The Quantum Mechanical Time Reversal Operator.

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Abstract

The analysis of the reversibility of quantum mechanics depends upon the choice of the time reversal operator for quantum mechanical states. The orthodox choice for the time reversal operator on QM states is known as the Wigner operator, T*, where * performs complex conjugation. The peculiarity is that this is not simply the unitary time reversal operation, but an anti-unitary operator, involving complex conjugation in addition to ordinary time reversal. The alternative choice is the Racah operator, which is simply ordinary time reversal, T. Orthodox treatments hold that it is either logically or empirically necessary to adopt the Wigner operator, and the Racah operator has received little attention. The basis for this choice is analysed in detail, and it is concluded that all the conventional arguments for rejecting the Racah operator and adopting the Wigner operator are mistaken. The additional problem of whether the deterministic part of quantum mechanics should be judged to be reversible or not is also considered. The adoption of the Racah operator for time reversal appears prima facie to entail that quantum mechanics is irreversible. However, it is concluded that the real answer to question depends upon the choice of interpretation of the theory. In any case, the conventional reasons for claiming that quantum mechanics is reversible are incorrect.

The Quantum Mechanical Time Reversal Operator.

1. Introduction.

Callender [1] argues for two contentious conclusions, both of which I support: that non-relativistic quantum mechanics is *irreversible* (non-time reversal invariant, or non-TRI for short), both in its probabilistic laws, and in its deterministic laws. These claims contradict the current assumptions in the subject. The first point, the irreversibility of the probabilistic part of quantum mechanics, is the most important for understanding irreversible processes, and was already argued convincingly some fifty years ago by Watanabe in 1955 [2], [3], as confirmed by Healey [4] and Holster [5], although it has been overlooked by most authorities on the subject, such as Davies [6], Sachs [7], and Zeh [8]. Similar points have also been made independently, notably by Schrodinger [9] and Penrose [10]. But I will not deal with this problem here, since I have examined it in detail elsewhere [5].

Here I only examine Callender's second claim, i.e. that the deterministic dynamics of quantum mechanics is also non-time reversal invariant. The problem here is more subtle, and we will find that the orthodox analysis suffers from a number of deep-seated conceptual confusions.

Callender [1], distinguishes between *TRI* (Time Reversal Invariance) and *WRI* (Wigner Reversal Invariance), the latter being generally *interpreted* as time reversal invariance in quantum mechanics, while the former is generally dismissed in quantum mechanics as logically incoherent. *TRI* is symmetry under the simple transformation: $T: t \rightarrow -t$ alone. *WRI* is symmetry under the combined operation: $\Theta = T^*$, where T is the simple time reversal operator, and * is complex conjugation. Callender first observes that:

> "If one surveys the literature concerning this issue, one finds many arguments that attempt to blur the difference between WRI and TRI. Probably the most frequent claim is that in quantum mechanics the physical content is exhausted by the probabilities. As Davies puts it, 'a solution of the Schrodinger equation is not itself observable' so Wigner's operation can restore TRI while leaving 'the physical content of QM unchanged' [Davies 1964, 156]." [1] p 262-263.

After briefly dismissing this kind of argument, Callender then observes:

> "This is not the place to go through all the misguided attempts to blur the distinction between WRI and TRI, but another popular argument claims that WRI is necessitated by the need to switch sign of momentum and spin under time reversal. Here the reply is that there is no such necessitation. In quantum mechanics, momentum is a spatial derivative ($-i\hbar\,\partial\Psi/\partial x$) and spin is a kind of 'space quantisation'. It does not logically follow, as it does in classical mechanics, that the momentum or spin must change signs when $t\rightarrow -t$. Nor does it logically follow from $t\rightarrow -t$ that one must change $\psi\rightarrow\psi^*$." [1], p. 263.

These are indeed the two main kinds of reasons given for interpreting *WRI* as time reversal invariance. I will concentrate here on analyzing the second reason, which in one form or another is regarded as conclusive by most authorities. The first kind of reason, as Callender observes, generally appeals to invalid operationalist or positivist principles, and I will briefly return to this in the final section. But the second reason is the main concern here, because it appears to follow from a straightforward argument, which I will analyse in detail. This conventional argument, although widely accepted by quantum physicists, is unsound. The failure of this kind of argument reflects deeper misunderstandings both of the logic of time reversal and the interpretation of quantum mechanics.

2. Background. The *T*-Reversed Theory.

I note firstly that the problem has a longer history than Callender [1] appears to be aware of, and has previously been discussed in some detail by O. Costa de Beauregard [11] in the relativistic context. The viability of the *T*-operator appears to have been first advanced by Racah [12]. de Beauregard, also citing Watanabe and Jauch and Rohrlich [13], claims that the use of *T* for the time reversal operator is supported by Feynmann's 'zigzag' model, which interprets anti-particles as 'particles traveling backwards in time':

> While the well-known motion reversal operation Θ is obviously quite consonant with the Schrodinger advancing time, and the Tomonaga-Schwinger advancing s philosophy, the Racah time reversal operation *T* is generally discarded with little comment as being non-physical. It can be consistently used, however, as recognized by Watanabe and by Jauch and Rohrlich. We intend to show here that (as defined in the framework of the Dirac electron

> theory) *T exactly* is the geometrical reversal of the time axis which is appropriate in the four dimensional space-time geometry, and is thus *naturally akin to the Feynmann zigzag philosophy*. [11], p.524.

