)-rad(a)}\ ,\qquad p\neq q\neq 1$$

As pointed out previously, the RHS of *(1.2')* could be reconciled to its counterpart as of (1.2) by merely holding a anywhere near zero while assuming *rad (0)=0*. At this rate, however, either *p=1* is implied (the same holding for q) or both are uncertain *a la* L'Hopitale. In this event:

$$[rad']^{-1}\equiv[rad'(a)]^{-1}=\frac{a}{rad(a)}*\frac{1}{q}\xrightarrow[a\to 0]{}\frac{a'}{rad'(a)}*\frac{1}{q}$$

Which implies *q=1*, the same going for p from comparing,

$$\frac{1}{p}*\lim_{a\to 0}\frac{(a+b)}{rad(a+b)}=\lim_{a\to 0}\frac{b}{rad(a+b)-rad(a)}$$

Zooming in on Operational Linkage: Beyondness as Simplicity

Consider taking one step further beyond the special conjecture and toward the underlying linkage between addition and multiplication (or, more generally, across operations, as will be proposed in the next section) by first considering the singular case featuring a degenerate basis:

$$\{p_k\}\to p,\sum_{k}^{K=n}p_k\to np\equiv\Sigma,\ \prod_{k}^{K=n}p_k=p^n\equiv\prod$$

One should have no difficulty showing that,

$$(3)\ \Sigma^n=n^n\prod$$

$$\prod^{\frac{1}{n}}=\frac{\Sigma}{n}$$

$$\prod\Sigma=\Sigma^n\prod$$

Interestingly, the latter suggests a striking glimpse at the long-offered (Shevenyonov, 2016c), utterly general *ordual calculus* premises:

$$(A,a)^{\rho-1} \sim (a,A),\ (A,a)^{\rho} = (a,A)^{\frac{\rho}{\rho-1}} = (X,X)$$

Now substitute *x* for *n* for generality and consistency's sake. As long as the sum is tantamount to the product (which could be seen as an *n>2* generalization of duality, $\rho * \frac{\rho}{\rho-1} \equiv \rho + \frac{\rho}{\rho-1}$), it follows that:

$$\sum^{x} = x^{x}\prod = x^{x}\sum$$

Unless the sum (or indeed the product) is unity, zero, or very large in absolute terms,

$$(3.1)\ \sum^{x-1} = x^{x}$$

Now, since $\sum = xp = p * x$, the [singular] basis can be inferred at, $p = x^{\frac{1}{x-1}}$. This inter-linkage between sums and products, or linearity and non-linearity can now be depicted as,

$$(C)\ px = p^{x}$$

again, as a generalization of $p + x = px$.

It obtains tentatively that,

$$(3.2)\ \sum^{x} = x^{\frac{x^2}{x-1}}$$

$$(3.3)\ \sum^{x-1} = x^{x} = [\sum^{1-x}]^{-1}$$

$$(3.4)\ \sum^{x+1} = x^{\frac{x^2+x}{x-1}} = \sum^{x} * \sum^{1/x}$$

$$(3.5)\ \sum^{x\pm m} = \sum^{x} * \sum^{\pm m/x}$$

$$(3.6)\ \sum^{mx} = \sum^{x+(m-1)x} = \sum^{x} * \sum^{m-1}$$

$$(3.7)\ \sum^{x^m} = \sum^{mx} * [\sum * \sum^{\frac{x^m-1}{1-x}}]$$

$$(3.8)\ \sum^{\varphi(x)} = \sum^{kx} * [\sum^{\frac{\varphi(x)}{x^k}} * \sum^{\frac{x^k-1}{1-x}}] \sim x^{-\frac{1}{x}}\ \forall k, \varphi(*)$$

$$(3.9)\ \sum * \sum^{\varphi} = \sum^{x} * \sum^{\varphi/x} \underset{\varphi\to 1}{\longrightarrow} \sum^{x+1}$$

$$(3.10)\ \sum^{mx} \underset{m\to 0}{\longrightarrow} \sum^{x} * \sum^{-1} \sim \sum^{\varphi} * \sum^{-\varphi/x}$$

