

Do Birds Flow? Using Dimensional Analysis to Model Bird Dynamics

Can the techniques of Fluid- and Thermo- Dynamics be applied to the study of birds?

What follows is a speculative investigation of this possibility.

Dimensionless terms are used in both fluid- and thermo- dynamics to model various phenomena such as rates of heat and mass transfer, the onset of turbulence, flow patterns, amounts of lift and drag, etc. using various terms like Reynold's Number, Nusselt's number and Prandtl's Number.

But what about using birds instead of heat or fluids?

For the moment, assume dimensional analysis *can* be applied – what results?

By analogy, consider Reynold's Number:

$$\text{Re} = \frac{v * \rho * l}{\mu} \quad (\text{units: } \frac{(\text{m/s})(\text{kg/m}^3)(\text{m})}{(\text{kg/ms})})$$

It is principally used to relate systems of different size. For instance, two differently sized airfoils of the same geometric proportions have the same flow patterns if their Reynold's numbers are the same. Hence the preoccupation of hydro- and aero-dynamists in playing with smaller scale models of larger systems. (See, for instance, the entry on 'Dynamic Similarity' in PITT or any introductory text in Fluid Mechanics or Thermodynamics).

So – does an analog for birds exist?

Consider the various quantities involved in bird study – the area of observation (A), the time duration of observation (T), the number of birds counted (N), etc. From such, the following dimensionless term can be constructed:

$$\frac{V * D * \text{sqr}(A)}{(N/T)} \quad (\text{units: } \frac{(\text{sqr}(\text{ha})/\text{hour})(\text{birds}/\text{ha})\text{sqr}(\text{ha})}{(\text{birds}/\text{hour})})$$

where: V= average velocity of birds (units: (sqr(ha)/hour))

D = local bird density (birds/ha)

A = area of observation (ha)

N = number of birds seen (birds)

T = time duration of observations (hours)

From this, various expressions can be constructed. For instance, from it you can obtain certain expressions, like:

$$N \text{ is proportional to } V * D * \text{sqr}(A) * T$$

Which gives a proportional expression for bird visitations to an area A over time T (more precisely $N = (2/\text{sqr}(\pi)) * V * D * T$).

Replacing $\text{sqr}(A)$ with L “characteristic length” gives:

$$N \text{ is proportional to } V * D * L * T ,$$

which taking the constant of proportionality as one, is just the idealised expression for counts in a transect of width L, observer speed V, time T and local bird density D.

These somewhat trivial examples suggest the ecological-analog of Reynold's number *may* have some connection with bird observations in the field.

What other dimensionless terms *might* exist?

Using quantities bird observers can measure plus some based on the birds themselves, the following 3 simple dimensionless terms can be made:

$$\text{i) } N/(D*A) \quad \text{ii) } \text{sqr}(A)/(V*t) \quad \text{iii) } t/T$$

where we define t as the average time a bird spends in area A during observation time T.

How can these dimensionless terms be used?

Let's try the Wilson-MacArthur Law ($S=cA^z$ where S is the number of species in area A and c, z are empirically derived constants).

For purposes of demonstration, imagine trying to derive this law using dimensional analysis.

Take the units of S as 'birds'. Obviously it is dependent on the local density D and area A. Notice $D * A$ has units of 'birds/ha * ha' or 'birds'.

So put $S = \text{a constant} * (D * A)$

Using examples from fluid mechanics as a guide, we can try to derive this constant using various powers (p, q, r below) of the above 3 simple dimensionless terms. That is,

$$S = (N / DA)^p (\text{sqr}(A) / Vt)^q (t / T)^r * (D * A)$$

Since (N/T) is just the number of birds visiting over time T, this should be constant on average over sufficient time. Thus p equals r and we can write:

$$S = (Nt / DAT)^p (\text{sqr}(A) / Vt)^q (DA) = \frac{(N / T)^p t^{p-q} D^{1-p} A^{1+\frac{q}{2}-p}}{V^q}$$

So here we have a more detailed version of the Wilson-McArthur Law that incorporates both ecological and observer quantities.

