AN ESTIMATE OF THE MASS OF NEUTRINOS FROM NUCLEAR MASS DEFECTS.

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ABSTRACT

During the synthesis of a deuteron nucleus from a neutron and a proton, an electron, positron pair and an antineutrino, neutrino pair are released. The electrostatic energy (order of MeV) of the former pair is released as gamma radiation and is equivalent to the mass defect. Based on the largest mass defect per nucleon (MDPN) as the criterion of nuclear stability, it is estimated that in general, for a nuclide X(Z,N) with a fractional mass defect f, the MDPN(nu) due to neutrinos/antineutrinos alone is equal to fk(n)[Z/(Z+N)], where the proportionality constant k(n) = 0.7 micro atomic mass unit. Since f is of the order of 0.01, MDPN(nu) is in nano-atomic mass units, which is the right order of the mass of neutrinos.

INTRODUCTION

The mass of the tiniest of particles, the "neutrinos", is expected to be about 10^{-9}th of that of a hydrogen atom! A precise knowledge of this mass is sought by cosmologists and physicists [1-7] to answer many queries about our Universe.

Painstaking experiments planned to answer the question [2-3], are jeopardized [8] by the over-riding difficulties and uncertainties. In this context [8,9], this report brings for the first time a different "possible" approach: estimates of the mass defects per nucleon attributable to neutrinos/antineutrinos, calculated using the largest mass defect per nucleon criterion for nuclear stability. Using the atomic mass data for the 105 most stable nuclides, three ranges of masses (all of which are less than a few eV) have been found, "possibly" pertaining to the three kinds [1,5], the electron (lightest), muon and tau neutrinos.

Outline of the idea:

The conversion of a neutron (n) to a proton (p^+) and the converse process are known to be accompanied by the release of (electrons, e^-, and) antineutrinos (ν) and (positrons, e^+, and) neutrinos (ν) respectively [6,7,10-12]:

$$n \;{-}{-}{-}{>}\; p^+ + e^- + \nu \quad \text{(spontaneous decay)} \qquad \ldots(1)$$

$$p^+ \;{-}{-}{-}{>}\; n + e^+ + \nu \qquad \ldots(2)$$

Therefore, during the synthesis of a nucleus, X^{Z+}, by the fusion of N neutrons and Z protons,

$$Nn + Zp^+ \;{-}{-}{-}{>}\; X^{Z+} + (\nu + \nu) \qquad \ldots(3)$$

the masses of neutrinos and antineutrinos can be expected to form a finite fraction of the mass defects (MD) or the binding energy (BE) (energy equivalent of MD) of the nuclides.

Outline of the method and results obtained:

The MD for a nuclide X(Z,N) of atomic mass A, characterized by Z electrons, Z protons and N neutrons of rest masses m_e, m_p and m_n each respectively, is:

$$MD = A_{Z,N} - A = fA_{Z,N} \qquad \ldots(4)$$

where $A_{Z,N}$ (>A) stands for the sum $Z(m_e+m_p)+Nm_n$, f= (1-r), where r is the ratio $A/A_{Z,N}$. f is similar to the packing fraction [$(A_{Z,N}-M)/M$, where M = Z+N, is the mass number]. The values of the rest masses are [13]: m_e+m_p = 1.007825032u and m_n = 1.008664924u. One atomic mass unit (u) is exactly 1/12th the atomic mass of a ^{12}C isotope (standard), and it's energy equivalent is 931 MeV.

In Table 1 are given the atomic mass (A) data taken from [13] for the most abundant stable (or the one with the longest half-life) nuclide of every element from Z=1 to Z=105. Nuclides marked with an * occur in only one isotopic form. The neutron number N in column 3 is the rest mass based number [14], $N \geq (A-Zm_H)/m_n > N-1$, where m_H stands for the sum m_p+m_e.

The mass defect per nucleon, MDPN, is an index of nuclear stability and is given by

$$MDPN = MD/(Z+N) = fA_{Z,N}/(Z+N) = fm_{Z,N} \qquad \ldots(5)$$

where $m_{Z,N}$ stands for the mean mass per nucleon, $A_{Z,N}/(Z+N)$. These are given in column 5, Table 1. Nuclides with large MDPN are considered more stable than others. Note that Fe has the largest MDPN.