I will follow de Beauregard and refer to the transformation: $T: t \rightarrow -t$ applied to quantum states as the *Racah operator,* and the orthodox *T** as the *Wigner operator.* de Beauregard's analysis is extremely interesting, but it is about relativistic quantum mechanics, and he does not argue that *T* is appropriate in non-relativistic quantum mechanics, or analyse the underlying logic of this choice in any detail, which is the aim here. I comment on his views in the final section.

I will define the deterministic part of ordinary, non-relativistic quantum mechanics as 'QM', and the first point is that:

Wigner Invariance: $T^*(\mathrm{QM}) = \mathrm{QM}$, or equivalently: $T(\mathrm{QM}) = {}^*(\mathrm{QM})$

Racah Non-Invariance: $T(\mathrm{QM}) \neq \mathrm{QM}$, and: ${}^*(\mathrm{QM}) \neq \mathrm{QM}$

It is readily seen that the time dependant Schrodinger equation is unchanged by the transformation *T**, but changed to an anti-symmetric form by *T* alone, and by * alone, by looking at the simple Schrodinger equation for a free particle, and its transformations:

Theory:	**Images of Schrodinger Equation:**	**Simple Solutions**
QM	$\partial\Psi/\partial t = i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A \exp((i/\hbar)(px - p^2t/2m))$
T(QM)	$-\partial\Psi/\partial t = i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A \exp((i/\hbar)(px + p^2t/2m))$
T^*(QM)	$-\partial\Psi/\partial t = -i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A \exp((i/\hbar)(-px - p^2t/2m))$
*(QM)	$\partial\Psi/\partial t = -i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A \exp((i/\hbar)(-px + p^2t/2m))$

The 'simple solution' here represents a particle with a precise momentum and kinetic energy, but with no position defined. More realistically, free particles are 'wave packets', represented by linear sums of simple solutions, with uncertainty in both momentum and position; but these have the same forms of transformation as illustrated by the simple solution, and the simple example suffices for the purposes of this paper. The class of these simple solutions for T^*(QM) is the same as for QM because p can be positive or negative. But the class of solutions for *(QM) (or equally T(QM)) is not the same as for QM because p^2 must be positive.

To see the main relationships between the transformations, we can take QM (or equivalently, T^*(QM)) to represent a class $\{\Psi\}$ of solutions to the Schrodinger wave equation, and T(QM) (or equivalently, *(QM)) to represent a 'dual' class, $\{T\Psi\}$, of T-transformed solutions. These are disjoint classes. There is a perfect 1-1 correspondence between them, as illustrated in Fig. 1.

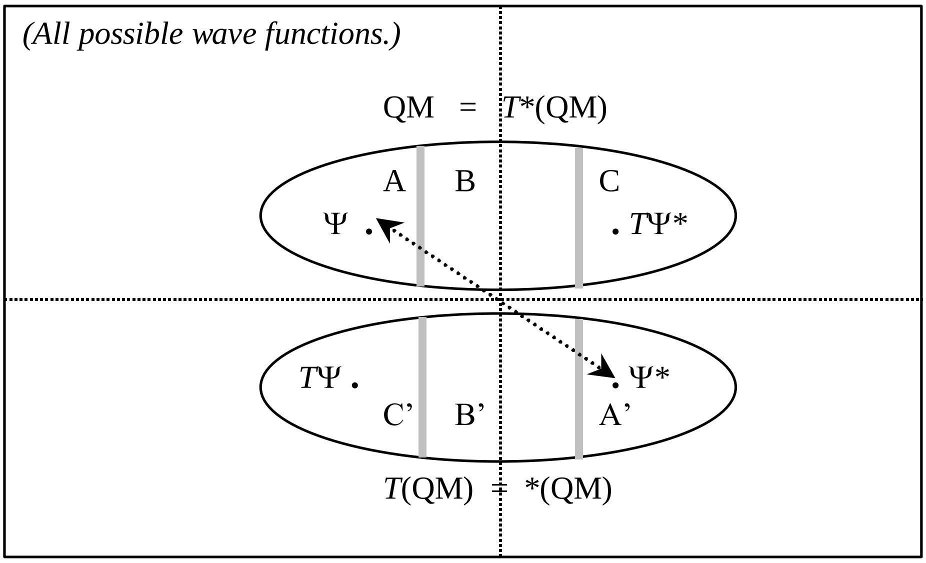

Figure 1.

Points in this Venn diagram represent logically possible complex-valued wave functions (mappings from points of space, $\boldsymbol{r}$, to complex numbers, z).
T-images are given by reflections in the horizontal dotted line.
*-images are given by reflections through the center point.
T^*-images are given by reflections through the central vertical line.
The top ellipse represents all the solutions to QM. This is identical to the set of solutions to: T^*(QM). The bottom ellipse represents all the solutions to T(QM), which is identical to the set of solutions to *(QM). This is disjoint from QM.

The wave functions in Fig. 1 have also been stratified into three kinds:

- A represents non-equilibrium thermodynamic processes (or retarded waves, dispersing from a centralized source).
- B represents equilibrium thermodynamic processes (with maximum dispersion).
- C represents non-equilibrium ‘anti-thermodynamic’ processes (or advanced waves, converging to a centralized source).
- A’, B’, and C’ are the *T*-reversed images with the corresponding dispersion properties. Note that: *T*(A) = C’, while: *(A) = A’.

The purely deterministic part of quantum mechanics allows solutions from A, B, and C. In reality, we do not find processes from C in our environment, only from A and B. This reflects the empirical fact that our world is rich in ‘irreversible processes’ on the classical scale. However, this is not explained by deterministic quantum mechanics, whether we adopt the Racah or the Wigner operator for time reversal, because both retarded and advanced waves are equally compatible with QM and with *T*(QM). (It is the irreversibility of the *probabilistic* part of quantum mechanics that is relevant to this; see [2], [3], [5]).