$$(3.11)\ \sum^{mx(x-1)} = \sum^{x^{-m}}$$

Of course, one might have wished for a more generalized, perhaps identity-based setup bearing on the fudge factor:

$$(C')\ \sum^{x} \equiv kx^{x}\prod = kx^{x}\sum$$

$$\sum^{x-1} = kx^{x}$$

At this rate, the invariant (albeit perhaps not necessarily one with respect to *x*, hinting at *k=k(x)*) could be tantamount to the [special-case] basis,

$$\sum^{x} * x^{-x} = x^{\frac{1}{x-1}} = p = k\prod$$

in which light,

$$k = k(x) = p^{1-x} = \frac{1}{x}$$

$$\sum = x, \qquad \prod = \sum^{-x}$$

Needless to say, this only holds under the two versions of the *pi-sigma calculus* married, which additionally presupposes unity sums, products, and basis for that matter—even though the unit operators have exhibited some nontrivial patterns and relationships throughout. Moreover, all of the above structures might resemble the [extended-form] Gamma-like patterns:

$$\Gamma(n+1) \sim n! \sim \int_0^{\infty} x^{n} e^{-x}\, dx$$

The underlying product could be assessed at, $x^{n}e^{-x} \sim \Gamma(n+1) - \Gamma(n) \sim (n-1)\Gamma(n)$

One alternative way around the issue could be to approach values by exponentiating them around an *x average*:

$$x_i \equiv x * e^{\Delta_i}, x = \frac{1}{n} * \sum_{i}^{n} x_i$$

$$\prod_{i}^{n} x_i \equiv \prod = x^{n} * e^{\sum_i^n \Delta_i} = n^{-n} * (x * \sum_{i}^{n} e^{\Delta_i})^{n}$$

$$(4)\ n^{n} e^{\Sigma\Delta} = \sum^{n} e^{\Delta}$$

$$\sum e^{\Delta} = \frac{e^{\frac{1}{n}\Sigma\Delta}}{\frac{1}{n}}$$

The above in differentials does resemble the [special-case] pi-sigma basis in levels (3) as above. On the other hand, under an *n* large, the linkage between linearity and nonlinearity is straightforward:

$$(4.1)\ \lim_{n\to\infty} \sum e^{\Delta} = \sum \Delta$$

By induction, it could be surmised for higher-order differences:

$$(4.2)\ \ n^n e^{\sum \Delta^k} = \sum^n e^{\Delta^k}\ \forall k$$

To discern any further utility, consider ways in which the *prime numbers asymptotic distribution* (PNT) could be inferred from the above considerations (if only insofar as the above pi-sigma notations may apply to *prime* products and sums much the way they do to *any* values—let alone this would bridge the gap between the ABC and the distant yet entangled areas).

As before, the values will be assessed in terms of their underlying logarithmic differentials, with the particular threshold actually referring to a *maximum* prime. In this case, the number of terms will pertain to that of the prime values in the implied distribution:

$$\pi(x_{max}) \sim \frac{x_{max}}{\log(\mathrm{x_{max}})} \sim K = n$$

In essence, it shall be maintained that the desired number of primes (as per a particular nature or value of the implied maximum) is *n*—a kind of backwards induction or dual stance.

Suffice it to gauge whether:

$$\frac{n}{x} * e^{-\Delta_{max}} \sim (logx + \Delta_{max})^{-1}$$

Based on (4) and (4.2), it should be self-explanatory that $\sum^n e^{\Delta} < \sum^n e^{\Delta_{max}}$. Therefore,

$$n > \frac{\sum \Delta}{\Delta_{max}} = \frac{\sum \Delta}{logx_{max} - logx}$$

By invoking $\sum\Delta = \sum logx_i - nlogx$, check whether

$$n > \frac{\sum logx_i}{logx_{max}}$$

In other words, with an eye on the exact same denominator, the entire scrutiny has come down to comparing the numerators, i.e. control if

$$\sum logx_i < x_{max}$$

At any rate, the LHS will fall short of $n * logx_{max}$ which, bearing in mind the original distribution as a matter of fact, suggests

$$\sum logx_i < n * logx_{max} \sim x_{max} \;\leftrightarrow\; \sum logx_i < x_{max} \;\; QED$$

Alternatively, the above amounts to comparing,

$$T < x^n \; vs. T' \equiv e^x$$

As has been noted before, this ratio amounts to just under, $(n-1)\Gamma(n)$, or above unity for $n>2$ (evidently a weak prerequisite for any asymptotic regularity ever to hold—let alone for an operation like sum or product to be well-defined or at all applicable).

