

Title Goldbach Conjecture

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Abstract The Goldbach Conjecture states:

Every even number greater than 4 can be written as the sum of two primes.

Examples:

$$\begin{aligned}6 &= 3+3 \\8 &= 3+5 \\10 &= 3+7 \quad 5+5 \\&\vdots \\&\vdots \\1224 &= 71+1153 \quad 73+1151 \quad 191+1033 \quad 193+1031\end{aligned}$$

We will call the two primes summing to a particular number a Goldbach Pair (GP) for that number.

Method

This treatment uses two simple facts and seems to confirm the Conjecture without providing an obvious method for discovering GP primes.

Proof

There are two self-evident facts:

- i. Every positive integer $E > 2$ can be written as the sum of two smaller positive integers.
- ii. If an even number E is a GP its component primes must be less than E .

Consider any even number $E > 6$ where all even numbers less than E are GP's. (C)

Then $E = E_1 + E_2$ $\{(E_1, E_2) \text{ even}; 2 \leq E_1 \leq E_2 < E\}$ by (i)

Therefore we can write:

$$E = (A-a) + e \quad \{e \text{ even}; 2 \leq e; (A, a) \text{ prime}; 3 \leq a < A \leq E-3\}$$

Thus $E-e = (A+a) - 2a$

So that if $e = 2a$

It follows $E = (A+a)$ and ii) confirms $(A+a)$ is a GP

An example using $E=12$ which can involve at most 3 primes and which satisfies (C) is:

$$\begin{aligned}12 &= (A-a) + e \\12 &= 2 + 10 \\&= (7-5) + (5+5) \\&= 7+5\end{aligned}$$