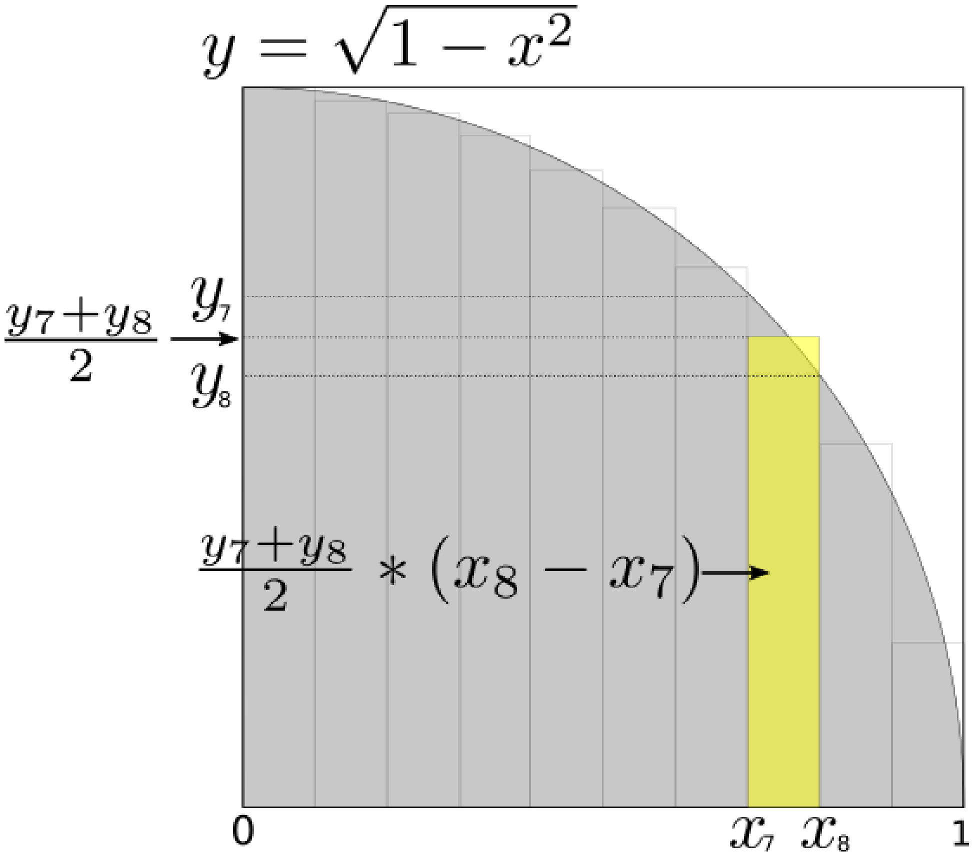
Approximate A Slice of π Essay

簡易圓周率估計

Cres Huang



The area of a sector with radius 1 enclosed by a square area of 1

ISBN 978-957-43-4649-3

Approximate A Slice of π Essay

Edition: 2017(01)-preview
2017/06
cres@mail.org

Abstract

One of the most important numbers in mathematics is π. We know it is a constant, but unable to show it in exact numbers. There are many ways to approximate it. This study illustrates another easy way to obtain the approximation of π.

π is a natural constant. However, it is an irrational number to us. I believe the calculation of π shows the fact that mathematical models can never perfectly represent nature. It is not coincident that nature fits the mathematics. Instead, it is the logically constructed mathematical models can describe the nature sufficiently. Same reasoning that a language has no logics in it. To describe our thoughts in communicable terms, any language will have to arrange words by the logics of our mind. Art, music, words, mathematics, or any form of description is language. We can say universe is musical, artistic, mathematical, and on. Nevertheless, our observation and measurement can only be approximate.

Contents

List of Figures

1 Introduction

π[3] is one of the most important numbers in mathematics. We know it is a constant, but unable to show it in exact numbers. It is defined as the ratio of a circle's circumference to its diameter[1]. There are many methods of computing the approximation. This study shows another simple graphical representation of approximating it.

2 A Slice of π

It is also the ratio of the disc with a radius of r enclosed by a square area of $4r^2$, Figure 1.

