Brief Solutions to Collatz Problem, Goldbach Conjecture and Twin Primes

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I published some solutions [1] a time ago to Goldbach Conjecture, Collatz Problem and Twin Primes; but I noticed that there were some serious logic voids to explain the problems. After that I made some corrections in my another article [2]; but still there were some mistakes. Even so, I can say it easily that here I brought exact solutions for them out by new methods back to the drawing board.

1 Goldbach Conjecture

The main question about Goldbach Conjecture is pretty clear that is each even number sum of two prime numbers?

1.1 The solution

Being p is a prime number, for definition of $p_4 > p_3 \wedge p_4 - p_3 = 2n + 2 \wedge n > 2 \wedge n \in \mathbb{Z}^+$; first number group which is created by n pieces non-prime consecutive positive odd whole numbers, the smallest odd whole number it has is p_3+2 or p_4-2n and the biggest odd number it has is p_3+2n or p_4-2, contains greater numbers than last number group which contains $n-1$ pieces consecutive non-prime odd numbers and none of the numbers n group contains, for definition of $p_2 > p_1 \wedge p_2 - p_1 = 2(n-1) + 2$ is before n group, and the smallest odd number it has is p_1+2 or $p_2-2(n-1)$ and the biggest one it has is $p_1 + 2(n-1)$ or $p_2 - 2$. Also $n-1$ groups which contain $n-1$ pieces non-prime consecutive odd numbers and have greater numbers than n groups have are possible; but these groups can never exist until n groups emerge because of the above stated reasons and definitions; because n groups also contain 2 pieces $(n-1)_1$ and $(n-1)_2$ consecutive groups which $n-2$ pieces elements of them are common with n groups.

The difference between $n-1$ and $(n-1)_m$ is not to be of prime number there after last number of first group and before first number of last group of emerging two groups; therefore when two groups emerged as $(n-1)_1$ and $(n-1)_2$, it means that n group already had been emerged spontaneously, and is first n group. Being $(n-1)_1$ is the first and $(n-1)_2$ is the second group, as last element of $(n-1)_2$ group is always bigger than all numbers of $(n-1)_1$, it means $(n-1)_1$ had already been emerged before n group. Also as $n-1$ group must exist at the place before $(n-1)_1$ group, then numbers of n group are always greater than numbers of $n-1$ group for the definition of n and $n-1$ groups. Here all possible $n-k$ groups for $m, k \in \mathbb{Z}^+$ definition are unique; but $(n-k)_m$ groups which have common element with another groups are not unique.

Being $k = 0$, for $n - k = n$ groups, 1 piece of selected n pieces consecutive odd numbers from 3 to numbers of $n+1$ group has to be prime number; so if some tables are made like table 1 and table 2 for each n, they will help about the main question.

When similar tables for the other n groups are made, and when an addition is done by order between selected n pieces of the smallest consecutive x number group elements and elements of y groups that each group of y contains n pieces of consecutive odd numbers, the result will be like in the tables. Here, the first group of y is the same with x column. The first and the smallest number of the next y group is greater by 2 than the first and the smallest number of the previous y group. The sum of b is always an even number as well.

Table 1: The table for n=2

x			**y**				
3	3	5	7	9	11	13	...
5	5	7	9	11	13	15	...
	x	**+**	**y**	=	**b**		
	6	8	**10**	**12**	14	...	2a+4
	10	**12**	14	16	18	...	2a+8

Table 2: The table for n=3

x			**y**					
3	3	5	7	9	11	13	...	
5	5	7	9	11	13	15	...	
7	7	9	11	13	15	17	...	
	x	**+**	**y**	=	**b**			
	6	8	10	12	**14**	**16**	...	2a+4
	10	12	**14**	**16**	18	20	...	2a+8
	14	**16**	18	20	22	24	...	2a+12

As the first y group is also x group and as also the biggest number of x group is always $2n+1$, the biggest number of first b group must always become $2(2n+1) = 4n+2$. This number is the first number which starts to be common with all the other b even numbers which are formed by different x elements for the same n value in the tables; because it is in the last emerging line. Each even number after this number can be formed absolutely as n pieces the number itself is included as well. As 1 piece of selected consecutive n pieces odd numbers must be prime number until $n+1$ group, minimum 1 piece of the even numbers which are in each even number group has n pieces of the same even number must be sum of two prime numbers.

