Complete Solutions to Quadratics with a TI-83 Plus

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Introduction

The TI-83 Plus and TI-84 Plus don't allow for the conversion of a user inputted variable, say $A = 3$, to the string $\sqrt{3}$ [1]. The quadratic formula is, apart from this, a good programming exercise. In this article we give a work around that gives complete exact solutions to quadratics.

The discriminant

The discriminant of the quadratic formula indicates five types of solutions. Given the quadratic $ax^2 + bx + c$, if d, the discriminant, $b^2 - 4ac$, is 0, there is one real solution $-b/2a$. If $d > 0$ there are two real solutions and if $d < 0$ there are two complex solutions. The two root solutions can be further divided into the perfect square and square root cases. If $fPart(\sqrt{|d|}) = 0$, then d is a perfect square and solutions are of the form

$$\frac{-b \pm i^M \sqrt{|d|}}{2a}, \tag{1}$$

where $M = 0$ for the real case and $M = 1$ for the complex. The square root: if $fPart(\sqrt{|d|}) \neq 0$, then d is not a perfect square and solutions are of the form

$$\frac{-b}{2a} \pm i^M \frac{N\sqrt{P}}{2a}, \tag{2}$$

where $\sqrt{|d|}$ is simplified to $N\sqrt{P}$ and $M = 0$ for the real case and 1 for the complex.

The TI-83 has a command to reduce the fractions in (1) and (2). The perfect square case is an easy application. The square root case requires a separate program to simplify $\sqrt{|d|}$ and then $Frac\triangleright$ can be applied to

$$\frac{-b}{2a} \text{ and } \frac{N}{2a}$$

with the final answer

$$\frac{-b}{2a} Frac \triangleright \pm i^M \frac{N}{2a} Frac \triangleright \sqrt{P}.$$

Structure of code

The viewing window of the TI-83 calculator is rather small, so it is best to break up variable assignments. Also, depending on the case, the solution needs to be clear. The following code exhibits these guiding ideas.

```
prompt A,B,C
B^2-4AC store D
Abs(D) store E
SQRT(E) store F
fPart(F) store G

If ((D >= 0) AND (G = 0))
THEN
Disp "REAL"
Disp "PURE SQUARE"
Disp (-B+F)/(2A) triangle FRAC
Disp (-B-F)/(2A) triangle FRAC
END

IF ((D<0) AND (G=0))
THEN
DISP "COMPLEX"
DISP "PURE SQ"
DISP -B/(2A) + i F/(2A) triangle FRAC
```

```
DISP -B/(2A) - i F/(2A) triangle FRAC
END

IF ((D>0) AND (G != 0))
THEN
DISP "REAL"
DISP "RADICAL"
SSR  //sub-routine below
Disp "-B/(2A)=", -B/(2A) Frac
Disp "H/(2A)=", H/(2A) Frac
Disp "SRT=", J
END

IF ((D<0) AND (G != 0))
THEN
DISP "COMPLEX"
DISP "RADICAL"
SSR
Disp "-B/(2A)=", -B/(2A) Frac
Disp "H/(2A)=", H/(2A) Frac
Disp "SRT=", J
END
```

Sub-routine

SSR (Simplify square root)

```
B^2-4AC store D
Abs(D) store E
SQRT(E) store F
fPart(F) store G

\\SIMPLIFY SQUARE ROOT RETURNS H and J,
\\as in H sqrt(J), INPUT E (ABS OF D), F (SQRT OF E)
IPART(F) STORE M
FOR (X,1,M)
 IF (FPART(E/X^2) = 0)  //E IS ABS VALUE OF D
```

```
 THEN
  X STORE H  //BUILDING H
 END  //IF
END //FOR
E/H^2 STORE J
Return
```

Evolution and test quadratics

The best way to evolve this program is to get the *structure* code to work using displays of text for the sub-routines called. Use the test cases in Table 1 to confirm that this top level is working. Then add in more code, testing as you go. There are five cases to test.

ONEROOT	$X^2 - 2X + 1$	1
RPS	$X^2 + 5X + 6$	-2, -3
RPS	$20X^2 - 23X + 6$	3/4, 2/5
RPS	$4X^2 + 12X - 16$	1, -4
RSQ	$X^2 - 4X + 2$	$2 + \sqrt{2}$, $2 - \sqrt{2}$
RSQ	$9X^2 - 30X + 18$	$5/3 \pm \sqrt{7}/3$
IPS	$X^2 - 4X + 8$	$2 + 2i$, $2 - 2i$
IPS	$9X^2 - 30X + 34$	$5/3 \pm i$
ISQ	$X^2 - 4X + 6$	$2 + i\sqrt{2}$, $2 - i\sqrt{2}$
ISQ	$9X^2 - 30X + 32$	$5/3 \pm i\sqrt{7}/3$

Table 1: Test cases.