There is a perfect isomorphism between the classes: $\Psi \leftrightarrow \Psi^*$, because Ψ and Ψ^* differ only in the ‘direction of rotation’ of the imaginary phase of the wave (represented by the sign in the Schrodinger equation). This direction is not directly measurable – only the *relative directions* of the complex rotation of separate particles or systems are measurable, so we cannot combine waves from QM with waves from *(QM), when we combine different particles into composite systems (or we would get the wrong kinds of interference effects). But the choice to use the *class* QM rather than the *class* *(QM) (or equivalently T(QM)) to represent particles can be regarded as an arbitrary convention in the first place. If Schrodinger had chosen to use *(QM) instead of QM, then we would simply have to ‘reverse’ all the usual deterministic laws, by taking the appropriate images under * of the equations for energy, momentum, etc. In this sense, at least, *(QM) can be used to represent a perfectly sensible theory, isomorphic to QM.

Now let us examine the deterministic laws satisfied by *(QM), or equivalently *T*(QM), rather than QM. The transformed wave equation is as given above: it is anti-symmetric with the usual Schrodinger equation for QM. For the free particle in QM:

(1) $$\partial\Psi/\partial t = i\hbar/2m\ \partial^2\Psi/\partial x^2$$

In the *T*-transformed theory, we have instead:

T(1) $$-\partial\Psi/\partial t = i\hbar/2m\ \partial^2\Psi/\partial x^2$$

This is obvious and well recognized. But what of the laws for the energy and momentum operators? These relate the classical properties of energy and momentum to the wave functions. In ordinary QM the key laws are:

(2)	$\mathbf{H}\Psi = i\hbar\,\partial\Psi/\partial t$	Kinetic Energy (zero potential)
(3)	$\mathbf{P}\Psi = -i\hbar\,\partial\Psi/\partial x$	Momentum

Along with:

(4)	$\mathbf{H}\Psi = \mathbf{P}^2\Psi/2m$	Classical Energy-Momentum Relation

And then Eqs. 3 and 4 entail:

(5) $$\mathbf{H} = -\hbar^2/2m\ \partial^2/\partial x^2$$

Substituted in Eq. 2 this gives: $-\hbar^2/2m\ \partial^2\Psi/\partial x^2 = i\hbar\ \partial\Psi/\partial t$, which is just Eq. 1 rearranged.

Then what are *the time reversed images* of Eqs.2-4? For the ‘dual’ version *(QM), or equivalently *T*(QM), the operators should be given by:

T(2)	$\mathbf{H}_*\Psi = -i\hbar\,\partial\Psi/\partial t$	Kinetic Energy (with zero potential)
T(3)	$\mathbf{P}_*\Psi = i\hbar\,\partial\Psi/\partial x$	Momentum
T(4)	$\mathbf{H}_*\Psi = \mathbf{P}_*^2\Psi/2m$	Classical Energy-Momentum Relation

I have labeled these $\mathbf{H}_*$ and $\mathbf{P}_*$, to make clear that these are distinct mathematical operators to **H** and **P** – they are what these operators defined as giving the classical energy and momentum transform to in the reversed theory.

We will work through this in more detail later, but it is easy enough to see why these must be adopted. In *(QM), we take the wave: Ψ* to represent a particle with the same classical properties as Ψ in QM – this is the basic isomorphism. Alternatively, in *T*(QM), *T*Ψ represents a particle with the time-reversed classical properties represented by Ψ in QM. Now, for instance, consider the special solution, Ψ, stated earlier for QM. Using Eq.2 we have: $\mathbf{H}\Psi = E\Psi$, with: $E = p^2/2m$ as the classical kinetic energy of the particle with the original wave function Ψ. We know that this is also the classical kinetic energy of the time reversed particle, represented by *T*Ψ, in the *T*-transformed theory *T*(QM). But the time differential term in (2) has the behavior: $\partial(T\Psi)/\partial t = -\partial\Psi/\partial t$, so to obtain the correct result, we must *define the classical kinetic energy operator* $\mathbf{H}_*$ *for the time reversed theory* by *T*(2) above, instead of by (2). (The same result follows by considering the wave Ψ*.)

Similarly, using Eq.3 we have: $\mathbf{P}\Psi = p\Psi$, where *p* is the classical momentum (in the *x*-direction) of the particle with the original wave function Ψ. We know that this is the *negative* classical momentum of the time reversed particle, represented by *T*Ψ. The space differential term in (3) has the behavior: $\partial(T\Psi)/\partial x = \partial\Psi/\partial x$, so to obtain the correct result, we must *define the classical momentum operator* $\mathbf{P}_*$ *for the time reversed theory* by *T*(3) above, instead of by (3). (Again, the same result follows by considering the wave Ψ*, which must have the same momentum in the theory *(QM) as Ψ in QM, but the differential term in (3) has the behavior: $\partial(\Psi^*)/\partial x = -\partial\Psi/\partial x$).

Thus, in the new theory, *T*(QM), we find that the 'classical operator laws' (2) and (3) are both 'reversed', to *T*(2) and *T*(3). By contrast, (4), giving the relation between momentum and kinetic energy, remains the same. And Eqs. *T*(3) and T(4) entail:

T(5) $\quad \mathbf{H}_* = -\hbar^2/2m\, \partial^2/\partial x^2$

Substituted in $T(2)$, this gives: $\hbar^2/2m\ \partial^2\Psi/\partial x^2 = i\hbar\ \partial\Psi/\partial t$, which is the *T*-reversal of (1), as required.

