On quantization of interacting velocity gauge fields

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Abstract

The many velocity gauge fields interaction Lagrangian density is extended to a continuous spectrum Lagrangian then quantized and normalized. The Euler-Lagrange equation is transformed to the Yang-Mills field, integral and curvature form operators. The Yang-Mills field form operator quantization gives the equivalent Dirac equation, the integral form operator quantization gives perturbation series similar to the path integral formulation of Quantum mechanics and the curvature form operator is used to get the Einstein-Hilbert Lagrangian density; operators are quantized with mass eigenvalue. Finally, the normalization of the wave function gives a charged massive particle Lagrangian density with six representations.

I. Introduction

The many velocity gauge fields interaction Lagrangian density introduced in article [1] is extended to a continuous spectrum Lagrangian then quantized and normalized. First, the source terms in the Lagrangian density of N interacting fields are included in the connection and summations are converted to integral over a small value $d\epsilon$. Second, the solution of the Euler-Lagrange equation is converted to the Yang-Mills field, curvature and integral form operators and quantized with mass eigenvalue. The Yang-Mills field operator quantization gives the Dirac equation, the integral form operator quantization gives the perturbation series similar to the path integral formulation of Quantum mechanics and the curvature form operator is used to get the Einstein-Hilbert Lagrangian density. Finally, the normalization of the wave function gives a charged massive particle Lagrangian density with six representations.

II. Lagrangian

From the Lagrangian density in article [1], including the source term in the connection and summing over a small value $d\epsilon$, the Lagrangian is evaluated to

$$L = \int d\epsilon \left[\alpha^{\mu}\left[\partial_{\mu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\phi\right]^{T}\left[\alpha^{\nu}\left[\partial_{\nu} + \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{1}$$

Solving the Euler-Lagrange equation for the Lagrangian in equation (1) yields

$$\alpha^{\mu T}\left[\partial_{\mu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\alpha^{\nu}\left[\partial_{\nu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi = \frac{1}{i\hbar}\int d\epsilon\left[\alpha^{\mu T}\phi_{\mu}\alpha^{\nu}\left[\partial_{\nu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{2}$$

III. Yang-Mills equations form

Let us define the field F and substitute in equation (2) as

$$\alpha^{\nu}\left[\partial_{\nu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi = F \tag{3}$$

$$\alpha^{\mu T}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]F = \alpha^{\mu T}\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}F \tag{4}$$

Equation (3) and (4) have the Yang-Mills equations form with source term, but they are different as they are operator identities to be quantized in the next sections.

IV. Dirac equation form

Let us quantize the differential operator in equation (4) by introducing the mass eigenvalue as

$$\alpha^{\mu T}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]F = -\frac{m^2c^2}{\hbar^2}\phi \tag{5}$$

Defining the wave function as

$$\psi = \begin{bmatrix} i\hbar F \\ mc\phi \end{bmatrix} \tag{6}$$

Using equation (6), equations (3) and (4) can be written as

$$i\hbar\Gamma^{\mu}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\psi = mc\psi \tag{7}$$

Equations (7) is the equivalent to the Dirac equation with interaction term and different gamma matrices introduced in article [1].

V. Integral equation form

Similarly to equation (5), an integral equation with mass eigenvalue part of the equation (4) is given by

$$\alpha^{\mu T}\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}F = -\frac{m^2c^2}{\hbar^2}\phi \tag{8}$$

From article [2], equations (8) can be written as

$$\frac{i\hbar}{mc}\int e^{-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}}F = \phi \tag{9}$$

Expending equation (9) in power series yields

$$\frac{i\hbar}{mc}\int\sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n}F=\phi \tag{10}$$

Substituting equation (10) in the wave function in equation (6) gives

$$\psi\ = i\hbar\left[\frac{F}{\int\sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n}F}\right] \tag{11}$$

VI. Curvature Form

Evaluating equation (2) with

$$\alpha^{\mu^T}\alpha^{\nu}+\alpha^{\nu^T}\alpha^{\mu}=2\eta^{\mu\nu} \tag{12}$$

$$\Delta=\eta^{\mu\nu}\left[\partial_{\mu}\partial_{\nu}-2\left[\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\partial_{\nu}\right] \tag{13}$$

$$R=\alpha^{\mu^T}\alpha^{\nu}\left[\partial_{\mu}\left[\frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]-\left[\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\left[\frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\right] \tag{14}$$

Using equations (3), (12), (13) and (14), equation (2) can be evaluated to

$$[\Delta-R]\phi=\alpha^{\mu^T}\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}F \tag{15}$$

Quantizing the curvature operator in equation (15) as in equations (5) and (8) gives

$$[\Delta-R]\phi=-\frac{m^2c^2}{\hbar^2}\phi \tag{16}$$

VII. Normalization

Since $\psi^{\dagger}\psi$ is the probability density function, it is normalized as

$$\int dx^4\psi^{\dagger}\psi=1 \tag{17}$$

The variation of equation (17) yields

$$\delta\int dx^4\psi^{\dagger}\psi=0 \tag{18}$$

Using equations (3) and (6) in differential form and without operators, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian densities

$$\mathcal{L}=\frac{m^2c^2}{\hbar^2}\phi^{\dagger}\phi+\left[\alpha^{\mu}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\phi\right]^{\dagger}\left[\alpha^{\nu}\left[\partial_{\nu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{19}$$

$$\mathcal{L} = \frac{m^2 c^2}{\hbar^2}\phi^\dagger\phi + F^\dagger F \tag{20}$$

Using equations (3) and (11) in integral form, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian densities

$$\mathcal{L} = \left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^n F\right]^\dagger \left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^n F\right] + \left[\alpha^\mu\left[\partial_\mu - \frac{1}{i\hbar}\int d\epsilon\phi_\mu\right]\phi\right]^\dagger \left[\alpha^\nu\left[\partial_\nu - \frac{1}{i\hbar}\int d\epsilon\phi_\nu\right]\phi\right] \tag{21}$$

$$\mathcal{L} = \left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^n F\right]^\dagger \left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^n F\right] + F^\dagger F \tag{22}$$

Using equations (3), (16) and (19) in curvature form, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian densities

$$\mathcal{L} = \phi^\dagger R\phi - \phi^\dagger\Delta\phi + \left[\alpha^\mu\left[\partial_\mu - \frac{1}{i\hbar}\int d\epsilon\phi_\mu\right]\phi\right]^\dagger \left[\alpha^\nu\left[\partial_\nu - \frac{1}{i\hbar}\int d\epsilon\phi_\nu\right]\phi\right] \tag{23}$$

$$\mathcal{L} = \phi^\dagger R\phi - \phi^\dagger\Delta\phi + F^\dagger F \tag{24}$$

VIII. Conclusion

In summary, the velocity gauge fields were quantized with mass eigenvalue and normalized, using the Yang-Mills field, curvature and integral form operators. The Yang-Mills field form operator quantization gave the equivalent Dirac equation, the integral form operator quantization gave perturbation series similar to the path integral formulation of Quantum mechanics and the curvature form operator quantization was used to get the Einstein-Hilbert Lagrangian density. Finally, the normalization of the wave function gave the Lagrangian density of the charged massive particle with six representations.

IX. References

[1] Rukundo, JPR, 2017. On velocity gauge field approach of interactions. Quaternion Physics, [Online]. 1, 5. Available at: https://quaternionphysics.files.wordpress.com/2017/11/on-velocity-gauge-field-approach-of-interactions8.pdf [Accessed 26 November 2017].

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