Virtual Term is needed to solve the Navier-Stokes Fluid Millennium Prize problem

Dmitri Martila

Physics Institute, University of Tartu,

*Ülikooli 18, 50090 TARTU, Estonia**

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Abstract

The Millennium Prize problem is solved because inconsistency of Navier-Stokes fluid and the perfect fluid is found. In several examples, the inconsistency of known Physics of fluid is shown. To make the fluid consistent, the author is endorsing the conservation law with a matter-type mathematical modification, which the author calls "virtual matter". [1] This matter is part of virtual reality because mathematically exists and is physically needed.

*Electronic address: eestidima@gmail.com

A. Universe

Consider the closed Friedmann Universe with metric

$$ds^2 = -dt^2 + a^2 \left(dr^2 + \sin^2 r \, [d\theta^2 + \sin^2\theta \, d\phi^2]\right), \tag{1}$$

where the scale factor is given by $a = a(t)$. From this metric and the Einstein equations

$$G_\mu^\nu + \Lambda \, \delta_\mu^\nu = 8\pi \, T_\mu^\nu \tag{2}$$

one obtains that any matter kind must satisfy the equation

$$T_\mu^\nu = \mathrm{diag}(\rho, p, p, p)\,. \tag{3}$$

For Universe filled with "perfect fluid" at rest, one has the density of fluid as mass divided by volume:

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^3. \tag{4}$$

Therefore, from $T^\nu_{\mu\,;\nu} = 0$ we conclude the equation of state $p = 0$ [8]. Therefore, the only possible tensor in the inertial tetrad is the dust tensor. However, that is not possible, because due to the strong equivalence principle, the curvature of spacetime does not alter the physics in a small free-falling inertial laboratory. Therefore, for a Universe filled with fluid ("perfect" or viscous, or simply with any isotropic matter), the mathematically consistent state is dust. It is already the needed Solution to Millenium Prize Problem! It is the inconsistency of the Navier-Stokes equation! The Clay Institute has promised a million for a solution to this problem. So behold, I am giving it to you! The answer to the problem of consistency is "No".

I. MORE PROOFS OF NS INCONSISTENCY

A. What is the curvature of spacetime we shall work in?

You may say: we consider a perfect fluid in flat spacetime with Descartes coordinates (x, y, z). However, it holds for the general case of curved spacetime, because of the following argument. Consider the perfect fluid in an inertial tetrad. Then there holds $\Gamma^\nu_{\mu\alpha} = 0$ and, therefore, e.g. $g_{\nu\,\alpha\,;\mu} = g_{\nu\,\alpha\,,\mu} = 0$, with

$$T^{\nu\mu}_{;\mu} = T^{\nu\mu}_{,\mu} = 0\,. \tag{5}$$

Latter is shown in the Appendix. I am considering it as the solution to known "Energy localization problem" in General Relativity.

B. Calculation

The energy-momentum tensor of the perfect fluid [3, 4] reads

$$T^{\nu\mu} = (\rho + p)\, u^\nu\, u^\mu + p\, g^{\nu\mu} \tag{6}$$

with $u^\nu\, u_\nu = -1$. Then $T^{\nu\mu}_{\;\;,\nu} = 0$ means

$$0 = u_\mu\, T^{\nu\mu}_{\;\;,\nu} = \frac{d\rho}{d\tau} + (\rho + p)\,\Theta\,, \tag{7}$$

where $\Theta = u^\nu_{\;,\nu}$ and

$$\frac{d\rho}{d\tau} = \frac{\partial\rho}{\partial x^\nu}\, u^\nu\,. \tag{8}$$

Let us calculate the density current [2, 3]

$$J^\mu = -T^{\nu\mu}\, u_\nu = \rho\, u^\mu\,. \tag{9}$$

Then

$$J^\mu_{\;,\mu} = \frac{d\rho}{d\tau} + \rho\,\Theta\,, \tag{10}$$

and so from Eq. (7)

$$J^\mu_{\;,\mu} = -p\,\Theta\,. \tag{11}$$

However, it is known that in flat spacetime the continuity equation for density current $J^\mu_{\;,\mu} = 0$ holds [3–5]. Therefore, $p = 0$ which is a violation for the fluid: it is no longer fluid but dust! One can derive the same result for a Navier-Stokes viscous fluid, but because the perfect fluid is the case of a Navier-Stokes fluid, the Navier-Stokes fluid is already proven to be inconsistent. In other words, this can be regarded as part of the solution to the more general problem of a fluid with viscosity, showing also $p = 0$ by another approach (see Appendix) [6].

C. The Navier-Stokes problem

Let the viscous coefficients are time and space functions: $\eta = \eta(x^\nu)$, $\zeta = \zeta(x^\nu)$. If the fluid is electrically neutral, then the potential field acting on the fluid is zero, $\vec{U} = 0$.

Nevertheless, the fluid can experience pushing from the sides of the fluid (the wings of an airplane are pushing air around the plane).