Here, if also $4n+4$ number after $4n+2$ number is included, if all of emerging n pieces of the same $4n+4$ even numbers are not the numbers of $n+1$ group in a table for the same n value, as all results of $4n+4$ and $4n+2$ form set of even num-

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bers greater than 8, then it means each even number which is greater than 8 absolutely must be sum of two prime numbers.

The equation between a values that first a namely a_1 which gives even numbers in the first line for $a = [1, \infty)$ over $2a + 4$, and a_n which accepts the first even as $4n + 2$ and gives the numbers of $2a + 4n$ is (1)

$$a_1 = a_n + 2n - 2 \tag{1}$$

Here, result of a_1 for $a_n = 1$ is also equal to number of used odd numbers in the first y line to form $4n + 2$ number or is equal to number of used numbers which are different than each other in the tables for the same n value, outside of using a_n to form $2a_n + 4n$ namely $4n + 2$ number; because it is also $x + y$ operation number of forming $4n + 2$ even number in the first line; therefore a_1 number over (1) for $a_n = 2$ is required to form $4n + 4$, and it must be (2)

$$N = 2n \tag{2}$$

1.2 The result

As the result for the above stated information, assume that $4n+2$ and $4n+4$ numbers cannot be sum of two prime numbers for each n value. Last n pieces of y consecutive odd numbers which are required to form $4n + 2$ and $4n + 4$ numbers for required n value, must be n group numbers; thus only $2n - n = n$ pieces y consecutive odd numbers can be used it means being 3 is the first usable number on the first y line; but already this means that all numbers except 1 are non-prime consecutive odd numbers for each n value that is impossible. Even if it is assumed that, for the worst possibility n and $n - 1$ groups emerge together by the same numbers as non-unique groups, also the information stated above that this assumption is impossible. Already if it is impossible even for n group, when it is assumed that there are another groups,

$$2n - \sum_{n=2}^{n} n \tag{3}$$

number of used or usable consecutive y odd numbers will decrease by (3), and is impossible for $n > 1$ definition. As and if we do not know prime separation, for the worst possibility of number of existent primes in $2n$ pieces usable consecutive odd numbers on the first y line, assume that for n, there are consecutive unique n groups until last $2n$ number even if actually they cannot be fitted such that as emerging number of the numbers will be bigger than $2n$ by this way; but if it is right, then amount will not be important that you can assume that there are $(2n)^2$ pieces n groups if essence of the function provides this that it is provided here; because the number of the non-prime numbers will increase greater than the primes for assumed n pieces unique groups; so as unique groups are between two primes, there must be $2n$ pieces prime numbers. As this $2n$ is also equal to number of the used numbers on the first y line that it is not important which numbers of $2n$ pieces numbers are prime or not prime here, absolutely minimum 1 piece of each n pieces the same even $4n + 2$ numbers and minimum 1 piece of each n pieces the same even $4n + 4$ numbers which emerge in each table separately are absolutely sum of two prime numbers. Also it means that all even numbers greater than 8 are sum of two prime numbers. This is also proof of infinite number of twin primes.

2 Twin primes

Select any unique number group which has n pieces of consecutive non-prime odd numbers. This group has to exist between 2 prime numbers according to the definition stated at the beginning between prime numbers and group numbers; because otherwise there will occur a group like $n + 1$ group instead of n group that actually infinite number of non-prime numbers can be consecutive. For example, let us take consecutive multiples of 3, 5, 7, 9 and 11 for $n = 5$ group. Being a is an odd number, any multiples of odd numbers become $a(2x + 1)$ for required x; so over $(6x_1 + 3) + 2 = 10x_2 + 5$, it becomes $x_1 = 5x$. Over $(6x_1 + 3) + 4 = 14x_3 + 7$, it becomes $x_1 = 7x$. Over $(6x_1 + 3) + 6 = 18x_4 + 9$, it becomes $x_1 = 9x$. Over $(6x_1 + 3) + 8 = 22x_5 + 11$, it becomes $x_1 = 11x$. Results of $6x_1 + 3$ which are odd multiples of 3 become $30x + 3$, $42x + 3$, $54x + 3$ and $66x + 3$ for the stated x_1 values. If also these are made equal to each other, being $(11 \cdot 9 \cdot 7 \cdot 5 \cdot x) = m$ and the first number of the group is multiple of 3, consecutive multiples of the group numbers become by order $6m + 3$, $6m + 5$, $6m + 7$, $6m + 9$ and $6m + 11$. For more consecutive odd multiples, we can increase the number of used numbers in a group forever.

