Bell's theorem refuted mathematically: du Sautoy cannot be right.

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Abstract

Here begins a precautionary tale from a creative life in STEM. Bringing an elementary knowledge of vectors to Bell (1964)—en route to refuting Bell's inequality and his theorem—we aim to help STEM students study one of the strangest double-errors in the history of science. To that end we question Marcus du Sautoy's claim that Bell's theorem is as mathematically robust as they come.

1 Preamble

1.1. (i) Bell's (1964) theorem is widely regarded as 'the most profound discovery of science' (Stapp 1975:271), 'one of the few essential discoveries of 20th Century physics' (vdMST 1992: v). (ii) The theorem claims that two sensible equations (which we endorse; together or apart) cannot be equal under QM. (iii) Proving the contrary, we refute Bell's theorem. (iv) For we show that the two equations are together valid under QM and EPR-Bohm (EPRB): the experiment at the heart of Bell's essay.

1.2. (i) du Sautoy (2016:170) claims that 'Bell's theorem is as mathematically robust as they come.' (ii) With an elementary proof—best read next to Bell (1964)—we also refute his claim.

1.3. (i) Bell (1964) is freely available online (see References): after download, please identify its unnumbered formulae as (11a), (14a)-(14c), (15a), (21a)-(21e), (23). (ii) Let Bell-(#) be shorthand for Bell 1964:(#). (iii) We show that Bell's move (14a)-(15) is absurd: Bell-(15) being Bell's famous inequality, see (1); infamous ours is (2). (iv) For the record: Bell-(19), (21), (21b)-(23) are also absurd.

1.4. (i) By absurd (▲) we mean mathematically false (math-false). (ii) The contrary here is QM-true (■). (iii) For, given its unqualified experimental success in this area, QM is our gold-standard for results here. (iv) An expression that is QM-false is also math-false here. (v) Taking mathematics (math) to be the best logic, our logic may flow for several lines before we comment; often with ▲ or ■.

1.5. (i) In Bell (1964), P denotes an expectation (an average). We, reserving P for probabilities, often denote expectations via $\langle\cdot\rangle$; see LHS Bell-(3). (ii) Here, to make our work easier to follow wrt Bell (1964), we replace Bell's $P(\vec{a}, \vec{b})$ with $E(a, b)$—which is no bad thing—our a and b being unit-vectors.

1.6. (i) As background, Watson (2017d) introduces EPRB at ¶¶2.1.-2.7. (ii) nb: Watson (2017d), a draft that refutes other Bellian claims, is not relied upon here: it is background. (iii) Nevertheless, as an interesting aside—for discussion elsewhere; since our results here are independently established—Watson 2017d:(17)-(24) refutes Bell's theorem without reference to quantum theory (QT).

1.7. (i) The key to our work here (as it is there) is this: rejecting inferences that are false in quantum settings, we posit the principle of true local realism: the union of true locality (after Einstein) and true (non-naive) realism (after Bohr). (ii) Given our QM-true results, we gladly address issues re our approach. (iii) To that end—to aid discussion, improvement, correction; with all our paragraphs, equations, figures, etc, numbered—constructive and pointed critical comments are especially welcome.

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1.8. Believing our analysis to be elementary—and knowing it to be QM-true—here's how we set the scene for now; with $A_i = A(a, \boldsymbol{\lambda}_i) = \pm 1$, based on Bell-(1); etc.

$$\text{Bell's inequality: } |E(a,b) - E(a,c)| - E(b,c) - 1 \quad \leq \quad 0. \blacktriangle \tag{1}$$

$$\text{Our inequality: } |E(a,b) - E(a,c)| - |1 - E(a,b)E(a,c)| \quad \leq \quad 0. \blacksquare \tag{2}$$

$$\text{Bell's theorem: } E(a,b) = -\textstyle\int d\lambda\, \rho(\boldsymbol{\lambda}) A(a, \boldsymbol{\lambda}) A(b, \boldsymbol{\lambda}) \quad \neq \quad -a \cdot b. \blacktriangle \tag{3}$$

$$\text{Our theorem: } E(a,b) = \frac{1}{n}\sum_{i=1}^{n} A_i B_i = -\textstyle\int d\lambda\, \rho(\boldsymbol{\lambda}) A(a, \boldsymbol{\lambda}) A(b, \boldsymbol{\lambda}) \quad = \quad -a \cdot b. \blacksquare \tag{4}$$

· (1) is Bell-(15), Bell's (1964) inequality formatted per ¶1.5: (1) is doubly absurd and false.