The norm of four-velocity is given $u^\nu u_\nu + 1 = 0$. By taking the covariant gradient, one gets

$$0 = (u^\nu u_\nu + 1)_{;\alpha}\, u^\alpha = a^\nu u_\nu + u^\nu a_\nu = 2\, a^\nu u_\nu\,, \tag{12}$$

where 4-acceleration $a^\nu = u^\nu_{;\alpha}\, u^\alpha$.

The four-current density is

$$J^\nu = -T^{\nu\mu}\, u_\mu = \rho\, u^\nu\,, \tag{13}$$

where the energy-momentum tensor $T^{\nu\mu}$ of the viscous fluid is taken from Ref. [3]. One obtains

$$J^\nu_{;\nu} = \frac{d\rho}{d\tau} + \rho\,\Theta\,, \tag{14}$$

where $\Theta = u^\nu_{;\nu}$ [7].

But on the other hand, because of $T^{\nu\mu}_{;\nu} = 0$ one has

$$(-T^{\nu\mu}\, u_\mu)_{;\nu} = -T^{\nu\mu}\, u_{\mu\,;\nu} = -\beta + \eta\, a^\nu a_\nu\,, \tag{15}$$

where

$$\beta = p\,\Theta + (2\eta/3 - \zeta)\,\Theta^2 - 2\,\eta\, u_{\nu\,;\mu}\, u^{(\nu\,;\mu)}\,, \tag{16}$$

and where $2\, u^{(\nu\,;\mu)} = u^{\nu\,;\mu} + u^{\mu\,;\nu}$.

Moreover, we have

$$u_\mu\, T^{\nu\mu}_{;\nu} = -\frac{d\rho}{d\tau} - \rho\,\Theta - \beta = 0\,. \tag{17}$$

In the derivations the following facts were used:

$$0 = (u^\beta u_{\beta\,;\alpha})^{;\alpha} = u^{\beta\,;\alpha} u_{\beta\,;\alpha} + u^\beta u^{;\alpha}_{\beta\,;\alpha}\,, \tag{18}$$

$$a^{;\alpha}_\alpha = (u^\beta u_{\alpha\,;\beta})^{;\alpha} = u^{\beta\,;\alpha} u_{\alpha\,;\beta} + u^\beta u^\alpha_{;\beta\,;\alpha}\,, \tag{19}$$

where contravariant "covariant derivatives" are made with help of metric tensor, e.g., $u^{;\alpha}_{\beta\,;\alpha} \equiv u_{\beta\,;\alpha\,;\gamma}\, g^{\gamma\,\alpha}$.

Therefore, from Eqs. (13)–(17) one obtains $a^\nu a_\nu = 0$. From Special Relativity it is known that $a^\nu a_\nu$ is zero only if the three-acceleration is zero: $\vec{a} = (0, 0, 0)$. The latter implies that the motion is force-free, and the streamlines of the fluid are geodetics $a^\nu = 0$ at every point of spacetime. Therefore, without experiencing any acceleration, the fluid is static and experiences no non-compensated pushing from the edges (no flying airplane then). In conclusion, the general (mathematically consistent) solution of the N-S equation is the pressure-free dust, $p = 0$.

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[1] Dark Matter and Dark Energy Explained by Fix to Vanishing of Falling Matter, viXra:1911.0425

[2] L.D. Landau, E.M. Lifshitz. The Classical Theory of Fields: Course of Theoretical Physics. Vol. 2, Butterworth-Heinemann, 1975.

[3] A.P. Lightman, W.H. Press, R.H. Price, S.A. Teukolsky. Problem Book in Relativity and Gravitation. Princeton University Press, 1975.

[4] L.D. Landau, E.M. Lifshitz. Fluid Mechanics: Course of Theoretical Physics. Vol. 6, Pergamon Press Verlag, 1966, 47–53; J.N. Reddy. An Introduction to Continuum Mechanics. Cambridge, 2008, 212–214.

[5] G.G. Stokes, "On the theories of internal friction of fluids in motion, and of the equilibrium and motion of elastic solids", Trans. Cambridge Philos. Soc, Vol. 8, 1845.

[6] The case $\Theta = 0$ in Eq. (11) would mean that $\rho =$ const for any pressure p (cf. Eq. (10)). But it is not possible for $p \approx 0$, because then we have dust with varying density. Therefore, $\Theta \neq 0$ and, thus, our result $p = 0$. Moreover, absolutely rigid object is not allowed, because the speed of interactions is finite.

[7] The expression $J^{\nu}_{;\nu} = K$ turns into inertial tetrad into known $J^{\hat{\nu}}_{,\hat{\nu}} = K$. Thus, must hold $K = 0$, because is known, that $J^{\hat{\nu}}_{,\hat{\nu}} = 0$.

[8] Here and in the following the index with semicolon means the covariant derivation using Christoffel symbols, while the index with comma means ordinary derivative with respect to the spacetime coordinate x^{ν}.