This completes the demonstration of how the standardized result follows from the proposed approach of featuring the linkage between operations. Better yet, it can be shown that:

(A) Both ABC and prime distribution lend themselves to shared origins
(B) The latter amounts to so much as a special case of the pi-sigma calculus without having to embark on any utter generality—which latter will, too, be suggested (Shevenyonov, 2016q)

It is natural to start with the aforementioned (4) through (4.2) *equivalence* (indeed, in line with the identity based approach) as opposed to an incomplete inequality as above:

$$n \equiv \frac{\sum e^{\Delta^k}}{e^{\frac{1}{n}\sum \Delta^k}} < \frac{\sum \Delta}{\Delta_{max}}$$

Likewise, for k=1, it holds that

$$n \sim \frac{\sum (x_i/x)}{(\prod^{\frac{1}{n}} x_i)/x} = \frac{\sum}{\prod^{\frac{1}{n}}}$$

Evidently, this refers back to (3), which pi-sigma premises could be seen as the ultimate object behind the asymptotic prime distributions, as hypothesized above.

Outlook on Operations as Unspecified Objects: Beyond the Divide

The present analysis of the long-standing problems offers but an early glimpse of the broader yet simpler perspective, as has been proposed in my previous expositions. In the forthcoming papers, it will be proposed that operations confront no binding borderlines, in the first place.

This attempt is dedicated to the late (awkward and preposterous as this state might sound) Mariam Mirzakhani, myself poised to come to learn from and fully appreciate her work one day. It is unfortunate that the same cannot carry over to the contributions by Teichmueller, for reasons we all know full well. I might never be in a position to even consider this alternative (or is it an emerging core yet?) area, unless specifically challenged to, much to my own regret and due to that other tragic lot being of a different and more discretionary nature than the former case.

References

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APPENDIX

Any pair of candidate coprimes could conveniently be formalized by building on what can be dubbed a *prime basis* $PB \sim \{p_k\}$ alternatively denoted in terms other than vector or set-theoretic as:

$$PB \equiv \prod_{k}^{\infty} p_k$$

All of the individual p_k in this *value potential* are primes whose set could be complete or boundless, even though it may effectively be restricted to a particular upper bound power M beyond which any applicable powers in the [Boolean-filtered] number structures below would identically be zeros:

$$m_k = m_k * (m_k < M)$$

Alternatively, it is the k that is effectively confined to a desired threshold:

$$X \equiv \prod_{k} p_k^{m_k}, \;\; m_k \equiv 0 \;\; \forall k > K$$

Since coprimes are characterized by an inherently interdependent or dual nature, the implied *power-conjugates* could be thrown in to arrive at a zero power in the respective basis position of the one number anytime its counterpart in the other is nonzero:

$$\frac{a}{b} \sim \frac{X_i}{X_j} = \frac{\prod_k p_k^{m_{ki}}}{\prod_k p_k^{m_{kj}}} = \prod_{k} p_k^{m_{ki}-m_{kj}} \equiv \prod_{k} p_k^{\Delta m_k}$$

$$m_{kj} \equiv 0 \; iff \; m_{ki} \neq 0 \;\; \leftrightarrow \;\; \Delta m_k = \pm m_k \;\; \forall k = \overline{1, K}$$

It is straightforward to see that the prime basis obtains as the rad of a product (featuring a full-fledged basis as a combination of coprime *semi-bases*):

$$PB = rad(ab) = rad(a)rad(b) = rad(\prod_{k} p_k^{m_{ki}} * \prod_{k} p_k^{m_{kj}}) = rad \prod_{k} p_k^{m_k} = \prod_{k} p_k$$

The working convention is that a zero effective power amounts to an omitted respective p_k layer of the basis in the semi-basis—in contrast to any nonzero exponent. On the other hand, this calls for value-*specific prime basis*:

$$PB_i * PB_j = rad(X_i) * rad(X_j) = PB$$

Needless to say, a prime *basis* need not amount to a prime [*number*], if only because *K* may not be tantamount to unity (it will be shown how prime singularity has similar as well as divergent implications compared to the more general prime-basis case with reference to the effective power).

