A considerable extension of the "Betti-Morse" and Morse theorem.

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Abstract

We formulate an considerable extension of the usual Morse theorem.

1 Proof

Theorem: Consider compact n dimensional manifolds $\mathcal{M}$ which can be foliated into $n-1$ dimensional closed (edges are allowed for but such extensions are for "technical" people) subspaces, some possibly with a finite number of singular points, such that the foliation *possibly* maximally collapses to an initial $\mathcal{I}_k$ or final surface $\mathcal{F}_l$ (both are allowed). The maximal oollapse is obviously uniquely defined (for example, the maximal collapse of a two dimensional torus is a circle whereas the maximal collapse of a two sphere is a point. Then, such spacetimes allow for Morse functions whose level surfaces for critical points associated to the "boundary of the foliation" might not exist. These level surfaces have either zero or n positive eigenvalues in the Hessian and should be dealt according to the following recipe. In case the intial and final have dimensions less than $n-1$, both are critical, the initial one with critical points whose Hessian has n positive eigenvalues and the final one with the same number of critical points whose Hessian has 0 positive eigenvalues for Morse functions which are increasing between the initial and final boundary. All other critical points, in the interior of the foliation, pertain to distinct Betti numbers. In case one of the "boundaries" has dimension $n-1$ and the other one not, one must make sure the right critical points are associated to the degenerate surface meaning that if the initial boundary has dimension less than $n-1$, the function is increases from the intial to the final boundary and the other way around for the disjoint case. The final case, where no collapse takes place is well known. In all those cases, the Betti numbers equal the number of critical points with corresponding number of negative eigenvalues.

This is a huge extension of the standard theorem as this one applies to *all* manifolds in the sense that suitable "Morse foliations" need to be extracted in case they would not be of the Morse type.

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