# Standing Wave and Reference Frame

Eric Su
eric.su.mobile@gmail.com
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A standing wave consists of two identical waves moving in opposite direction. In a moving reference frame, this standing wave becomes a traveling wave. Based on the principle of superposition, the wavelengths of these two opposing waves are shown to be identical in any inertial reference frame.

According to Doppler Effect, a moving wave detector will detect two different frequencies on these two waves. Consequently, the wave detector will detect different speeds from both waves due to the same wavelength but different frequencies. The calculation of the speed of the microwave in the standing wave is demonstrated with the typical household microwave oven which emits microwave of frequency range around 2.45 GHz and wavelength range around 12.2 cm.

## I. INTRODUCTION

The superposition of two waves produces either a traveling wave or a standing wave. A standing wave in one inertial reference frame becomes a traveling wave in another inertial reference frame. The two waves that make up the standing wave will acquire new properties in another inertial reference frame. Property such as speed, frequency or wavelength can be calculated with the principle of superposition.

The results show that the wavelengths of both waves are identical to each other in any inertial reference frame. However, their speeds and frequencies differ from each other.

## II. PROOF

Consider one-dimensional interaction.

### A. Superposition of Waves

Two waves overlaping with each other will result in a new wave. Let wave 1 be

$$sin(k_1x + w_1t) \tag{1}$$

and wave 2 be

$$sin(k_2x + w_2t) \tag{2}$$

Apply principle of superposition to overlap both waves to get a new wave

$$sin(k_1x + w_1t) + sin(k_2x + w_2t) \tag{3}$$

Apply reparameterization to simplify calculation.

$$k_1 = \frac{k_1 + k_2}{2} + \frac{k_1 - k_2}{2} \tag{4}$$

$$k_2 = \frac{k_1 + k_2}{2} + \frac{k_2 - k_1}{2} \tag{5}$$

$$w_1 = \frac{w_1 + w_2}{2} + \frac{w_1 - w_2}{2} \tag{6}$$

$$w_2 = \frac{w_1 + w_2}{2} + \frac{w_2 - w_1}{2} \tag{7}$$

Let

$$f(x,t) = \frac{(k_1 + k_2)x}{2} + \frac{(w_1 + w_2)t}{2} \tag{8}$$

$$g(x,t) = \frac{(k_1 - k_2)x}{2} + \frac{(w_1 - w_2)t}{2} \tag{9}$$

Wave 1 can be written as

$$sin(k_1x + w_1t) = sin(f + g) \tag{10}$$

Wave 2 becomes

$$sin(k_2x + w_2t) = sin(f - g) \tag{11}$$

Superposition of both wave 1 and wave 2 produces

$$sin(f + g) + sin(f - g) \tag{12}$$

$$= 2sin(f)cos(g) \tag{13}$$

### B. Standing Wave

A standing wave is formed by two identical waves moving in opposite direction. Both waves have the same frequency, wavelength and amplitude.

$$w_1 = w_2 = w \tag{14}$$

$$k_1 = -k_2 = k \tag{15}$$

Superposition of both waves produces

$$2sin(f)cos(g) = 2sin(wt)cos(kx) \tag{16}$$

The new wave is a standing wave cos(kx) with new amplitude 2sin(wt).

### C. Reference Frame

Let this standing wave be stationary in reference frame $F_1$. Let another reference frame $F_2$ be in a relative motion to $F_1$.

Every point in the standing wave moves with zero velocity in $F_1$. Therefore, every point moves with the same velocity in $F_2$.