As in fluid mechanics, the values of p, q have to be derived empirically from measurements.

A simple "proof-of-concept" series of measurements were thus taken.

From 23 march 2000 to 19 April 2000 observations were made for 2 hours per day over an area of approximately ¼ ha, which gave a rough estimate of $S=4.42 * T^{0.4399}$ (T in hours) for birds in an open forest 10 kilometres west of Coonabarabran NSW.

From unpublished work by Hugh Ford, $S=13.64 * A^{0.166}$ for birds in temperate forests in SE Australia for A = 20 ha to 200,000 ha.

This makes $1+q/2-p=0.166$ and $p=-0.4399$ giving $q=-2.5478$.

Substituting these values for p, q in the above gives:

$$S = (N/T)^{-0.4399} t^{2.1079} D^{1.4399} V^{2.5478} A^{0.166}$$

So estimating values for N, T, t, D, V should (fingers crossed) give a half-decent version of the Wilson-MacArthur Law for the observation area. Alas, I don't have definite estimates for all these variables, but the following subjective estimates, mostly based on long-term experience of the site, might be considered sufficiently "ballpark":

1. (N/T) = 10 birds/hour
2. t = 1/6 hours
3. D = 10 birds/ha
4. V = 4*sqr(ha)/hour = 400 metres/hour

Substituting these values into the above gives:

$$S = (10)^{-0.4399} (1/6)^{2.1079} (10)^{1.4399} (4)^{2.5478} * A^{0.166} = 7.83 * A^{0.166}$$

Given the crudity of the estimates, this compares favourably with Ford's $S = 13.64 * A^{0.166}$.

To those unfamiliar with dimensionless analysis, the above may look a "bit flaky". But it is easy to find derivations "*of like spirit*" in any Introductory Fluid Mechanics or Thermodynamics text. And I am pretty sure no-one fully understands why such techniques often (but not always) work in such areas – hence their "relegation" to engineering rather than (say) physics. I guess the final test of any method is "Does it work?" For engineers it's a case of – if so, be thankful. Hopefully the above will be considered in a like frame of mind and the above example might show the possibilities dimensional analysis might provide in relating *observable* quantities to *ecological* ones.

It would be interesting to undertake further, more accurate analysis using better quality data than the above over a variety of habitats and latitudes. For instance, which factors dominate in what areas? Are the exponents p, q similar everywhere? Or are there seasonal or spatial variations? Does the exponent of D remain greater than 1 for tropical areas? Could this "explain" higher biodiversity levels from such "rich" (high D) areas? ("...energy and biomass production alone cannot explain the tropical dominance of biological diversity" E.O. Wilson "The Diversity of Life" p200). Or, if not "explain", at least provide a new perspective on.

Whatever the case, the current (unconscious) practice of assuming bird counts are proportional to estimates of bird density is almost certainly wrong.

To date, dimensional analysis has been mainly used for zoological matters (eg: allometry, metabolism, locomotion, etc) to the exclusion of ecology/field studies that incorporate observer effects. If nothing else, dimensional analysis as practiced in engineering circles, may prove useful as a hypothesis generating technique. Its potential deserves greater consideration and testing, not just in the study of bird populations but elsewhere in ecology as well.

Comments welcome (D Williams everythingflows@hotmail.com)

References:

1. PITT Valerie H (ed) 1982 The Penguin Dictionary of Physics (see "Dynamic Similarity" and "Dimensional Analysis")
2. FORD, Hugh 2000 pers comm. Bird Seminar Gunnedah NSW
3. Wikipedia entry on 'Dimensional Analysis': http://en.wikipedia.org/wiki/Dimensional_analysis
4. WikiBooks 'Fluid Mechanics chapter 4': http://en.wikibooks.org/wiki/Fluid_Mechanics/Ch4