Let us now turn to the key objection to using *T* as the time reversal operator.

3. The key objection to using *T* for Time Reversal.

The key objection is that *T* does not have the right formal properties to properly represent time reversal in QM, whereas *T** does. In particular, it is said that *T* does not transform energy or momenta in the correct way for time reversal - for the energy of the reversed state of a particle must go be unchanged, whereas the momentum must reverse, but (it is said) the use of *T* to transform to time reversed wave functions does not satisfy this requirement. This point is repeated over and over again in different forms. It is common in ordinary text-books, e.g.:

> ...we find that [the Heisenberg equation of motion] can be invariant only if
>
> $$THT^{-1} = -H$$
>
> This, however, is an unacceptable condition, because time reversal cannot change the spectrum of *H*, which consists of positive energies only. If *T* is taken to be anti-unitary [ie. *T** adopted], the * operator changes the i to -i [in the Heisenberg equation] and the trouble does not occur. (Gasiorowicz, [14], p.27).

Or Messiah:

> We are thus led to define a transformation of dynamical variables and dynamical states, which we shall call *time reversal*, in which **r** and **p** transform respectively into **r** and **-p**. ... [*T**] obviously satisfies [the] relations. Therefore we may take [*T**] as our time reversal operator. ([15], p.667).

An expanded version of this argument goes:

(i) The energy of a time reversed wave function must be the same as the original energy – i.e. the time reversed spectrum must be the original, positive, spectrum.

(ii) The energy of a QM wave function is given by: $\mathbf{H}\Psi = i\hbar\,\partial\Psi/\partial t$.

(iii) However, $T\Psi$ has the negative of the energy of Ψ, for, by (ii):

$\mathbf{H}(T\Psi) = i\hbar\,\partial(T\Psi)/\partial t$

$= -i\hbar\,\partial\Psi/\partial t$

$= -\mathbf{H}\Psi$,

contradicting (i).

(iv) Hence *T* cannot be the time reversal operator. It does not have the right formal properties, since it reverses energy, which time reversal cannot do.

(v) However, *T** does uniquely have the appropriate formal properties.

(vi) Hence *T** is the only reasonable choice for time reversal.

The problem with this argument is in step (iii), because the law: $\mathbf{H}\Psi = i\hbar\,\partial\Psi/\partial t$ represents the energy *in quantum mechanics;* but we are no longer considering quantum mechanics; we are transforming to *the time reversed theory, T(QM).* And in this theory, the law for the energy operator transforms to its reverse: $\mathbf{H}\Psi = -i\hbar\,\partial\Psi/\partial t$.

Our wave function $T\Psi$ is *not a QM wave function.* It is not a QM wave function precisely because its energy equation is *not* the QM equation, as defined in (ii). The operator **H** in equation (ii) defines *the classical correlate of energy for a QM wave function.* Since $T\Psi$ is not a QM wave function, why should we assume that the classical correlate of its energy is defined in the same way?

Step (iii) looks convincing, but it is circular, *because it only applies under the prior assumption that QM is time reversible,* i.e., that each time-reversed-Ψ is also a QM wave function. On this assumption, then it would indeed be true that *T* cannot be the time reversal operator. I.e: *If* $T\Psi$ *obeys QM given that* Ψ *obeys QM, then T is not the time reversal operator, because* $\mathbf{H}\Psi = -\mathbf{H}(T\Psi)$. However, this is equally stated as the converse fact: that *if T is the time reversal operator, then QM is irreversible, because* $\mathbf{H}\Psi = -\mathbf{H}(T\Psi)$*!* And this gives the correct conclusion which is simply that *T is the time reversal operator and QM is irreversible*.

The orthodox analysis has fallen into a peculiar kind of circular fallacy in reasoning about this. What has not been recognized is the simple fact that when we take the transformed *theory T*(QM), we find that we naturally have to transform the classical-quantum correspondence principles along with the wave functions themselves, to obtain the empirical meaning of the theory. I examine an explicit treatment of this point next, to illustrate how weak the orthodox argument is.

4. Transforming the Classical-Quantum Correspondence Laws.

The previous kind of argument against *T* is drawn out at length by Sachs [7], following Wigner. The key point of his argument is to distinguish the laws (2) and (3) as 'definitions of kinematic variables', and effectively to hold that since they are definitions (or logical truths), they cannot alter under time reversal. Sachs contrasts laws of this kind with 'dynamic equations', which can alter under reversal transformations. It is true that *definitions or logical truths never alter under time reversal* (or any general transformations). But the weakness in Sachs' argument is that *he fails to show that laws like (2) and (3) are definitions.* He simply claims this without discussion. Sachs begins by making a distinction between:

> "…three elements of the formal structure we have come to expect to encompass all dynamic theories of physics. These are:
> A. The mathematical manifold within which the motions or states of the physical system are to be described.
> B A set of kinematic observables (measurable quantities whose definitions are independent of forces or interactions)…
> C. The general structure of the dynamic equations giving the causal relationships between the kinematic variables." ([7], p.31).