· (2) corrects (1): QM-true, (2) is the result that Bell should have reached; see (7)-(17).

· (3) is Bell's theorem: inferred by Bell via (1) and QM, (3) is absurd because (1) is absurd.

· (4) is our refutation of Bell's theorem (3): (4) is QM-true, so (4) & (2) are QM-true together.

1.9. Sharing Bell's indifference (1964:195)—whether $\boldsymbol{\lambda}$ is continuous or discrete (we can work with both)—we include both possibilities in (4). The variant of Bell-(2) in (3) & (4) is Bell-(14): it is the form that Bell uses in his principal analysis; (14a)-(14c).

2 Introduction

2.1. A longer Introduction would discuss Bell's nomination for a Nobel Prize in Physics—providing context from his other achievements in QM—including the technical developments (by others) that followed his theorem. [We, however, having never left Einstein's side wrt locality (and for another day): we refute Bell's work on local-causality—which includes his theorem—at Watson (2016d).]

2.2. From Bell (2004, cover): 'John Bell ... is particularly famous for his discovery of the crucial difference between the predictions of conventional quantum theory and the implications of local causality, a concept insisted on by Einstein.' [And as proven by us: the point is that there is no difference!]

2.3. We believe a wider knowledge of Bell's drive to resolve his 'action-at-a-distance' (AAD) dilemma (Bell 1990:7)—and of his 'don't be a sissy' have-a-go attitude (Mermin 2001:1)—will bring many students to life, and to a life, in STEM. [Accepting locality, contra Bell (1990:13), we reject AAD.]

3 Analysis

3.1. (i) En route to showing (1) to be doubly absurd—and thus (3) also—let's first warm-up by confirming a neglected fact: (1) is absurd under QM; and thus math-false. (ii) What's more, using our QM-true but (see ¶1.6) independently derived (4): (1) is math-false without reference to QT.

$$|a \cdot c - a \cdot b| + b \cdot c - 1 \quad \leq \quad 0. \blacktriangle \tag{5}$$

3.2. For, testing (1), hence (5), using RHS (4) with an EPRB example: if $a \cdot b = b \cdot c = \frac{1}{2}$ & $a \cdot c = -\frac{1}{2}$,

$$\text{then LHS (1) and LHS (5)} = |-\tfrac{1}{2} - \tfrac{1}{2}| + \tfrac{1}{2} - 1 = \tfrac{1}{2} \nleq 0. \blacktriangle \tag{6}$$

3.3. (i) Thus, via RHS (4)—with (5) math-false—Bell's (1) and Bell's (3) are refuted; the latter via (4), which is (3)'s contradiction. (ii) Why is it so? As we'll see: Bell's derivation of (1) is itself absurd!

3.4. (i) To see this, we start on the same line as Bell: and (nb), it ends one line later (as follows). (ii) We apply LHS of QM-true (4)—which is (nb) that all-important valid-for-us Bell-(2)—to LHS of valid Bell-(14a). (iii) nb: to be clear re the origin of Bell's errors: Bell-(14a) is valid; Bell-(14b) is absurd, thus QM-false. (iv) In short, we need go no further than Bell's move from his (14a) to his (14b) to refute Bell's theorem; though—to reveal other, perhaps heretofore unseen absurdities—we do.

3.5. (i) We randomly distribute $3n$ particle-pairs—using up to $3n$ detector-pairs—over randomized detector-settings $(a, b), (a, c), (b, c)$; (a, b) denoting the angle between a and b, etc. (iii) Seeking generality, we allow each particle-pair to be unique: and uniquely indexed; $i = 1, 2, ..., 3n$. (iv) Finally—for convenience in presentation; and to an adequate accuracy hereafter—we allow n to be such that:

$$\text{Bell 1964:(14a)} = E(a,b) - E(a,c) \tag{7}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A_i B_i - \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i} \tag{8}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A_i B_i \left[1 - A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i}\right]. \tag{9}$$

$$\therefore |E(a,b) - E(a,c)| = \left|\frac{1}{n}\sum_{i=1}^{n} A_i B_i \left[1 - A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i}\right]\right| \tag{10}$$

$$\leq \left|\frac{1}{n}\sum_{i=1}^{n} 1 \left[1 - A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i}\right]\right| \tag{11}$$

$$\leq \left|\frac{1}{n}\sum_{i=1}^{n} 1 - \frac{1}{n}\sum_{i=1}^{n} A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i}\right| \tag{12}$$

$$\leq \left|1 - \frac{1}{n}\sum_{i=1}^{n} A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i}\right| \tag{13}$$

$$\leq |1 - E(a,b)E(a,c)|. \blacksquare \tag{14}$$

$$\neq \text{Bell 1964:(15)} = 1 + E(b,c). \blacktriangle \tag{15}$$

$$\text{nb: Bell's inequality (15), for any } a, b, c \; : \; |a \cdot c - a \cdot b| + b \cdot c - 1 \leq \tfrac{1}{2} \nleq 0. \blacktriangle \tag{16}$$

$$\text{Whereas our (14), for any } a, b, c \; : \; |a \cdot c - a \cdot b| - |1 - (a \cdot b)(a \cdot c)| \leq 0. \text{ QED.} \blacksquare \tag{17}$$