Using the accepted largest MDPN as the criterion of nuclear stability, the equation for the atomic mass of any nuclide,

$$A = Zm_H + N'm_n \qquad \text{...(6)}$$

where $N \geq N' > N-1$, was shown [14] to give the highest MDPN.

Therefore, the mass defect MD = $(N-N')m_n$ and the MDPN defined by Eq.5 is equal to [14],

$$MDPN = (N-N')m_n/(Z+N) \qquad \text{...(7)}$$

This equation implies that the formation of a nuclide of atomic mass A from the total mass $A_{Z,N}$ takes place at the expense of an amount of mass $(N-N')m_n$ during the fusion of the neutrons with protons. This is also in accord with the earlier view [12], that during the fusion of neutrons and protons to form a nucleus, the protons are restored at the expense of (the unstable) neutrons.

As neutrons break-up into protons, electrons and antineutrinos (cf: Eq.1), the mass m_n in Eq.7 splits into m_H and Δm_n. Thus,

$$MDPN = \underbrace{(N-N')m_H/(Z+N)}_{MDPN(1)} + \underbrace{(N-N')\Delta m_n/(Z+N)}_{MDPN(2)} \qquad \text{...(8)}$$

where $\Delta m_n = m_n - m_H$ (= 0.00084u). The values of MDPN(2) are of the order of 10^{-6}u as can be seen from column 6, Table 1. As $\Delta m_n/m_H$ = 8.4×10^{-4}, MDPN(2) << MDPN(1).

The MDPN resulting from the break-up of a mass Δm_n from the mean mass per nucleon, $m_{Z,N}$, (cf: Eq.5), is given by,

$$\text{MDPN} = \underbrace{f[m_{Z,N} - \Delta m_n]}_{\text{MDPN}(1')} + \underbrace{f\Delta m_n}_{\text{MDPN}(2')} \quad \ldots(9)$$

where each of the two terms on the right are nearly equal to the corresponding ones in Eq.8.

For nuclear stabilty, Eq.(9) should conform with Eq.(8), which is based on the highest MDPN criterion. Therefore, the terms MDPN(1′) and MDPN(2′) gain and lose masses respectively and become equal to MDPN(1) and MDPN(2). Thus:

$$\begin{aligned} \text{MDPN} &= \text{MDPN}(1) + \text{MDPN}(2) \\ &= \text{MDPN}(1') + \{[fZ/(Z+N)](\Delta m_n)^2/m_n\} \\ &+ \text{MDPN}(2') - \{[fZ/(Z+N)](\Delta m_n)^2/m_n\} \end{aligned} \quad \ldots(10)$$

The second term on the right within each of the { } brackets is of the order of 10^{-9}u, since it is the product of the fraction [fZ/(Z+N)], which is about 10^{-2}, and the constant, $k(n) = \Delta m_n^2/m_n$ = 6.995×10^{-7}u. Note that this term is in the expected range of the mass of neutrinos/antineutrinos! On considering this as the mass defect due to neutrinos/antineutrinos, MDPN(ν),

$$MDPN(\nu) = [fk(n)Z/(Z+N)] \quad ...(11)$$

the actual values for various nuclides are as shown in the last column in Table 1. The +MDPN(ν) and -MDPN(ν) terms probably indicate that the antineutrinos released by neutrons in term (2′) are absorbed by the protons in term (1′), and therby the MDPN conforms with the sum MDPN(1)+MDPN(2) of Eq. (8).

Figure 1 shows a plot of MDPN(ν) vs the atomic number, Z. The three regions probably indicate the three types of light neutrinos, the electron (lightest), muon and the tau neutrinos (or different proportions of the three kinds of neutrinos).

From Eq.11, the total mass defect of the nuclide $X_{Z,N}$ due to neutrinos, MD(ν) [= (Z+N)MDPN(ν)] is found to be,

$$MD(\nu) = fk(n)Z \quad ...(12)$$

Figure 2 shows that MD(ν) varies smoothly with Z (since fZ varies smoothly with Z). Note again the three regions.