A standing wave in $F_1$

$$2sin(wt)cos(kx) \tag{17}$$

becomes a traveling wave in $F_2$.

$$2sin(w't')cos(k'x' + u't') \tag{18}$$

where

x' is space coordinate in $F_2$.

t' is time coordinate in $F_2$.

x' and t' are chosen so that

$$(x', t') = (0, 0) \tag{19}$$

if

$$(x, t) = (0, 0) \tag{20}$$

This traveling wave in $F_2$ is made of two waves corresponding to wave 1 and wave 2 in $F_1$.

$$2sin(f')cos(g') \tag{21}$$

$$f'(x', t') = \frac{(k'_1 + k'_2)x'}{2} + \frac{(w'_1 + w'_2)t'}{2} \tag{22}$$

$$g'(x', t') = \frac{(k'_1 - k'_2)x'}{2} + \frac{(w'_1 - w'_2)t'}{2} \tag{23}$$

Both equation (18) and equation (21) represent the traveling wave in $F_2$.

$$2sin(w't')cos(k'x' + u't') = 2sin(f')cos(g') \tag{24}$$

Equation (24) demands

$$sin(w't') = sin(f') \tag{25}$$

or

$$sin(w't') = cos(g') \tag{26}$$

Therefore, from equation (22) and (23),

$$k'_1 + k'_2 = 0 \tag{27}$$

or

$$k'_1 - k'_2 = 0 \tag{28}$$

The wavelengths of both waves in $F_2$ are identical.

By the definition of standing wave, the wavelengths of both waves in $F_1$ are identical.

**Therefore, the wavelengths of both waves in a standing wave are identical in any inertial reference frames.**

For two waves in oppostite direction,

$$k'_1 + k'_2 = 0 \tag{29}$$

The traveling wave in $F_2$ is

$$2sin(w't')cos(k'x' + u't') = 2sin(f')cos(g') \tag{30}$$

$$= 2sin(\frac{(w'_1 + w'_2)t'}{2})cos(\frac{(k'_1 - k'_2)x'}{2} + \frac{(w'_1 - w'_2)t'}{2}) \tag{31}$$

Therefore,

$$w' = \frac{w'_1 + w'_2}{2} \tag{32}$$

$$k' = \frac{k'_1 - k'_2}{2} = k'_1 = -k'_2 \tag{33}$$

$$u' = \frac{w'_1 - w'_2}{2} \tag{34}$$

### D. Doppler Effect

Let a wave detector be stationary in $F_2$.

According to Doppler Effect, this wave detector will detect higher frequency in the wave moving toward the wave detector. The frequency of wave moving away from the wave detector will be lower to the wave detector.

Therefore, a stationary wave detector in $F_2$ detects different frequencies in both waves.

The speed of a wave is equal to its frequency multiplied by its wavelength.

**Due to the same wavelength but different frquencies, the speeds of both waves differ in $F_2$ but remain identical in $F_1$.**

Let the velocity of the wave detector in $F_1$ be -V, then the velocity of the traveling wave in $F_2$ should be V.

$$\frac{u'}{k'} = V \tag{35}$$

From equation (34),

$$w'_1 - w'_2 = 2k'V \tag{36}$$

From equation (32),

$$w'_1 + w'_2 = 2w' \tag{37}$$

Combine equation (36) and (37) to get

$$w'_1 = w' + k'V \tag{38}$$

$$w_2' = w' - k'V \tag{39}$$

The velocity of wave 1 in $F_2$ is

$$v_1' = \frac{w_1'}{k_1'} = \frac{w_1'}{k'} = V + \frac{w'}{k'} \tag{40}$$

The velocity of wave 2 in $F_2$ is

$$v_2' = \frac{w_2'}{k_2'} = \frac{w_2'}{-k'} = V - \frac{w'}{k'} \tag{41}$$

**The velocity and the speed of a wave to the wave detector depends on the relative motion between the wave and the wave detector.**

### E. Time Transformation

The traveling wave in $F_2$ diminishes if its corresponding standing wave in $F_1$ diminishes. Therefore,

$$sin(wt) = sin(w't') \tag{42}$$

$$\frac{t}{t'} = \frac{w'}{w} \tag{43}$$

If there are two standing waves represented by $w_a$ and $w_b$ in $F_1$, the following condition should be satisfied by $w_a$ and $w_b$ of arbitrary value.

$$\frac{w_a'}{w_a} = \frac{t}{t'} = \frac{w_b'}{w_b} \tag{44}$$

Therefore, $\frac{t}{t'}$ is a constant, Q, in $F_2$ and is independent of w.

$$t = t'Q \tag{45}$$

**Time in one inertial reference frame is proportional to time in another inertial reference frame.**

### F. Wave Cavity

A standing wave can be formed inside an optical cavity such as laser or a microwave cavity such as microwave oven.