The critical distinction is between what Sachs calls the *kinematic observables* and the *dynamic laws* of quantum mechanics. He holds that the kinematic observables are defined by the general commutation relations, while the dynamics are determined by the laws governing the Hamiltonian operator, *H.* Having stated these (his equations 3.3-3.6), he then observes a key principle in his analysis:

> "To establish the form of *T* we must impose two conditions: first, that it be a kinematically admissible transformation, and second, introducing the physical content, that it conform to the requirements of the correspondence principle – namely, operators representing classical kinematic observables must transform under *T* in a manner corresponding to classical motion reversal." ([7], p.34)

We should note to begin with that Sachs does not propose any *direct definition of the time reversal transformation,* but instead works through a set of indirect principles governing time reversal on the basis of what he supposes are general principles for the formal interpretation of a physical theory. Yet he does not even try to give any justification for these general principles - he merely states them as if they are well-known facts. And his procedure also contradicts the usual assumption that the concept of time reversal is *objective*: that the symmetry itself is conceptually defined independently of any given theory, and in analyzing the time reversal invariance of a particular theory we examine whether it objectively satisfies this symmetry. This symmetry is defined in all other branches of physics as symmetry under the transformation: T: $t \rightarrow -t$, but Sachs' procedure instead requires us to define 'time reversal' in quantum mechanics in a special way that is 'appropriate' to quantum mechanics. There is a dangerous preconception involved here that the 'appropriate' definition *should* render quantum mechanics as a time reversal invariant theory. But this robs the notion of time reversal invariance of *objectivity,* because in his treatment, we can pick and choose among different definitions of what this symmetry means, until we find one that quantum mechanics satisfies. This is not a promising start.

He follows this with a dense argument to show that the time reversal operator must be anti-unitary. This begins with a 'derivation' of "the conclusion that T must include the operator K, which takes any complex number z into its conjugate complex" ([7], p.35). In the next main step he invokes another principle, that:

> "Another main requirement on a kinematically admissible transformation is that, *in the absence of forces or interactions* (i.e. in the absence of causal effects), the dynamic equations must be left invariant". ([7], p.35).

This allows him to conclude that "H_0 [the Hamiltonian operator including only kinetic terms] is invariant under time reversal. " ([7], p.36). But this already begs the question – in fact it directly rules out T immediately as a 'kinematically admissible transformation', because the dynamics equation, (1) for the free particle is not invariant under T. He then gives a further mathematical argument to conclude that the time reversal operator must be anti-unitary, and must be identified with the Wigner operator T^* or TK.

Sachs' argument here is a confused piece of analysis, and it rests on a number of unproved assumptions. He appeals to a variety of 'principles' without justifying them. He has provided his own summary of mathematical arguments first put forward by

Wigner in 1932; but the conceptual basis for the underlying principles of time reversal remain opaque.

The particular flaw in Sachs' argument, and others of the same general kind, is the assumption that the form of the 'kinematic laws' (or the classical correspondence principles) of quantum mechanics *must remain invariant* under time reversal of the theory. He wants to allow that the form of *dynamic laws* in *T(QM)* (essentially the Schrodinger equation) are evaluated independently, on the basis of their mathematical structure *plus the time reversal transformation on the states.* But the derivation of the 'appropriate' time reversal transformation on states is obtained from the requirement that the 'kinematic variables' in the time reversed theory are defined by the same relationships as in the original theory.

But why should this be so? *If T(QM) is really an irreversible theory,* then the time reversed wave functions *will fail to obey the ordinary laws of quantum mechanics*, and why shouldn't they fail to obey what he (mistakenly, in my view) calls the 'kinematic principles' (i.e. the usual classical correspondence principles) of the theory?

The kind of argument Sachs recounts is deeply ingrained in the current account of quantum theory. The problems stem from the complicated role of observable operators, like **H** and **P**, and their connection with classical counterparts. The problem involves the distinction between *definitions* and *empirical or contingent laws.* In the next section I will consider this problem, and show why the equations (2)-(4) cannot be simply regarded as definitions, and left invariant under time reversal.

5. Definitions and Empirical or Theoretical Laws.

A basic logical problem in physics arises from the practice of using *implicit quantifiers* on variables in the statements of laws of physical theories. This often creates a muddle between definitions, and empirical or theoretical laws. In the case of Eq.2, for instance, the term Ψ is implicitly quantified. But compare the two following ways of interpreting the quantification:

(6) $(\forall \Psi)(\mathbf{H}\Psi = i\hbar\,\partial\Psi/\partial t)$ *Universal Quantifier*

Or alternatively:

(7) $(\forall \Psi \in \mathrm{QM})(\mathbf{H}\Psi = i\hbar\, \partial\Psi/\partial t)$ *Limited Quantifier*

Eq.6 is read with Ψ ranging over *all logically possible wave functions.* Interpreted this way, it is a *logical definition* of **H**, and we could simply rewrite **H** as a mathematical operator:

(8) $\mathbf{H} = i\hbar\, \partial/\partial t$

Eq.7 is read instead as saying that *for all wave functions,* Ψ, *that satisfy QM,* the operator **H** has the property that: $\mathbf{H}\Psi = i\hbar\, \partial\Psi/\partial t$. (We can expand this more formally if we like to: $(\forall \Psi)(\Psi \in \mathbf{QM} \Rightarrow (\mathbf{H}\Psi = i\hbar\, \partial\Psi/\partial t))$, but (7) is easier to read).

If the law (2) is intended as an empirical or theoretical law of QM, rather than merely as a definition, then this seems the natural reading.

How do we choose between these two readings? It depends on whether we intend to interpret **H** as merely a 'defined mathematical operator' – in which case it is merely a short-hand notation for the term: $i\hbar\, \partial/\partial t$, and Eq.2 states a tautology – or alternatively, whether we intend **H** as an quantity with a definite physical interpretation, e.g. if: $\mathbf{H}\Psi = E\Psi$, then E is the classical energy of the particle represented by Ψ.