On dividing Eq. (11) by Eq. (5), one finds that MDPN(ν) (or MD(ν)) forms a well-defined fraction of MDPN (or MD),

$$MDPN(\nu)/MDPN = MD(\nu)/MD = (Z/A_{Z,N})k(n) \quad ...(13)$$

independent of f. Therefore, this fraction can be calculated for any value of Z and N. Fig. 3 shows the linear dependence of this fraction on $Z/A_{Z,N}$.

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Table 1: Atomic Mass (A) from (13); and MDPN(ν), Eq. θ.

X	Z	N,rm	ATOMIC MASS A (u)	MDPN (u)	MDPN(2) (micro-u)	MDPN(ν) (nano-u)
H	1	0	1.007825	0.000000	0.00	0.00
He	2	2	4.002603	0.007594	6.32	2.63
Li	3	4	7.016004	0.006019	5.01	1.79
Be*	4	5	9.012182	0.006938	5.78	2.14
B	5	6	11.009306	0.007437	6.19	2.35
C	6	6	12.000000	0.008245	6.87	2.86
N	7	7	14.003074	0.008025	6.68	2.78
O	8	8	15.994915	0.008563	7.13	2.97
F*	9	10	18.998403	0.008351	6.95	2.74
Ne	10	10	19.992440	0.008623	7.18	2.99
Na*	11	12	22.989770	0.008708	7.25	2.89
Mg	12	12	23.985042	0.008868	7.39	3.08
Al*	13	14	26.981534	0.008944	7.45	2.99
Si	14	14	27.976926	0.009069	7.55	3.15
P*	15	16	30.973761	0.009105	7.58	3.06
S	16	16	31.972071	0.009118	7.59	3.16
Cl	17	18	34.968853	0.009147	7.62	3.08
Ar	18	22	39.962383	0.009227	7.68	2.88
K	19	20	38.963707	0.009186	7.65	3.10
Ca	20	20	39.962591	0.009180	7.65	3.18
Sc*	21	24	44.955910	0.009253	7.71	3.00
Ti	22	26	47.947947	0.009364	7.80	2.98
V	23	28	50.943964	0.009385	7.82	2.94
Cr	24	28	51.940512	0.009421	7.85	3.02
Mn*	25	30	54.938049	0.009410	7.84	2.97
Fe	26	30	55.934942	0.009437	7.86	3.04
Co*	27	32	58.933200	0.009413	7.84	2.99
Ni	28	30	57.935348	0.009374	7.81	3.14
Cu	29	34	62.929601	0.009396	7.82	3.00
Zn	30	34	63.929146	0.009378	7.81	3.05
Ga	31	38	68.925581	0.009366	7.80	2.92
Ge	32	42	73.921178	0.009367	7.80	2.81
As*	33	42	74.921597	0.009341	7.78	2.85
Se	34	46	79.916522	0.009351	7.79	2.76
Br	35	44	78.918338	0.009327	7.77	2.87
Kr	36	48	83.911508	0.009358	7.79	2.78
Rb	37	48	84.911792	0.009337	7.78	2.82
Sr	38	50	87.905617	0.009375	7.81	2.81
Y*	39	50	88.905849	0.009355	7.79	2.84
Zr	40	50	89.904702	0.009351	7.79	2.88
Nb*	41	52	92.906376	0.009301	7.75	2.84
Mo	42	56	97.905407	0.009270	7.72	2.76
Tc	43	55	97.907215	0.009243	7.70	2.81
Ru	44	58	101.904349	0.009240	7.70	2.77
Rh*	45	58	102.905504	0.009215	7.67	2.79
Pd	46	60	105.903483	0.009211	7.67	2.77
Ag	47	60	106.905093	0.009183	7.65	2.80
Cd	48	65	113.903359	0.000314	0.26	0.09
In	49	65	114.903879	0.000375	0.31	0.11
Sn	50	69	119.902199	0.000731	0.61	0.21