Conclusion

An interesting puzzle: how can one generate all the possible cases for arbitrary values. Just stipulate the right side of

$$\left(x - \frac{a}{b}\right)^2 = 0, \pm\frac{c^2}{d^2}, \pm\frac{c^2}{d^2}e$$

and all five cases emerge. So, a good puzzle is to ask students to generate a quadratic with roots $3/4 \pm 2/7$ and variations on these themes.

Screen captures

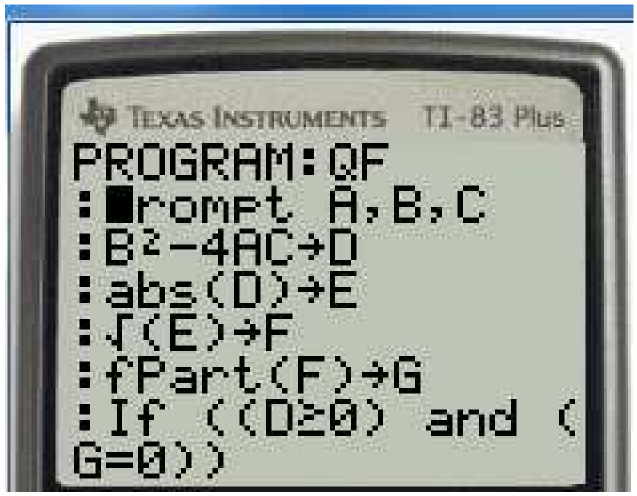

Figure 1: Caption for capture-1

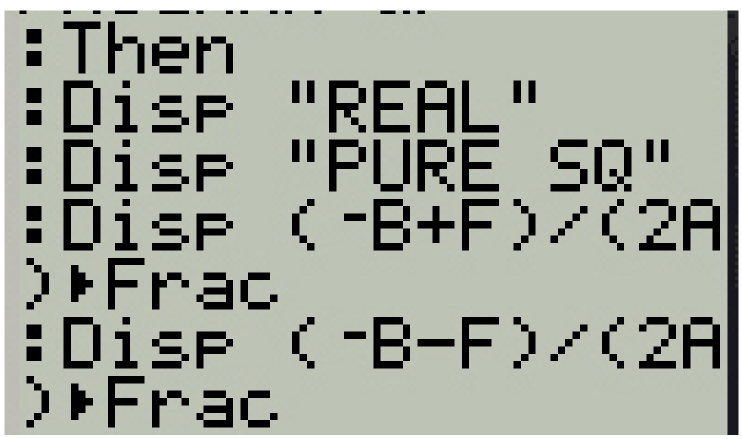

Figure 2: Caption for capture-2

References

[1] C. R. Mitchell, *Programming the TI-83 Plus/TI-84 Plus*, Manning, Shelter Island, NY, 2013.