On the latter interpretation, (2) applies to all QM wave functions – but it does not apply to any possible wave function we can define. It is not a logical truth. It is part of the physical content of the theory of quantum mechanics.

To illustrate the distinction further, consider the interpretation of the quantifier on Eq.1. We could take it as either:

(9) $(\forall \Psi)(\partial\Psi/\partial t = i\hbar/2m\, \partial^2\Psi/\partial x^2)$

Or:

(10) $(\forall \Psi \in \mathrm{QM})(\partial\Psi/\partial t = i\hbar/2m\, \partial^2\Psi/\partial x^2)$

Now (9) cannot possibly be true, because there are wave functions that do not satisfy (9) – as we have seen with $T\Psi$ for instance. Rather, (10) can be regarded as the definition of the class QM (for the limited simple theory of free 1-dimensional wave functions). But then, (10) by itself makes no reference to anything empirical. If we take (10) to define the class of theoretically possible wave functions in quantum mechanics, then we must engage additional laws like Eq.2-4, *interpreted as empirical laws about measurable classical observables,* to give us an empirical theory.

Note also that we cannot take all three of Eq. 2-4 as definitions. For suppose we take 2 and 3 as definitions of the operators **H** and **P**. We cannot also take 4 as a *definition,* because, given (2) and (3) are definitions, (4) *is simply not true of all logically possible* Ψ*s.* In particular, it is easily checked that (4) is not true of the wave function: $T\Psi$ = A $\exp((i/\hbar)(px + p^2t/2m))$.

Instead, what must be intended is that, *given that (2) and (3) are definitions of* **H** *and* **P**, then *the QM wave functions obey (4).* Then (4) appears to be a *contingent proposition,* which is true of *quantum mechanical wave functions,* but not true in general.

Given this interpretation of Eqs. 2-4, it is obvious that (2) and (3) are invariant under *T, being simply mathematical definitions,* but (4) is anti-symmetric under *T.* This means that the classical commutation relations for the time reversed theory, *T*(QM), contradict the relations in the original theory, QM – naturally enough, because QM is not *T*-invariant.

Yet this is exactly what Sachs' argument denies – or rather, his argument recognises that this would be the case if we adopted *T* for time reversal invariance, but *he denies that it is possible for the relation (4) to alter under time reversal,* and concludes that *T cannot represent time reversal.* Yet his only reason is that (2)-(4) are 'definitions of kinematic variables', and cannot change under 'admissible transformations'. This is simply wrong.

To analyse the problem properly, we first have to *interpret the meaning of the operators* **H** and **P** and the laws (2)-(4). The general result of the analysis will not affected by this – we will find inevitably that QM is *not* TRI, but *T* is nonetheless a perfectly sensible transformation to apply to the theory QM. What the interpretation affects is the detailed view of the *T* transformations on the operators **H** and **P**, or on the laws (2)-(4) – since if we interpret these in two different ways, they will naturally show different transformation properties.

Perhaps the most sensible interpretation, in line with physicists' normal practice, is to take (2)-(4) to be intended to represent *empirical laws,* and to take their transformations to be an alternative set of empirical laws. On this view, we can take **H** and **P** to be defined as mathematical operators, but we have to add something extra to Eq.2 and 3 to incorporate their *contingent or empirical* interpretation. What we add to (2) is an additional clause to the effect that:

(2.Extra) *If* $\mathbf{H}\Psi = E\Psi$ *then E would be the classically measured energy of the particle represented by* Ψ.

Now having *defined* **H** by (2), we can obtain its time reversed image, $T\mathbf{H}$¸ by considering that T(2) and (2) are both definitions or tautologies, and calculating:

$$\begin{aligned} T(2) &\equiv T(\mathbf{H} = i\hbar\,\partial/\partial t) \\ &\equiv (T\mathbf{H} = T(i\hbar\,\partial/\partial t)) \\ &\equiv (T\mathbf{H} = -(i\hbar\,\partial/\partial t)). \end{aligned}$$

I.e. the definition of $T\mathbf{H}$ is:

$$T\mathbf{H} = -(i\hbar\,\partial/\partial t) = -\mathbf{H}$$

Similarly for $T\mathbf{P}$:

$$\begin{aligned} T(3) &\equiv T(\mathbf{P} = -i\hbar\,\partial/\partial x) \\ &\equiv (T\mathbf{P} = T(-i\hbar\,\partial/\partial x)\,) \\ &\equiv (T\mathbf{P} = -i\hbar\,\partial/\partial x\,) \end{aligned}$$

I.e. the definition of $T\mathbf{P}$ is:

$$T\mathbf{P} = -i\hbar\,\partial/\partial x = \mathbf{P}$$

There is no surprise in this. But to obtain the *T* reversal of (2.Extra), we have to reason further: what would the time reversal of this empirical law state? Here we meet a special and entrancing difficulty: there is no formal method of calculating such reversals, because there is no formal method of representing *contingency* in physics. This will be somewhat mysterious to physicists, but it is quite clear to modern intensional semanticists or logicians, because physics has only an *extensional formalism,* whereas the concept of *contingency* requires us to go to a deeper level of *intensional semantics,* where we formal devices for quantifying over 'worlds' or 'systems' or something equivalent. (See papers in [16] for basic accounts of intensional logic). But no *intensional* formalism for physics is currently used, and consequently we cannot calculate the answer to our problem formally. Instead, we must do what physicists routinely do, and reason to the answer in an informal way.

Let us suppose that ordinary QM is wrong, and *T*(QM) is correct instead, so that a real particle is not modeled by the QM wave function Ψ after all, but by the corresponding *T*(QM) wave function Ψ^*.