I selected n_1, n_2, n_3 and n_4 consecutive odd numbers in $n = 4$ group like $p_1 n_1 n_2 n_3 n_4 p_2$ being p is prime number. Minimum one of these n numbers has to be multiple of 3; because separation of odd multiples of 3 is according to $6x + 3$, and so there are always 2 consecutive odd numbers between two consecutive odd multiples of 3. Here, if n_2 becomes odd multiple of 3, then p_2 must be the next multiple of 3 that this is only possible for $n = 5$. If n_3 becomes odd multiple of 3, then p_1 must be the previous multiple of 3 that this is also possible for $n = 5$. As it was said, it is possible to form groups have infinite number of consecutive non-prime odd number, namely $n = 4$ must exist anyway.

If n_1 becomes odd multiple of 3, then n_4 must be the next multiple of 3 and also n_5 becomes the next multiple of 3 after n_4 as n_0 became the previous odd multiple of 3 before n_1 over $n_0 n_x p_1 n_1 n_2 n_3 n_4 p_2 n_y n_5$.

If n_4 becomes odd multiple of 3, then n_5 becomes the next multiple, and n_0 and n_1 become the previous multiples of 3; thus n_1 and n_4 are pretty suitable to be odd multiple of 3.

Here, infinite number off odd consecutive n number can take place after n_5; so element number of the next group after $n = 4$ is not important; but n_y is always prime or not, this is important. Over $n_y = n_5 - 2 = (6x + 3) - 2$, it becomes $n_y = 6x + 1$. Hence, n_y never can be only prime number where

$x \in \mathbb{Z}^+ \wedge x > 0$. It is not prime for required x, and otherwise it is prime for emerging odd numbers between two n_y and n_{y+1} numbers which are a result of consecutive two x and $x+1$ values; so when it becomes $n_y = p_3$, it is a twin prime group between n_4 and n_5; thus twin primes are in infinite number.

3 Collatz Problem

The main question about the Collatz Problem is also pretty clear. When a positive whole number is selected, if the number is an even number then it is divided by 2; otherwise it is multiplied by 3, and after that 1 is added to the result. When the same operation with required option of the problem due to the condition of to be odd or even number of the result is repeated for the last results, can each positive integer which is different than 0 and 1 be reduced into 1?

3.1 The solution

If the input number is an even number, and if it is not an even number as 2^n as well for definition of $n \in \mathbb{Z}^+ \wedge n > 0$; being pn is process number, when the input number is divided by pn times 2 or directly by 2^{pn}, each positive even number absolutely turns into a positive odd number as they can be defined as $(2x+1) \cdot 2^n$ for definition of $x, n \in \mathbb{Z}^+ \wedge n > 1 \wedge x > 0$; thus we should only work over odd numbers.

$$a_{n+1} = \frac{3a_n + 1}{2} \tag{4}$$

Over (4), it must become $a_{n+1} = \{2, 5, 8, 11, ..., 3x - 1\}$ where $x \in \mathbb{Z}^+ \wedge x > 0$ for a limited interval. For the numbers which make a_n an odd number, it becomes $a_{n+1} = \{5, 11, 17, ..., 6x - 1\}$ for a limited interval and the same x definition. Also it becomes $a_n = \{3, 7, 11, 15, ..., 4x - 1\}$ over a_{n+1} odd numbers for the same conditions.

