A Short Note on Relativistic Kinetic Energy

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Abstract

In this short note we are looking into a few interesting details about relativistic kinetic energy.
Key words: Kinetic energy, rest-mass energy, mass gap.

1 At what velocity does a "particle" have relativistic kinetic energy equal to its own rest-mass energy?

Einstein relativistic kinetic energy is given by

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \tag{1}$$

To find the velocity that gives a kinetic energy equal to its own rest-mass energy, we have to solve the following equation with respect to v

$$\begin{aligned}
\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 &= mc^2 \\
\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} &= 2mc^2 \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= 2 \\
\frac{1}{1-\frac{v^2}{c^2}} &= 2^2 \\
1 &= 4-4\frac{v^2}{c^2} \\
4\frac{v^2}{c^2} &= 3 \\
v &= \frac{\sqrt{3}}{2}c
\end{aligned} \tag{2}$$

That is a rest-mass has a kinetic energy equal to its own rest-mass at velocity $v = \frac{\sqrt{3}}{2}c \approx 0.866 \times c$. Whether or not this gives any deeper insight in physics is hard to say, but we notice that $\frac{\sqrt{3}}{2} = \cos(30°)$. This velocity is also the point at which the Lorentz factor, $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, equals 2. That means for every second passing in the moving frame two seconds passes in the rest frame; frames related to this velocity are named the double-time in [1]. This also means that the relativistic mass is twice the rest-mass.

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2 At what velocity does a rest-mass have relativistic kinetic energy equal to the mass gap rest-mass energy?

Haug [2] has defined the mass gap as

$$m_g = \frac{\hbar}{\bar{\lambda}_g}\frac{1}{c} \approx 1.17337 \times 10^{-51} \text{ kg} \tag{3}$$

where $\lambda_g = 299792458$ m is the reduced Compton wavelength of the mass gap, the mass gap is unlike all other rest-masses depending on the observational time-window. (The mass gap is equal to the much larger Planck mass when we assume that it only lasts for one Planck second and the observational time window is only one Planck second).

What is the velocity a rest-mass must take to have a kinetic energy equal to the rest-mass energy of the mass gap? To answer that question, we must solve the following equation with respect to v

$$\begin{aligned}
\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 &= m_g c^2 \\
\frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 &= \frac{\hbar}{\bar{\lambda}_g}\frac{1}{c}c^2 \\
\frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1-\frac{v^2}{c^2}}} &= \frac{\hbar}{\bar{\lambda}_g}\frac{1}{c}c^2 + \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= \frac{\frac{\hbar}{\bar{\lambda}_g}\frac{1}{c}c^2 + \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2} \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= \frac{\hbar c\left(\frac{1}{\bar{\lambda}_g} + \frac{1}{\bar{\lambda}}\right)}{\hbar c\frac{1}{\bar{\lambda}}} \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= \frac{\left(\frac{1}{\bar{\lambda}_g} + \frac{1}{\bar{\lambda}}\right)}{\frac{1}{\bar{\lambda}}} \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= \frac{\bar{\lambda} + \bar{\lambda}_g}{\bar{\lambda}_g} \\
\frac{1}{1-\frac{v^2}{c^2}} &= \left(\frac{\bar{\lambda} + \bar{\lambda}_g}{\bar{\lambda}_g}\right)\left(\frac{\bar{\lambda} + \bar{\lambda}_g}{\bar{\lambda}_g}\right) \\
\frac{1}{1-\frac{v^2}{c^2}} &= \frac{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2}{\bar{\lambda}_g^2} \\
1 &= \left(\frac{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2}{\bar{\lambda}_g^2}\right)\left(1-\frac{v^2}{c^2}\right) \\
\frac{\bar{\lambda}_g^2}{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2} &= \left(1-\frac{v^2}{c^2}\right) \\
\frac{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2 - \bar{\lambda}_g^2}{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2} &= \frac{v^2}{c^2} \\
\frac{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda}}{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2} &= \frac{v^2}{c^2} \\
c\frac{\sqrt{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda}}}{\sqrt{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2}} &= v \\
v &= c\frac{\sqrt{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda}}}{\bar{\lambda} + \bar{\lambda}_g} \qquad (4)
\end{aligned}$$

We could also have approximated this, when $v << c$ we have the well known

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \approx \frac{1}{2}mv^2 \tag{5}$$

and this gives the following approximation solution

$$\begin{aligned} \frac{1}{2}mv^2 &= m_g c^2 \\ v &= c\sqrt{2\frac{m_g}{m}} = c\sqrt{2\frac{\bar{\lambda}}{\bar{\lambda}_g}} \end{aligned} \tag{6}$$

Since $\bar{\lambda}_g = 299792458$ m, this is numerically equivalent to

$$v = c\sqrt{2\frac{m_g}{m}} = \sqrt{2c\bar{\lambda}} \tag{7}$$

(but the units are wrong in $\sqrt{2c\bar{\lambda}}$.)

For example, for an electron we get

$$v = \frac{c\sqrt{2\bar{\lambda}_e\bar{\lambda}_g + \bar{\lambda}_e^2}}{\bar{\lambda}_c + \bar{\lambda}_e} \approx 0.01521 \text{ m/ s} \tag{8}$$

This velocity gives a kinetic energy equal to $\hbar$ numerically, but with the wrong units.