Then when we measure a particle as having a classical energy *E,* we are not measuring that: $\mathbf{H}\Psi = E\Psi$ for $\Psi \in$QM; instead we are measuring by an alternative operator, $\mathbf{H}_*$, where: $\mathbf{H}_*(\Psi^*) = E(\Psi^*)$ for $\Psi \in$QM and $\Psi^* \in T$(QM). But then it follows that, since $\mathbf{H}_*(\Psi^*) = E(\Psi^*)$ and $\mathbf{H}\Psi = E\Psi$, we must identify: $\mathbf{H}_* = \mathbf{-H} = T\mathbf{H}.$

This is what is represented by the law *T*(2) above. Similar reasoning gives us *T*(3), i.e. that: $\mathbf{P}_* = \mathbf{-P} = -T\mathbf{P}$. The relation *T*(4) follows similarly.

Hence we obtain the natural time reversed theory, *T*(QM), as Eq. *T*(1)-*T*(4), using the alternative operators $\mathbf{H}_*$ and $\mathbf{P}_*$ to represent the real empirical content, with the implicit meaning that:

T(2.Extra) *If* $\mathbf{H}_*\Psi = E\Psi$ *then E would be the classically measured energy of the particle represented by* Ψ.

And similarly for the momentum relation.

What we have obtained as the theory *T*(QM) is the natural 'dual' theory we would use if we choose to identify particle wave functions using the class of *T*-images of the usual QM wave functions that Schrodinger originally adopted, as illustrated in

Fig.1. The fact that this 'dual' theory exists and is equally as sensible as ordinary QM is hardly in dispute. It also seems natural that it correctly represents the time reversed image of ordinary QM. The arguments of Sachs and others that there is no coherent theory *T*(QM) are surely mistaken. But note the reason Callender gives in the second quotation above is also not correct: time reversal does reverse the classical property of momentum; the key point is that it transforms the form of the classical operators to do this.

6. Positivistic Arguments against *T*.

This brings us back, however, to the first reason mentioned by Callender in the quotations above for adopting *T** as the time reversal operator, and I will briefly comment on this. Essentially, it now appears that *the choice to use QM rather than T(QM) is arbitrary*, because there is no way of measuring the wave functions directly, and we cannot distinguish whether a wave function is 'really' Ψ or Ψ^*. On the positivist view, the two theories, QM and *T*(QM), *are indistinguishable in their physical predictions*, and so they should be taken to represent the same theory.

Two different kinds of arguments should be distinguished here. The first – and strongest - appeals to the 'probability interpretation' of quantum mechanics, in which *physical reality is denied to the wave function altogether*, and only the probabilities represented by the wave function are regarded as physically real. If this is correct, then the asymmetry between the theories QM and *T*(QM) is not a physical feature of the universe at all, because the wave functions are simply not physical things. But two points should be made. First, whether such an *interpretation* of quantum theory is ultimately sustainable is then the deeper question. I will not try to answer this here, except to observe that the popular arguments for this kind of interpretation are often based in turn on flimsy positivistic principles, and the problem of establishing an adequate 'probabilistic interpretation' is more difficult than it first seems, because the probabilities by themselves do not have enough detailed structure to represent the interference effects generated by superpositions of the complex wave functions. How do we dispense with the wave functions themselves, and yet retain the detailed information they represent that is required to predict interference effects correctly?

Supposing this can be done, however, there is a second crucial point to be made here: this view *does not deny that T, i.e. the Racah operator, represents time reversal* *in quantum theory!* Rather, *it dissolves the physical difference between T and the Wigner operator, T*, by imposing an interpretation of quantum mechanics where the waves* Ψ *and* Ψ** are seen as physically identical – or as modeling the same physical reality*. On this view, *T* should still be taken as the time reversal operator: the work of arriving at the conclusion that *quantum mechanics is nonetheless time symmetric* is really done by the *interpretation of the wave function*. This alerts us to the fact that whether (the deterministic part of) quantum mechanics is *TRI* is not solely dependant on adopting *T*, but also dependant on the *interpretation of the physical reality of the wave function.* The orthodox account conflates these two points.

A second kind of argument, however, is based on a more general kind of positivistic fallacy, which essentially involves arguing that the theories QM and *T*(QM) are *observationally indistinguishable, and should therefore be regarded as the same theory*. A version of the argument was given by Reichenbach:.

> There remains the problem of distinguishing between Ψ(q,t) and Ψ(q,-t). In order to discriminate between these two functions, we would first have to know whether [$E\psi = i\hbar\, \partial\psi/\partial t$ or: $E\psi = -i\hbar\, \partial\psi/\partial t$] is the correct equation. But the sign on the right in Schrodinger's equation can be tested observationally only if a direction of time has been previously defined. We use here the time direction of the macroscopic systems by the help of which we compare the mathematical consequences of Schrodinger's equation with observation. Therefore, to attempt a definition of time direction through Schrodinger's equation would be reasoning in a circle; this equation merely presents us with the time direction we introduced previously in terms of macrocosmic processes. ([17], pp. 209-210).

Note that Reichenbach does *not* deny that: *T*Ψ(q,t) = Ψ(q,-t) does in fact represent time reversal. Instead, he argues that the wave functions: Ψ(q,t) and *T*Ψ(q,t) are observationally indistinguishable, and that this undermines the conclusion that QM is irreversible.