The below is a table over a_n and a_{n+1} numbers by order over (4) for a limited interval being E is even and O is odd.

Table 3: a_n and a_{n+1} numbers

a_n	3	7	11	15	19	23	...
a_{n+1}	5	11	17	23	29	35	...
a_{n+2}	E	O	E	O	E	O	...

On the table of 3, for $12x - 7$ numbers from a_{n+1} numbers with the same x definition, it becomes (5).

$$18x - 10 = \frac{3(12x - 7) + 1}{2} \tag{5}$$

The result of (5) is absolutely even number for each x. For $12x - 1$ numbers from a_{n+1} numbers with the same x definition, it becomes (6).

$$18x - 1 = \frac{3(12x - 1) + 1}{2} \tag{6}$$

The result of (6) is absolutely odd number for each x. Right this point, the question is this that for (7),

$$a_{n+2} = \frac{3a_{n+1} + 1}{2} \tag{7}$$

when a_{n+2} becomes an even number and divided by 2^{pn}, does emerging odd numbers as a result always emerge before a_{n+1} in set of odd numbers due to number order or can it be bigger number than a_{n+1}?

As the answer, if the result of the operation of (7) becomes even, to realize of to be reduced of the result of $\frac{3a_{n+1} + 1}{2 \cdot 2^{pn}}$ operation into an odd number which is before a_{n+1}, the condition of (8) always has to be provided.

$$1 > \frac{\frac{3a_{n+1} + 1}{2 \cdot 2^{pn}}}{a_{n+1}} \tag{8}$$

If (8) is edited then as (9),

$$1 > \frac{1}{2^{pn+1}}\left(3 + \frac{1}{a_{n+1}}\right) \tag{9}$$

the inequality of (9) always provides this for the definition of $pn > 0 \wedge a_{n+1} > 1 \wedge pn, a_{n+1} \in \mathbb{Z}^+$.

As a result, when a_{n+2} is reduced into an odd number, the odd number is always before a_{n+1} odd, and is smaller than it. It means that the odd number as a result of $a_m = \frac{3a_{n+1} + 1}{2 \cdot 2^{pn}}$ is always smaller than a_{n+1} number. Even if $\frac{3a_m + 1}{2}$ becomes even number, again it can be reduced into a smaller odd number than both a_m and a_{n+1}, and it is acceptable for the other repeats as well.

Right this point, a second question emerges that is there a number which always gets bigger and does not become an odd number on (4) infinite chain.

As the answer, there is table for the numbers which do not emerge on table 3. If these numbers are included to the numbers on table 3 as well, the sum is set off odd numbers. There will no other odd number which is not included to the calculations.

Table 4: The numbers which are not in the previous table

5	9	13	**17**	21	25	**29**	...	4x+1

The numbers which are written thick are also in a_{n+1} line in table 3. The other numbers are the numbers which are not in table 3.

If some groups are made for $4x + 1$ numbers in table 4, there will only emerge 3 groups for $12x - 3$, $2x + 1$ and $12x - 7$ numbers for the same x definition. Being $a_{n+1} = 12x - 3$, (10) always gives even result.

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$$18x - 8 = \frac{3(12x-3)+1}{2} \tag{10}$$

Being $a_{n+1} = 12x + 1$, (11) always gives even number result as well.

$$18x + 2 = \frac{3(12x+1)+1}{2} \tag{11}$$

As $12x - 7$ numbers, they are already the same numbers with a_{n+1}, and at the result of (7) they always give even number being $a_{n+1} = 12x - 7$.

As even numbers, they can always be reduced into a smaller odd number than the odd number which makes them even number in the operation of (4) as it was proved; thus if each one of $a_n = 4x - 1$ numbers do not get greater by turning into an odd number when (4) is repeated for each n number where $n \in \mathbb{Z}^+$, it means all positive whole numbers different than 0 and 1 can be reduced into 1.