Sb	51	69	120.903822	0.000776	0.65	0.23
Te	52	77	129.906223	0.001301	1.08	0.36
I*	53	73	126.904468	0.001133	0.94	0.33
Xe	54	77	131.904155	0.001417	1.18	0.41
Cs*	55	77	132.905447	0.001456	1.21	0.42
Ba	56	81	137.905243	0.001714	1.43	0.49
La	57	81	138.906349	0.001750	1.46	0.50
Ce	58	81	139.905435	0.001801	1.50	0.52
Pr*	59	81	140.907648	0.001828	1.52	0.53
Nd	60	81	141.907719	0.001870	1.56	0.55
Pm	61	83	144.912745	0.001971	1.64	0.58
Sm	62	89	151.919729	0.002229	1.86	0.63
Eu	63	89	152.921227	0.002256	1.88	0.65
Gd	64	93	157.924101	0.002437	2.03	0.69
Tb*	65	93	158.925343	0.002463	2.05	0.70
Dy	66	97	163.929712	0.002621	2.18	0.74
Ho*	67	97	164.930319	0.002649	2.21	0.75
Er	68	97	165.930290	0.002681	2.23	0.77
Tm*	69	99	168.934211	0.002759	2.30	0.79
Yb	70	103	173.938858	0.002898	2.41	0.81
Lu	71	103	174.940768	0.002915	2.43	0.83
Hf	72	107	179.946549	0.003039	2.53	0.85
Ta	73	107	180.947996	0.003058	2.55	0.86
W	74	109	183.950932	0.003129	2.61	0.88
Re	75	111	186.955750	0.003188	2.65	0.89
Os	76	115	191.961479	0.003297	2.75	0.91
Ir	77	115	192.962923	0.003313	2.76	0.92
Pt	78	116	194.964774	0.003354	2.79	0.94
Au*	79	117	196.966551	0.003395	2.83	0.95
Hg	80	121	201.970625	0.003502	2.92	0.97
Tl	81	123	204.974412	0.003555	2.96	0.98
Pb	82	125	207.976636	0.003614	3.01	0.99
Bi*	83	125	208.980384	0.003616	3.01	1.00
Po	84	124	208.982415	0.003603	3.00	1.01
At	85	124	209.987131	0.003600	3.00	1.02
Rn	86	135	222.017569	0.003734	3.11	1.01
Fr	87	135	223.019731	0.003742	3.12	1.02
Ra	88	137	226.025402	0.003779	3.15	1.03
Ac	89	137	227.027747	0.003787	3.15	1.03
Th*	90	141	232.038050	0.003844	3.20	1.04
Pa	91	139	231.035878	0.003829	3.19	1.05
U	92	145	238.050784	0.003905	3.25	1.05
Np	93	143	237.048166	0.003893	3.24	1.06
Pu	94	149	244.064197	0.003961	3.30	1.06
Am	95	147	243.061372	0.003949	3.29	1.08
Cm	96	150	247.070346	0.003986	3.32	1.08
Bk	97	149	247.070298	0.003983	3.32	1.09
Cf	98	152	251.079579	0.004017	3.35	1.09
Es	99	151	252.082971	0.000000	0.00	0.00
Fm	100	155	257.095096	0.000119	0.10	0.03
Md	101	155	258.098427	0.000137	0.11	0.04
No	102	155	259.101046	0.000156	0.13	0.04
Lr	103	155	260.105571	0.000168	0.14	0.05
Rf	104	155	261.108912	0.000185	0.15	0.05
Ha	105	155	262.114370	0.000194	0.16	0.05

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* Nuclides which occur in only one isotopic form

Legends for Figures:

Figure 1:

The mass defects per nucleon due to neutrinos / antineutrinos, MDPN(ν) (cf: Eq.11) vs the atomic number Z.