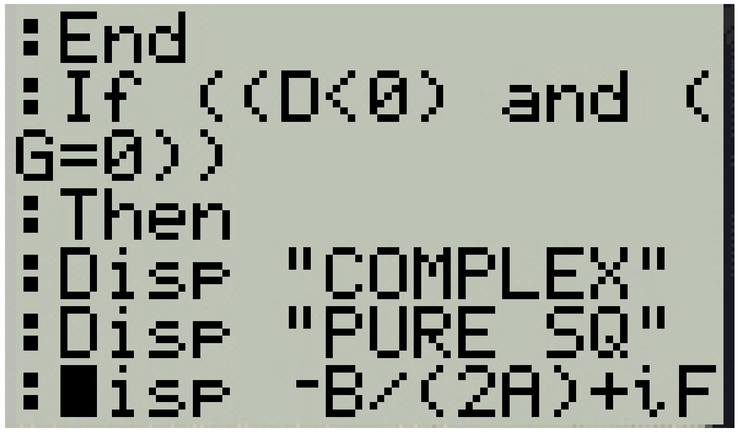

Figure 3: Caption for capture-3

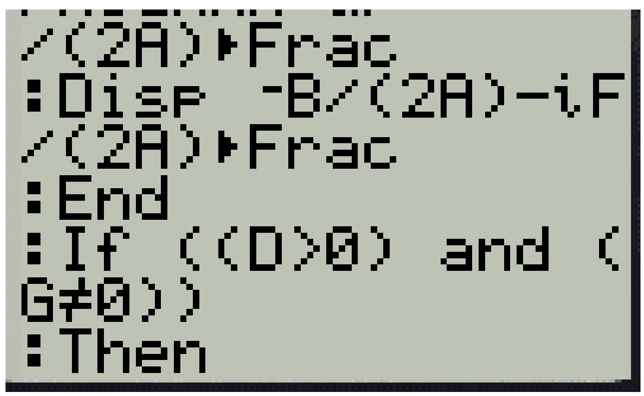

Figure 4: Caption for capture-4

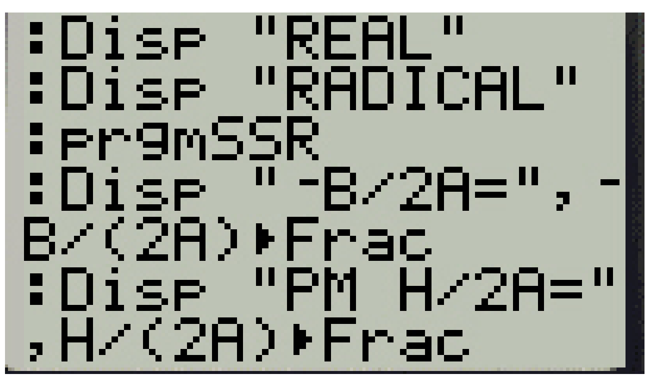

Figure 5: Caption for capture-5

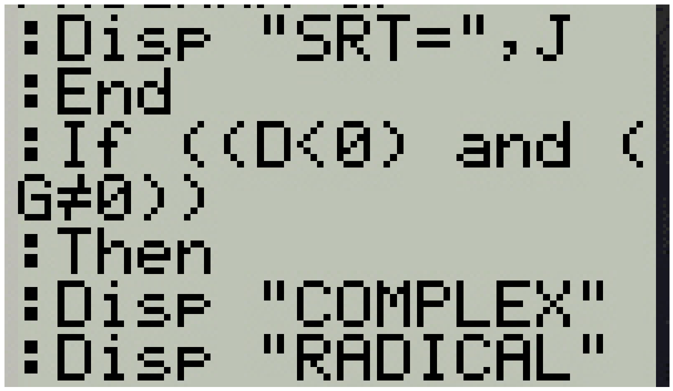

Figure 6: Caption for capture-6

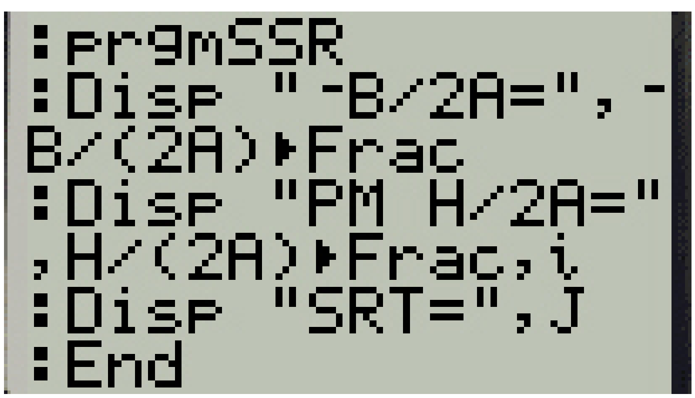

Figure 7: Caption for capture-7

Figure 8: Caption for ssr-capture-1

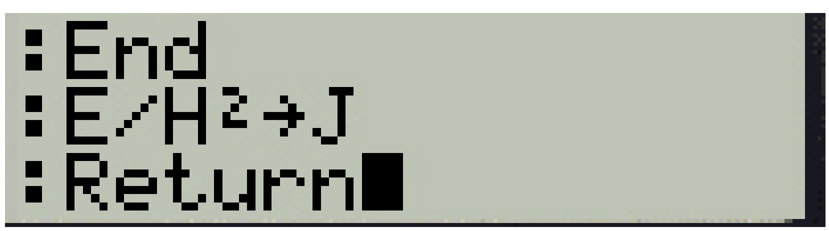

Figure 9: Caption for ssr-capture-2