A number of confusions can be found here, but there is one key point that needs to be made. Suppose that we have a time asymmetric theory, **T**, which we regard as observationally indistinguishable from its reversal, *T***T.** Positivistic reasoning suggests that **T** and *T***T** are therefore identical theories. But we can construct a weaker theory: (**T** *or T***T**)¸ i.e. the disjunction of **T** and its time reversal, *T***T**, and this weaker theory is obviously time symmetric. If we suppose that **T** and *T***T** are logically equivalent – or have identical meanings - this would entail that **T** is identical to the theory (**T** *or T***T**).

But this cannot be correct. Certainly, **T** entails (**T** *or T***T**), because any observational evidence for **T** is also evidence for: (**T** *or T***T**). On the other hand, can we have evidence that **T** is true, whereas *T***T** is not true, so that we can establish the stronger, *non-TRI* theory **T** by itself? The positivist account makes **T** and *T***T** appear indistinguishable: for how can we distinguish whether we are *really* in a **T**-universe, or in a *T***T**-universe? Assuming we cannot, we are led to take **T** to be equivalent to: **T** *or T***T**. But this is not a valid argument: instead it is an illustration of a typical kind of fallacy inherent in positivist-empiricist conceptions of meaning. (Indeed, Reichenbach's reasoning would remove the possibility of ever establishing a time asymmetric theory, because we could reduce any theory **T** to: (**T** or *T***T**).) Such fallacies have deeply infected the subject, and they are only properly dispelled by tackling modern semantics seriously.

This is not the place to analyse such fallacies, but there is a very important application to the present problem, which brings us back to the view of Costa de Beauregard, who argues that *T* is indeed the correct time reversal operator in *relativistic quantum mechanics – but that this theory is nonetheless TRI or reversible (unlike the non-relativistic theory).*

Note that the disjunction: (QM or *T*(QM)) should be read with the quantification:

(QM or *T*(QM)) $(\forall\ \Psi$ that are physically real$)(\Psi \in \text{QM})$ or
$(\forall\ \Psi$ that are physically real $)(\Psi \in T(\text{QM})$

Now both QM and *T*(QM) taken separately do entail something very significant: *that all Ψ's of real objects have common time orientations (in their complex rotations).*

However, the genuine reversible variant of QM is not: (QM or *T*(QM)), but the weaker disjunction, which I will call QMA:

QMA $(\forall\ \Psi)(\Psi \in \text{QM or } \Psi \in T(\text{QM})\ )$

QMA allows mixtures and superpositions of wave functions with opposite directions of complex rotation. QMA contradicts both QM, and (QM or *T*(QM)), and is truly TRI.

Now what is the evidence that QM, or (QM or T(QM)) is true, rather than QMA? It is the observation that ordinary particles always have a *common relative time orientation.* But this only appears necessary in non-relativistic QM. The interpretation of the *T* operation in non-relativistic quantum theory leaves particle types (e.g. electrons) as the same particle types (electrons), and only transforms the trajectories (not the charges), and in this case, QMA cannot be realistic – because we cannot have one *electron* that satisfies QM and another *electron* that satisfies *T*(QM). Costa de Beauregard's suggestion [11] means that if we take *T* to transform particles (e.g. electrons) into their anti-particles (positrons), with anti-particles having the opposite direction of complex rotation, then the theory will have the form of QMA after all – since anti-particles do exist. What is wrong with taking *electrons* to satisfy QM, and *positrons* to satisfy *T*(QM) and *T* to transform the *charges* of particles, so that electrons transform to positrons – as Feynmann's interpretation suggests?

If de Beauregard's arguments are correct, the fundamental transformations for the time reversal, charge reversal, and space reversal have been misconstrued. But while his arguments support the idea that *T* rather than *T** is the time reversal operator, the conclusion is that *relativistic quantum mechanics is nonetheless TRI*, or reversible, because the correct interpretation is like QMA, which has a quite different logical structure to QM. But further discussion of this point of view is beyond the scope of this paper.

7. Conclusions.

The orthodox account of time reversal transformations in quantum theory presented authoritatively in a wide range of textbooks and specialized treatises is conceptually inadequate. The arguments typically put forward that *T** rather than *T* must be adopted as the time reversal operator in quantum mechanics for logical reasons are mistaken. There is no reason to reject the *T* operator on such grounds. Alternative positivist arguments from the 'indistinguishability' of QM and *T*(QM) are also laden with errors. Arguments from the 'probabilistic' *interpretation* of the quantum wave function are more serious, but they are not reasons to reject that *T* is the time reversal operator, only to conclude that the irreversibility of QM in this respect is not a physical feature of the universe - because this interpretation denies that the wave function is itself physical.

These conceptual flaws in the account of time symmetry of quantum theory, when considered along with the decisive flaws in the account of time symmetry of the probabilistic component of quantum theory raised by Watanabe, Healey, Penrose, Callender, and Holster, should be a cause for deep concern. They show how poorly the conceptual foundations of quantum theory are understood. If there is any single culprit for this state of affairs, it is the complacency engendered by the positivist approach to conceptual analysis in physics. For despite being accepted as deeply inadequate by philosophers and logicians for over fifty years, positivism unfortunately remains as the central point of departure in many conceptual accounts of quantum physics, and is found at the center of the orthodox analyses of the subject of time reversal.

T is indeed the time reversal operator in ordinary quantum mechanics, and this theory fails to be time reversal invariant unless we adopt the probabilistic interpretation. However the arguments of Costa de Beauregard show that *relativistic* quantum theory may be convincingly interpreted as being TRI by adopting the Feynmann interpretation of anti-particles as ‘particles traveling backwards in time’, i.e. by adopting the view that time reversal transforms particles into anti-particles. This view needs to be explored in more detail.

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