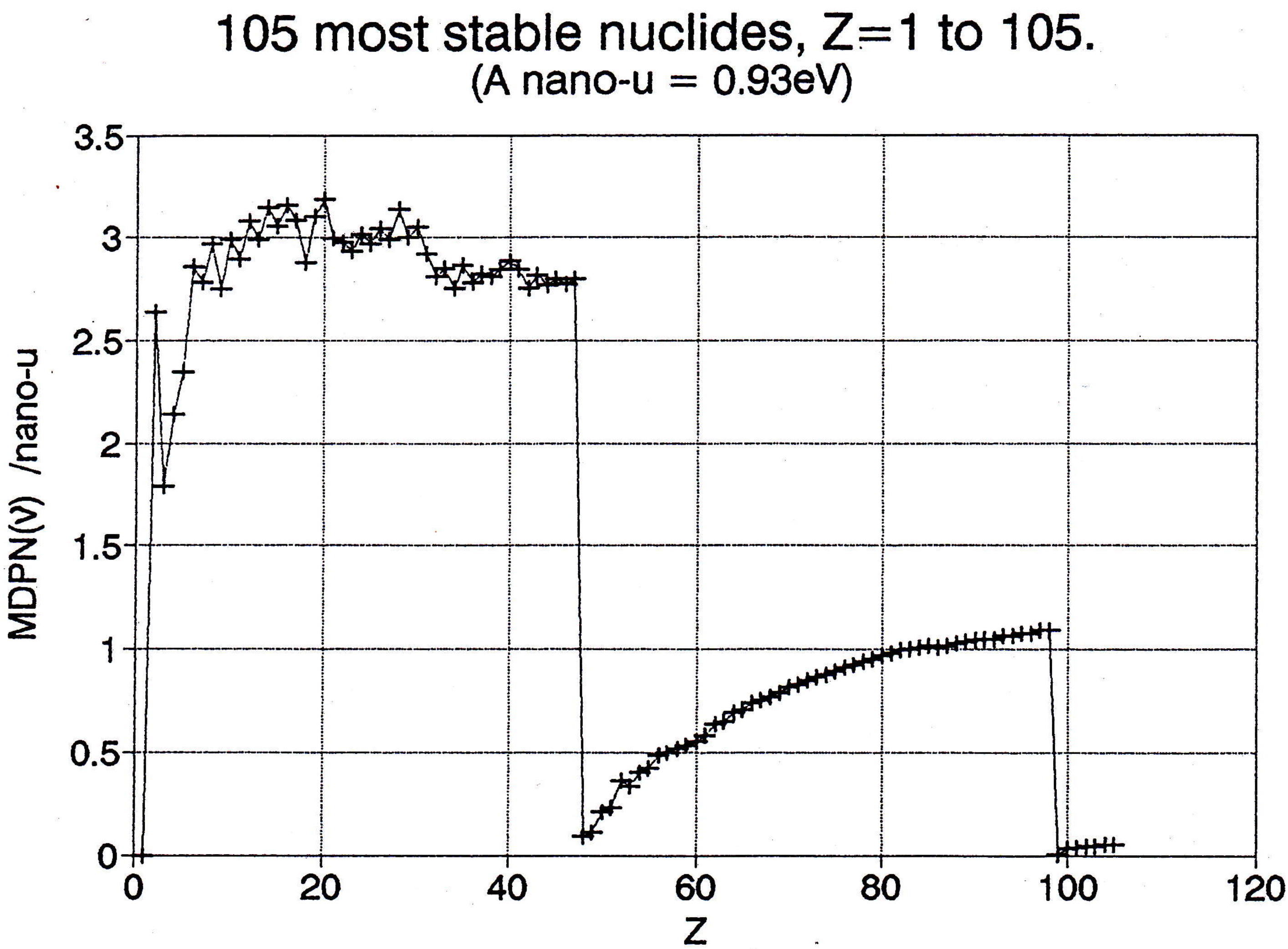
Figure 2:

The total mass defect of the nuclide due to neutrinos / antineutrinos, MD(ν) [= (Z+N)MDPN(ν), cf: Eq.12] vs the atomic number, Z.

Figure 3:

The linear dependence of the fraction of mass defect, MD(ν)/MD, due to neutrinos on the ratio $Z/A_{Z,N}$ (cf: Eq.13).

Fig. 1 (R.H.)



105 most stable nuclides, Z=1 to 105.
(A nano-u = 0.93eV)
MDPN(V) /nano-u
Z
0
20
40
60
80
100
120
0
0.5
1
1.5
2
2.5
3
3.5

Fig. 2 (R.H.)