Consider the standing wave inside a typical household microwave oven. The wavelength of the microwave emitted by the microwave oven is typically between 12 cm and 12.7 cm. The distance between two adjacent nodes can be visibly measured to be about 6.2 cm on a layer of chocolate or cheese.

Let the standing microwave be stationary in reference frame $F_1$. Let a wave detector move relatively toward the microwave oven at a speed of V in $F_1$. The wave detector is stationary in $F_2$.

According to Doppler Effect, the time for a single wavelength to pass through the detector in $F_1$ for $W_+$ is

$$t_+ = \frac{L}{C+V} \tag{46}$$

and for $W_-$ is

$$t_- = \frac{L}{C-V} \tag{47}$$

where
$W_+$ is the wave moving toward the detector.
$W_-$ is the wave moving away from the detector.
C is the speed of both $W_+$ and $W_-$ in $F_1$
L is the wavelength of both $W_+$ and $W_-$ in $F_1$

The corresponding time in $F_2$ can be obtained from equation (45)

$$t_+' = \frac{t_+}{Q} = \frac{L}{(C+V)Q} \tag{48}$$

$$t_-' = \frac{t_-}{Q} = \frac{L}{(C-V)Q} \tag{49}$$

Let L' be the wavelength of both $W_+$ and $W_-$ in $F_2$. The speed of $W_+$ in $F_2$ is

$$\frac{L'}{t_+'} = (C+V)Q\frac{L'}{L} \tag{50}$$

The speed of $W_-$ in $F_2$ is

$$\frac{L'}{t_-'} = (C-V)Q\frac{L'}{L} \tag{51}$$

### G. Wavelength Transformation

As the wave detector moves through a full wavelength of the standing wave in $F_1$, a full wavelength of the traveling wave passes through the wave detector in $F_2$.

$$\frac{L}{T} = V = \frac{L'}{T'} \tag{52}$$

T is the time required in $F_1$ while T' is the corresponding time in $F_2$.

$$\frac{L'}{L} = \frac{T'}{T} = \frac{t'}{t} = \frac{1}{Q} \tag{53}$$

$$Q\frac{L'}{L} = 1 \tag{54}$$

From equation (50), the speed of $W_+$ in $F_2$ is

$$\frac{L'}{t_+'} = (C+V)Q\frac{L'}{L} = C+V \tag{55}$$

From equation (51), the speed of $W_-$ in $F_2$ is

$$\frac{L'}{t_-'} = (C-V)Q\frac{L'}{L} = C-V \tag{56}$$

**The speed of microwave depends on the reference frame. It is identical to the speed of light in $F_1$. It is either greater or smaller than the speed of light in $F_2$.**

## III. CONCLUSION

The speed of the microwave in a standing wave is a relative value. It depends on the relative motion between the standing wave and the wave detector. To the wave detector, the microwave can be faster or slower than the speed of light.

Based on the principle of superposition, the wavelengths of two opposing waves in the standing wave are always identical to each other in any inertial reference frame. This leads to one important fact that time in one inertial reference frame is linearly proportional to time in another inertial reference frame. A related fact is that space in one inertial reference frame is linearly proportional to space in another inertial reference frame.

Consequently, the resulting velocity of any wave by transformation of reference frame is exactly the addition of the velocity of the wave in the original inertial reference frame to the relative velocity of the new inertial reference frame.

The results from the principle of superposition matches well with the conservation law of Translation Symmetry. One is the conservation of simultaneity[1]. The other is the conservation of distance[2].

Lorentz Transformation violates the conservation of simultaneity[1] and distance[2]. Therefore, Lorentz Transformation is not a proper transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[2][4].

As a direct property of Translation Symmetry, both time[5] and distance[2] are independent of reference frame.


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