3.6. (i) We conclude: Bell's mathematically-invalid inequality (15) –[cf (15) & (1) with (14) & (2)]– here tested under QM-true (4) at (16), yields absurdity: so Bell's theorem is again refuted (as is ever the case with us). (ii) Whereas, on the other hand: our (14) passes its test—and is QM-true—at (17) (else we would not present it). (iii) The basis for our—math-true and QM-true—logic-flow (7)-(17) follows; proceeding slowly, step-by-step:

- (7) is the agreed start-point, Bell-(14a): for Bell [us]; en route to his [our] inequality (1) [(2)].
- (8) $A_i B_i = \pm 1, A_{n+i} C_{n+i} = \pm 1$: from Bell-(1) and ¶1.9 (our shared indifference).
- (9) $\frac{1}{n}\sum_{i=1}^{n} A_i B_i A_i B_i = 1$; since $A_i B_i A_i B_i = 1$.
- (10) if $X = Y$ then $|X| = |Y|$.
- (11) $\frac{1}{n}\sum_{i=1}^{n} A_i B_i \leq \pm\frac{1}{n}\sum_{i=1}^{n} 1$; $|\pm Z| = |Z|$.
- (12) removing the inner-brackets and (proceeding slowly) using $\frac{1}{n}\sum_{i=1}^{n} 1 \cdot 1 = \frac{1}{n}\sum_{i=1}^{n} 1$.
- (13) by reduction; and now using $\frac{1}{n}\sum_{i=1}^{n} 1 = 1$.
- (14) by definition; see LHS (4). ■
- (15) Bell's troubling $\neq$ begins at that math-false and QM-false Bell-(14b). ▲
- (16) under our QM-true (4): Bell's famous inequality (15)—thus (1)—is absurd. ▲
- (17) under our QM-true (4): our inequality (14)—thus (2)—is QM-true. QED. ■

3.7. With our results QM-true: Bell's inequality (1) and its consequents (5)-(6) are refuted via (7)-(17). Bell's theorem (3) is refuted via its contradiction—(4)—in (15). nb: CHSH (1969) and others imply that (1) is Bell's theorem. Either way, it is refuted; CHSH is itself refuted at Watson 2017d:(41).

3.8. Given ¶3.6, with Bell's inequality and Bell's theorem doubly refuted—and only, in passing here, his later ideas (which Watson (2017d) refutes)—the key points of difference here (in our terms) are:

> 'The quantum mechanical expectation value $-a \cdot b$ [see RHS (3) & RHS (4)] cannot [sic] be represented, either accurately or arbitrarily, in the form of Bell-(2),' (p.199). (3) is true [sic], so (4) is not [sic] possible (p.196; line below Bell-(3). Any theory reproducing RHS (4)—the QM predictions—must be nonlocal [sic] (p.195). Therefore, since the QM predictions are well-founded, locality must be abandoned [sic]; thus, from Bell (1990:13): 'I step back from asserting that there is AAD and I say only that you cannot [sic] get away with locality. You cannot [sic] explain things by events in their neighbourhood.'

3.9. In sum—using the same technique that Bell used to derive his QM-false (1) & (3); but avoiding the pitfalls (our (2) & (4) being QM-true)—Bell's inequality (1) and Bell's theorem (3) are refuted.

3.10. For the record, the math-truth (and hence, here, the QM-truth) for (1)—and thus for (5)—is,

$$\text{for any } a, b, c \quad : \quad |a \cdot c - a \cdot b| + (b \cdot c) - 1 \leq \tfrac{1}{2}\,; \blacksquare \tag{18}$$

consistent with our result at (6). [Proving the limits of (16) & (17) is left as a STEM exercise.]