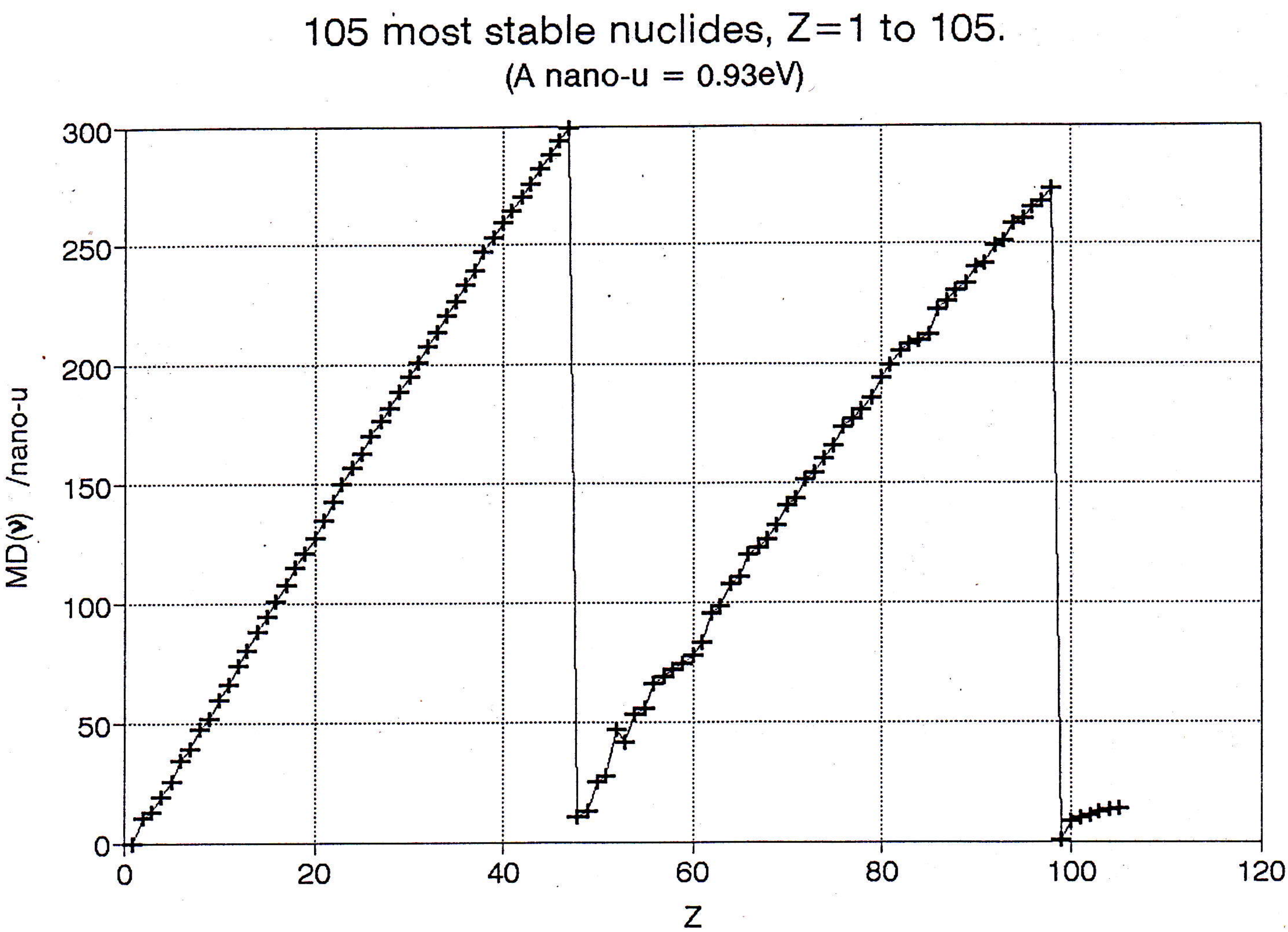
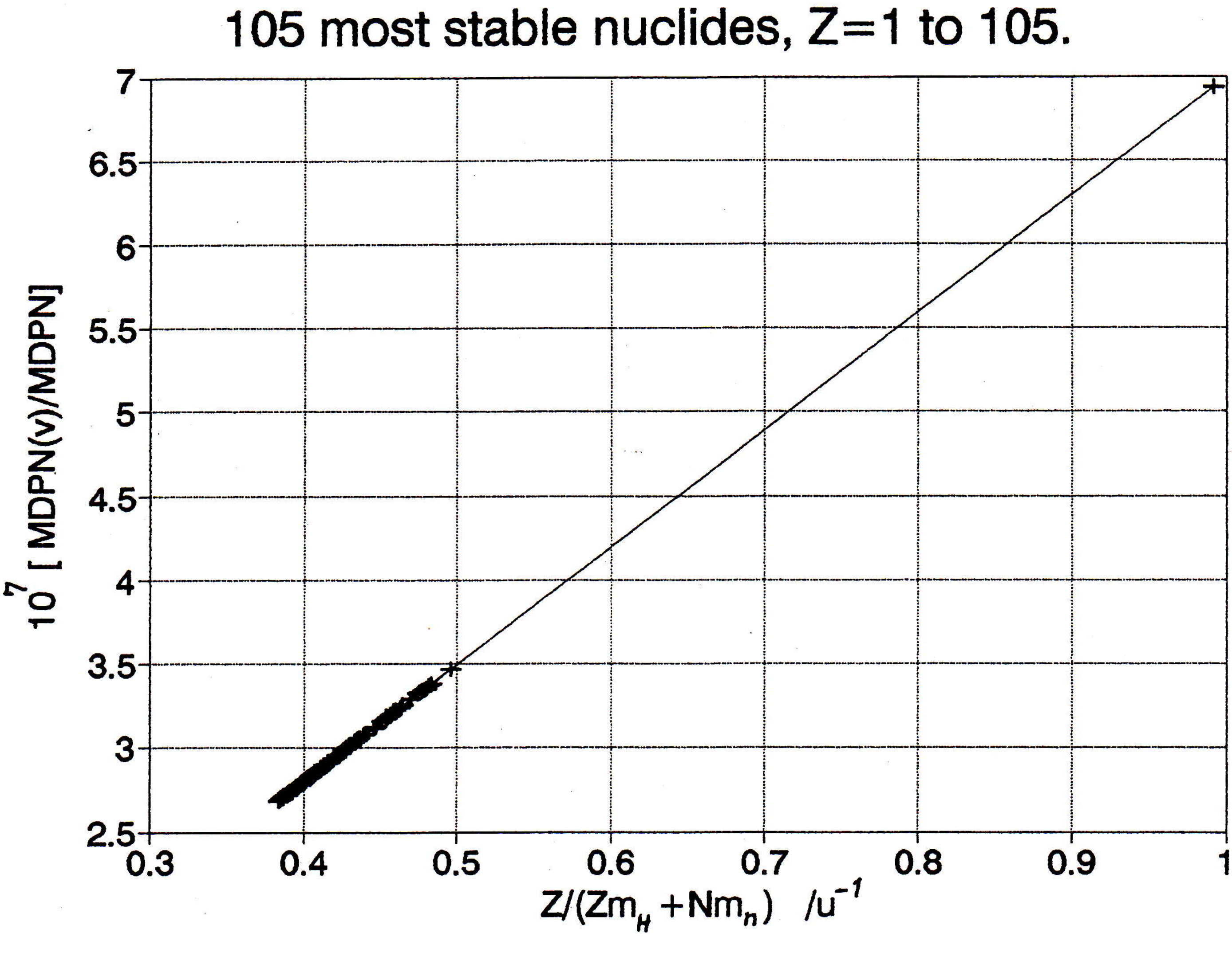

105 most stable nuclides, Z=1 to 105.
(A nano-u = 0.93eV)
MD(ν) /nano-u
Z
0
50
100
150
200
250
300
0
20
40
60
80
100
120

Fig. 3 (R.H.)



105 most stable nuclides, Z=1 to 105.
10^7 [MDPN(v)/MDPN]
Z/(Zm_H +Nm_n) /u^-1
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1