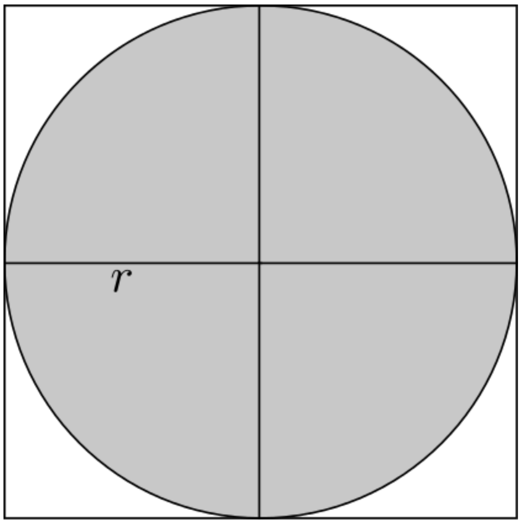


Figure 1: A disc enclosed in a square

The area of the disc is πr^2. The area of the square is $4r^2$. Hence the ratio of the disc and the square is $\frac{\pi r^2}{4r^2} = \frac{\pi}{4}$. We can simplify the calculation by slice a quarter sector from this round pie, Figure 2.

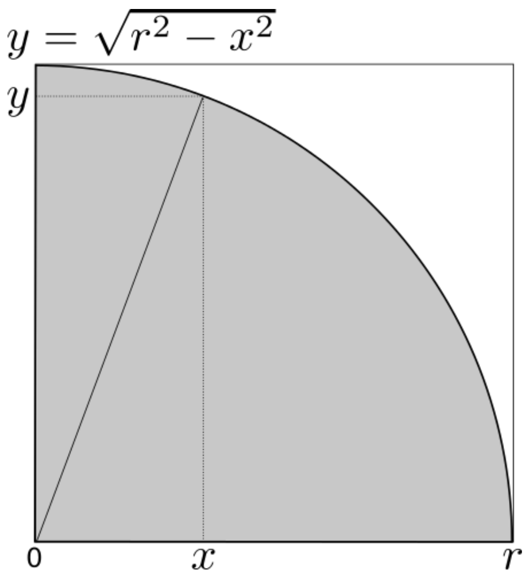


Figure 2: Quarter sector enclosed in a square

The area of the quarter sector is the integral $\int_0^r \sqrt{r^2 - x^2} \mathrm{d}x$.

2.1 Calculation The Approximation of π

Here is how the approximation is calculated.

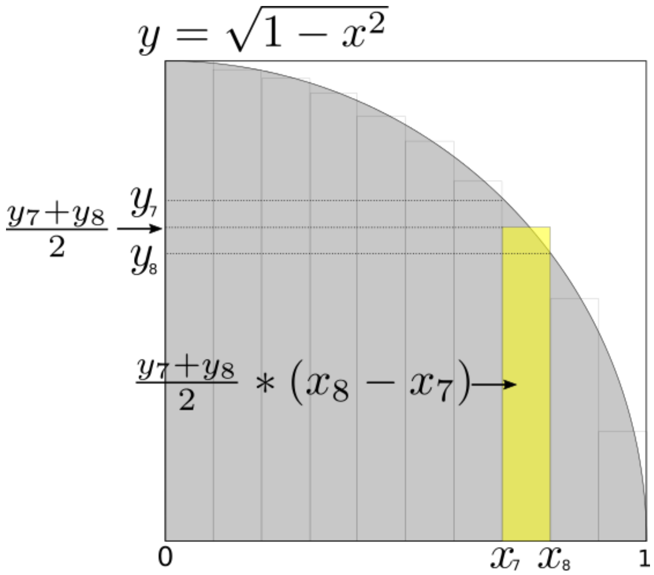


Figure 3: Area approximation of the quarter sector

By equally slice the quarter sector into thin pieces and calculate each area of rectangle at midpoint, shaded yellow in Figures 3. The total area of the rectangles will be close to the quarter sector. The thinner the slice the closer the approximation will get.