In table 3, $8x - 5$ numbers from a_n numbers turn into a_{n+1} number which gives even result over (7); thus the only chance is to give odd result always of $8x - 1$ numbers over (4) infinite chain. To be realized of this, the same numbers with a_n must emerge on a_{n+1} line in table 3. Also always the numbers which give odd result must emerge on a_{n+1} line between them. For the same x condition, $4x - 1$ and $6x - 1$ operations give a_n and a_{n+1} over (4) for the same x number; so the waited loop occurs or does not occur,

$$x_1 = \frac{6x_2}{4} \tag{12}$$

(12) shows this over the equality of $4x_1 - 1 = 6x_2 - 1$. For (12), it becomes $x_1 = 3t$ and $x_2 = 2t$ over $t \in \mathbb{Z}^+ \wedge t > 0$ condition; so the problem is reduced into the rule of table 5 below.

Table 5: The numbers for each t

2t	2	4	6	8	10	12	...
3t	3	6	9	12	15	18	...

Each number on $4x - 1$ and $6x - 1$ is also order number of a_n and a_{n+1} in table 5; thus the number has $3t$ order number on a_n or $2t$ line and the number has $2t$ order number on a_{n+1} or $3t$ line in table 5 are the same numbers.

As the odd numbers on $3t$ line in table 5 are a_{n+1} numbers which give even result in table 3, they are elected; thus table 5 turns into table 6.

Table 6: The other numbers for each t

4t	4	8	12	16	20	24	...
6t	6	12	18	24	130	36	...

In table 6, to occur of the infinite loop, when a $4t$ number is selected, also $6t$ number which is under it and on $6t$ line in table 6 must be even number, and also this $6t$ number must take place on $4t$ line again. This condition has to take place for one or more than one number to be broken of the Collatz's reducing chain, and then one or more than one number will not be reduced into 1; but this is impossible; because for $t_{n+1} = \frac{6t_n}{4}$ where $t > 0 \wedge t, n \in \mathbb{Z}^+$, for each t whole number,

$$t_{n+1,t} = \lim_{n \longrightarrow \infty} \frac{6t_{n,t}}{4} \tag{13}$$

(13) has to be provided for the condition of (14),

$$t_{n,t}, t_{n+1,t} \in \mathbb{Z}^+ \tag{14}$$

where $t_{n,t} = 4t$, t is the order number of the number which is waited of starting the loop from it, and n is the repeat number of (13) for each t. As this condition of (14), it cannot be provided for each t. For example, for $t_{1,1} = 4$, it becomes $6t_{1,1} = 4t_{2,1}$ and so becomes $t_{2,1} = 6$. For $t_{2,1} = 6$, it becomes $6t_{2,1} = 4t_{3,1}$ and so becomes $t_{3,1} = 9$. For $t_{3,1} = 9$, it becomes $6t_{3,1} = 4t_{4,1}$ and so becomes $t_{4,1} = 27/2$. As it can be seen, $t_{4,1} \notin \mathbb{Z}^+$ and so the condition of (14) cannot be provided.

3.2 The result

(13) cannot continue forever; because wee need a number which has infinite number of common divisors like 4^∞ or $(2x + 1) \cdot 4^\infty$ imaginary numbers. As it can be seen, only we can increase the repeat number by using a t number like 4^m where $m > 0 \wedge m \in \mathbb{Z}^+$ that if m gets bigger, then the repeat will increase; but there is no infinite repeat; hence, any whole number absolutely can be reduced into 1 by changing operation numbers of the Collatz's rule due to the used number.

For the repeats of (4), being 2^m is order number of the selected odd number of a_n in table 3 where $m > 0 \wedge m \in \mathbb{Z}^+$, it gives $m + 2$ pieces odd number, and then the last one gives even number on (4) for 2^m. Table 7 is a demonstration of this.

Table 7: The other numbers for each t

m	a_n				
1	7⟶	11⟶	17		
2	15⟶	23⟶	35⟶	53	
3	31⟶	47⟶	71⟶	107⟶	161
.					
.					

Also we can use an operation like (15), and we can derive another operation as well. The below is an example.

$$a = 7 + \sum_{m=3}^{m} 2^m \tag{15}$$

Here for (15), $m + 1$ pieces odd numbers or repeats on (4) emerges being $a = a_0$ which is first input number on (4). You can write your own operation as well.

References

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