We can generalize this to obtain a formula for the the velocity needed to produce a kinetic energy equal to $z \times m_g$, where z is an positive integer. We get

$$v_z = \frac{c\sqrt{z2\bar{\lambda}_e\bar{\lambda}_g + z^2\bar{\lambda}_e^2}}{\bar{\lambda}_g + z\bar{\lambda}_e} \tag{9}$$

So, for the electron to obtain a kinetic energy equal to its own rest-mass energy, we must have $z = \frac{c}{\bar{\lambda}_e}$. This gives

$$v_z = \frac{c\sqrt{z2\bar{\lambda}_e\bar{\lambda}_g + z^2\bar{\lambda}_e^2}}{\bar{\lambda}_g + z\bar{\lambda}_e} = \frac{\sqrt{3}}{2}c \approx 0.866 \times c \text{ m/s} \tag{10}$$

Also, it is reasonable to assume that z must be smaller than $\frac{c}{l_p}$, where l_p is the Planck length, see [3]. Based on this observation, a fundamental mass can never attain a kinetic energy equal to the rest-mass of a Planck mass, but rather it will fall just below that level. According to [4], the maximum kinetic energy allowed for a "fundamental particle" is $k_{e,max} = m_p c^2 - mc^2 = \hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)$.

3 Kinetic energy when $v = c$

Again from derivation 4 we have

$$\begin{aligned} \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 &= m_g c^2 \\ \frac{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda}}{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2} &= \frac{v^2}{c^2} \\ c\frac{\sqrt{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda}}}{\sqrt{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda} + \bar{\lambda}_g^2}} &= v \\ v &= c\frac{\sqrt{\bar{\lambda}^2 + 2\bar{\lambda}_g\bar{\lambda}}}{\bar{\lambda} + \bar{\lambda}_g} \end{aligned} \tag{11}$$

If we set $\bar{\lambda}_g = 0$, we find that $m_g = \frac{1}{0}\frac{1}{c} = \infty$ and we also get $v = c$. In other words, the formula is consistent with the observation that if $v = c$, then the relativistic kinetic energy goes to infinity as well. This is impossible, and the velocity for a rest-mass must be $v < c$, as first suggested by Einstein, see [5]. This strongly indicates there must be a limit on the reduced Compton wavelength of any rest-mass particle. Based on the assumption that the minimum reduced Compton wavelength is equal to the Planck length, Haug [6, 7, 8] has recently suggested an exact speed limit for anything with rest-mass equal to

$$v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \tag{12}$$

Formula 13 is fully consistent with this in the way it naturally shows that $v < c$. In addition, formula 12 gives an exact limit of how close v can move towards c.

4 Maximum kinetic energy

Haug [4] has, based on atomism, derived the maximum kinetic energy for any fundamental particle. Here we use a somewhat different approach and get the same result. We can set

$$c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} = c\frac{\sqrt{\bar{\lambda}^2+2\bar{\lambda}_2\bar{\lambda}}}{\bar{\lambda}+\bar{\lambda}_2} \tag{13}$$

and solve with respect to $\bar{\lambda}_2$. We then get

$$\bar{\lambda}_2 = \frac{l_p}{1-\frac{l_p}{\bar{\lambda}}} \tag{14}$$

This means we maximum can have a kinetic energy equal to

$$\begin{aligned}
\frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2 &= m_2c^2 \\
\frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 &= \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c}c^2 \\
\frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 &= \frac{\hbar}{\frac{l_p}{1-\frac{l_p}{\bar{\lambda}}}}\frac{1}{c}c^2 = \hbar c\left(\frac{1}{l_p}-\frac{1}{\bar{\lambda}}\right)
\end{aligned} \tag{15}$$

We know for any observed particle such as an electron, for example, that $\bar{\lambda} >> l_p$; then we have

$$\frac{\hbar}{\frac{l_p}{1-\frac{l_p}{\bar{\lambda}}}}\frac{1}{c}c^2 \approx m_pc^2$$

still, it is actually slightly smaller than m_pc^2. In the interesting case where $\bar{\lambda} = l_p$, we get

$$\begin{aligned}
\frac{m_pc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2 &= m_2c^2 \\
\frac{\frac{\hbar}{l_p}\frac{1}{c}c^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - \frac{\hbar}{l_p}\frac{1}{c}c^2 &= \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c}c^2 \\
\frac{\frac{\hbar}{l_p}\frac{1}{c}c^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - \frac{\hbar}{l_p}\frac{1}{c}c^2 &= \frac{\hbar}{\frac{l_p}{1-\frac{l_p}{l_p}}}\frac{1}{c}c^2 \\
\frac{\frac{\hbar}{l_p}\frac{1}{c}c^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - \frac{\hbar}{l_p}\frac{1}{c}c^2 &= \hbar c\left(\frac{1}{l_p}-\frac{1}{l_p}\right) \\
\frac{\frac{\hbar}{l_p}\frac{1}{c}c^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - \frac{\hbar}{l_p}\frac{1}{c}c^2 &= 0
\end{aligned} \tag{16}$$

The equation can only be satisfied by setting $v_{max} = 0$, and this is what Haug has claimed in a series of articles, focusing on the concept that the Planck mass particle is the collision point between light particles. In the precise moment of the reflection point, it stands still for one Planck second before dissolving into energy again. In other words, the Planck mass particle can never have kinetic energy, but only rest-mass energy, and it is at rest as observed from any reference frame. This means we must always have

$$k_{e,max} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 = m_p c^2 - mc^2 = \hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right) < m_p c^2$$

This basically shows that the derivations here are fully consistent with our other papers on this topic.

References

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