3.11. Also—from the note below Bell-(14b)—here's Bell's key error;

$$\text{Bell-(14a)} \neq \text{Bell-(14b); cf RHS (14) with RHS (15): which is true (nb) if (14b) is true!} \tag{19}$$

3.12. (i) In other words: Bell's move (14a) - (14b)—using Bell-(1) incorrectly—is the source of the $\neq$ in (19). (ii) Bellian absurdities thus arise from Bell's misuse of Bell-(1) after Bell-(14a). QED. ■

3.13. (i) Which brings us to another interesting (and fun) STEM exercise. (ii) Comparing (1) & (2), we see that the difference represents the physical significance of (19)'s $\neq$ and Bell's associated false move. (iii) Further, we see that the difference represents a constraint on the allowable detector-settings under du Sautoy/Bell. (iv) So: under what conditions does the du Sautoy/Bell theory go through? (v) In the light of ¶3.5, what is the physical significance of any such constraint?

3.14. So: (i) given the scenario in ¶3.5—given $3n$ particle-pairs, with from one to $3n$ detector-pairs available; at different sites—over varied settings equivalent to $(a, b), (a, c), (b, c)$; (ii) being cautious re false inferences; (iii) given Bellian absurdities and our QM-truths; (v) and given du Sautoy (2016:170)—'Bell's theorem is as mathematically robust as they come'—we here conclude with a question: (vi) Is Bell's theorem as mathematically robust as they come? Or have we missed something?

3.15. See Quora discussion: https://www.quora.com/Is-Marcus-du-Sautoy-right-re-Bells-theorem

4 Conclusions

> 'This was our dilemma: our analysis of EPRB led us to admit that, somehow, distant things are subtly connected, or at least not disconnected,' after Bell (1990:7). But there was hope: 'This action-at-a-distance business will pass. ... If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly. ... But I believe the questions will be resolved,' after Bell (1990:9).

4.1. (i) Eliminating 22 math-expressions from Bell (1964)—between Bell-(3)-(12) and after Bell-(15)—Bell's ideas would be clearer still if (3) is declared to be Bell's [invalid] theorem. (ii) There follows, from ¶1.4, this clarifying dictum: an expression that is QM-false here is math-false here; and vice versa. (iii) Indeed, it is passing strange that so many miss/dismiss this wake-up call: check Bell-(15)'s source.

4.2. Students should be encouraged to understand ¶3.5—nb: our use of unique particle-identifiers is more elementary (and much less daunting) than it sounds—to thus understand the basis for ¶4.1: and to then press on and find the many interesting errors in Bell-(14b)-(23).

4.3. (i) 'Britain's most famous mathematician'—du Sautoy (2016, cover)—may well say (p.170), 'Bell's theorem is as mathematically robust as they come.' (ii) But bound here, as we all are by QM and EPRB—even with us further bound by true local realism—we find no basis for that conclusion here.

4.4. Indeed, in closing, we launch our own impossibility theorems. (i) Given EPRB, QM, and true local realism [the union of true locality and true realism per Watson (2017d)]—ie, given the locality that we (with Einstein) accept; which Bell (1990:12-13) and many others reject—it is impossible that Bell-(14a) = Bell-(14b). (ii) du Sautoy cannot be right.

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6 References [DA = date accessed]

1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200.
http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf [DA20170328]

2. Bell, J. S. (1990). "Indeterminism and nonlocality." Transcript of 22 January 1990, CERN Geneva. Driessen, A. & A. Suarez (1997). Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God. A. 83-100.
http://www.quantumphil.org./Bell-indeterminism-and-nonlocality.pdf [DA20170328]

3. Bell, J. S. (2004). Speakable and Unspeakable in Quantum Mechanics. Cambridge, Cambridge University.

4. CHSH (1969). "Proposed experiment to test local hidden-variable theories." Physical Review Letters 23 (15): 880-884.

5. du Sautoy, M. (2016). What We Cannot Know. London, 4th Estate.

6. Mermin, N. D. (1988). Spooky actions at a distance: Mysteries of the quantum theory. The Great Ideas Today 1988. M. J. Adler. Chicago, Encyclopædia Britannica Inc: 2-53.

7. Mermin, N. D. (2001). "Whose knowledge?"
https://arxiv.org/pdf/quant-ph/0107151.pdf [DA20171116]

8. Stapp, H. P. (1975). "Bell's theorem and world process." Il Nuovo Cimento, 29B (2):270-276.

9. vdMST (1992). Bell's Theorem and the Foundations of Modern Physics A. van der Merwe, F. Selleri, G. Tarozzi, Eds. Singapore, World Scientific: v-vi.

10. Watson, G. (2017d). "Bell's dilemma resolved, nonlocality negated, QM demystified, etc."
http://vixra.org/abs/1707.0322 [DA20171107]