What is more, lest the coprime ratio might collapse to a prime [number], some sign alteration would be warranted:

$$\exists(k, \Delta k): sign\Delta m_k = -sign\Delta m_{k+\Delta k}$$

While at it, please note:

$$a = (ab * \frac{a}{b})^{\frac{1}{2}} = \prod_k p_k^{\frac{\mu_k}{2}},\ \ \mu_k = \begin{cases} 2m_k \\ 0 \end{cases}$$

$$b = (ab / \frac{a}{b})^{\frac{1}{2}} = \prod_k p_k^{\frac{\mu'_k}{2}},\ \ \mu'_k = 2m_k - \mu_k$$

It is straightforward to see that $rad(xy) = rad(x)rad(y)$ holds for coprimes only (as above) whereas, for lack of conjugacy, the more general (independent or *cardinale*) notion of values, even as one might be based on the very same premises, would suggest:

$$rad(ab) = rad\prod_k p_k^{m_{ki}+m_{kj}} = \prod_k p_k \equiv PB \neq rad\left(\prod_k p_k^{m_{ki}}\right) * rad\left(\prod_k p_k^{m_{kj}}\right) = \prod_k {p_k}^2 \equiv PB^2$$

At this stage, for ease of notation as well as manipulation (or indeed to circumvent the intricacies of vector operations and equivalence of the form, ${X_i}^{\{m_i\}^{-1}} = {X_j}^{\{m_j\}^{-1}} = \{p\}$), one may want to consider the notion of the *effective power*, to be defined in a straightforward fashion:

$$m \equiv \frac{log\prod_k p_k^{m_k}}{log\prod_k p_k} = \frac{\sum_k m_k log p_k}{\sum_k log p_k}$$

Readily verifiable,

$$rad(X) = rad\left(\prod_k p_k^{m_k}\right) = \prod_k p_k = X^{\frac{1}{m}} \leftrightarrow X^{1-\frac{1}{m}} * rad(X) = X = [rad(X)]^m$$

It is by the same token that the implied functional or recurrent representation could be rationalized with an eye on its solution or [non-vector] reduction:

$$f(m) \equiv \varphi * f(m-1) = \varphi^m * f(0), \qquad f(m) \equiv X, \qquad \varphi \equiv rad(X)$$

That said, not only might the above solution suggest a structure similar to the [reciprocal] *gamma distribution* or *factorial* (should the exponential stretching allow for a linear-transform equivalent as will be discussed in the *pi-sigma calculus* section), this analogy could be reinforced by inferring [alternatingly] that *rad (0)=1*. At this rate, some of the patterns under study may resemble structures of the *Euler beta* sort, or indeed their combinatorial counterparts. In pi-sigma terms, and based on the analogy $rad \sim (\prod *)^{\frac{1}{m}}$ yet to be covered in text,

$$X = f(m) = \varphi(m) * f(m-1) = \frac{(\sum *)^m}{m!} = \frac{m^m \prod *}{m!} \sim e^m * \sqrt{\frac{m}{2\pi}} \prod *$$

For the singular case (prime or m=1) with the sum tantamount to the product, this reduces to identity. Somewhat similar results could be obtained asymptotically for large underlying values (in place of the powers):

$$\Gamma(a+b) \sim \Gamma(a) * a^b \sim \sqrt{\frac{2\pi}{a}} * \frac{a^{a+b}}{e^a}$$

By treating the above as a functional equation and holding the factor fixed, the reduced form can be approximated as,

$$\Gamma(a) \sim a^a \; or \; a^{\frac{a^2}{2b}}$$

Alternatively, by substituting *(m, 1)* for *(a, b)*, the above can take on,

$$\Gamma(m) \sim \sqrt{2\pi} * \frac{m^{m+\frac{1}{2}}}{e^m}$$

While direct substitution of the sort might make little sense, it points to similar results throughout the levels of analysis.