For example, we equally slice the quarter sector into 100,000 pieces, $n = 100,000$, and approximate it's area in a spreadsheet:

$$\sum_{i=1}^{n} \left(\frac{\sqrt{1 - x_i^2} + \sqrt{1 - x_{i-1}^2}}{2} \right) * \frac{1}{n} \approx 0.78539815411134.$$

The ratio of the quarter sector and the square is $(\frac{\pi r^2}{4} : r^2)$ again, $\frac{\pi}{4}$. Hence,

$$\pi \approx 4 * 0.78539815411134 \approx 3.14159261644536.$$

Which means, the size of the largest disc fits inside of a square with the area of 4 is $\pi \approx 3.14159261644536$, and the proportion is $(\frac{\pi}{4}) \approx 78.539815411134\%$. The proportion is also true for the circumference of the disc (2π) over the perimeter of the square(8). Additionally, 78.54% of 4 gives us 3.1416.

3 An Interesting Thought About π

Circular, spherical, and hexagonal patterns are prevalent in nature. The circumference of a circle is $2r\pi \approx 6.28$. We can fit 6 circles of the same size around one at center snugly.

Figure 4: Bubbles

Together, they fit nicely into a balanced shape of hexagon[2].

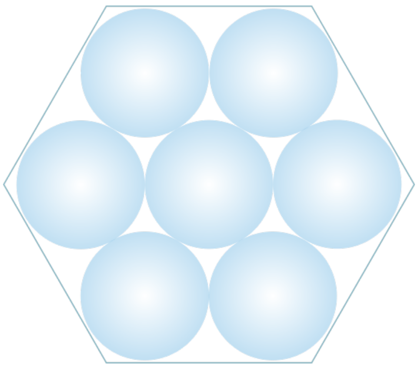


Figure 5: Bubble in a hexagon

As we see soap bubbles piling up, it compressed into a hexagonal array.

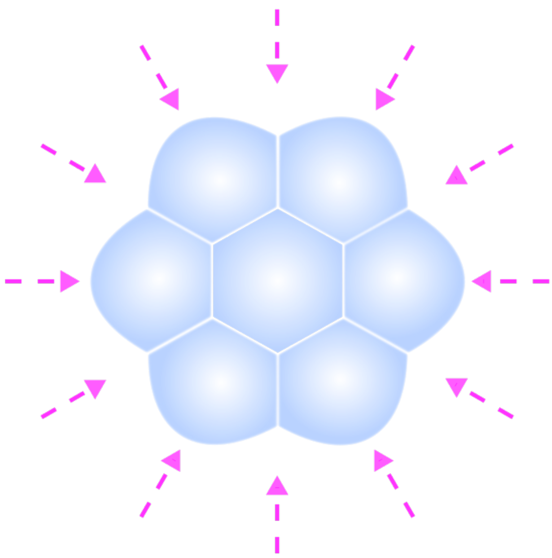


Figure 6: Hexagon

Sphere[4] is the most balanced shape started from micro scale. I believe it is another way of showing why nature prefers hexagonal shapes. Not only it's the best way to divide a space but also the strength of the structure. The efficiency and strength enable the structure to build fast and endure.

3.1 Point Dimension

I also believe that sphere is the fundamental shape of the universe. The only shape that is perfectly equilibrium is sphere. Logically and physically, no other shapes can reach the minimum origin. We can say it is point dimension. All dimensions are approaching zero, the smallest sphere point between existence and null.

4 Summary

π is a natural constant. However, it is an irrational number to us. I believe the calculation of π shows the fact that mathematical models can never perfectly represent nature. It is not coincident that nature fits the mathematics. Instead, it is the logically constructed mathematical models can describe the nature sufficiently. Same reasoning that a language has no logics in it. To describe our thoughts in communicable terms, any language will have to arrange words by the logics of our mind. Art, music, words, mathematics, or any form of description is language. We can say universe is musical, artistic, mathematical, and on. Nevertheless, our observation and measurement can only be approximate.

References

[1] *Circle*, Wikipedia, the free encyclopedia
https://en.wikipedia.org/wiki/Circle

[2] *hexagon*, Wikipedia, the free encyclopedia
https://en.wikipedia.org/wiki/Hexagon

[3] *Pi*, Wikipedia, the free encyclopedia
https://en.wikipedia.org/wiki/Pi

[4] *Sphere*, Wikipedia, the free encyclopedia
https://en.wikipedia.org/wiki/Sphere

Your advices and corrections are appreciated. Please send your correspondence to: cres@mail.org

ISBN 978-957-43-4649-3