The physical nature of the basic concepts of physics

6. Entropy

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Abstract

In my paper Part 1 on the physical nature of "Linear Momentum", I have demonstrated that the difference between thermal motion and congruent motion is characterized by the degree of isotropy of the velocities of the particles and in my paper Part 3 on the physical nature of "Work, Kinetic Energy and Planck's constant" I have demonstrated that the generation of work in particle systems consist in the transformation of isotropic motion into congruent translational motion.

In my paper Part 5 on the physical nature of "Pressure, Temperature and Thermal Energy" I came to the conclusion that the statistical Maxwell – Boltzmann speed distribution function is a consequence of the repetitive isotropic collisions, that are typical for thermal motion.

This allows me, on the basis of Clausius' and Boltzmann's absolute concepts of entropy, to propose a new proportionate entropy, in function of the degree of isotropy of the velocities of the elements of a particle system, which makes it possible to reveal the link between the second law of thermodynamics and Newton's first law of motion, and which in my next paper will allow me to explain the physical nature of the variable velocity of particle systems.

1. Entropy

1.1 Thermal entropy

1.1.1 The history of the Carnot cycle

The concept of 'entropy' finds its roots in thermodynamics, which is the scientific discipline that describes the transformation of heat into work.

The scientific approach of modern thermodynamics was initiated by the French engineer Sadi Carnot (1796 – 1832) through his work on the motive power of fire.

Sadi Carnot was the son of Lazare Carnot (1753 – 1823) who was a French engineer who served under Napoleon Bonaparte as governor of Antwerp (where there is still a 'Carnot' street named after him) and who was obsessed by the operation of machines, especially water wheels, by which the flow of water could be used to drive a machine. After Napoleon's defeat, Lazare Carnot moved to Magdeburg (Germany) where he saw one of the first steam engines [1] . His son, Sadi Carnot was struck by the analogy between the water wheel and the steam engine, in which the flow of heat instead of water was used to drive a machine.

Instead of focusing on the technical improvement of steam engines, Sadi Carnot focused on the theoretical efficiency of the heat process, which allowed him to develop the so-called 'Carnot cycle'. In his 1824 paper "*Réflexions sur la puissance motrice du feu et sur les*

machines propres à développer cette puissance” Carnot enunciated his important theorem that the efficiency of engines working in a (Carnot) cycle is a universal function of the temperatures T_h and T_c. His work had however little immediate impact at the time [i]. But, through the writings of Emile Clapeyron on Carnot's work in the 1840s, his ideas came to the notice of a young German student, Rudolf Gottlieb (who became later known as Clausius) and of a young British graduate William Thomson (who became later known as Lord Kelvin). Clausius modernized Carnot's arguments and in 1865 he introduced the concept of Entropy (S) for a Carnot process, using the equation: $dS = dQ/T$

1.1.2 The generation of work by means of a Carnot cycle

The purpose of the 'Carnot cycle' was to transform the heat of a steam vessel or a combustion process, into a maximum amount of work.[ii]

-        In section 3 of my paper Part 5 on the physical nature of pressure, temperature and thermal energy, I have demonstrated that the thermal kinetic energy of an ideal monatomic gas: $K_T = Nm_1q^2/2 = mq^2/2$, is a mathematical expression of half its total amount of isotropic translational momentum flow: $Q_T = Nm_1q^2 = mq^2$, in which q is the randomly orientated thermal RMS-velocity of the particles):

-        And in section 3.4 of my paper Part 3 on the physical nature of work, kinetic energy and Planck's constant, I have demonstrated that the 'kinetic energy of congruent/bulk motion' of a body: $K = Nm_1v^2/2 = mv^2/2$, is a mathematical expression of its reversibly transferrable congruent momentum flow, which is equal to half its total amount of congruent translational momentum flow: $Q_c = Nm_1v^2 = mv^2$.

The Carnot cycle for the cyclic transformation of heat into work, is a reversible process. This means that the changes are done with infinitesimal steps, to make sure that the system and its environment remain in equilibrium during each step of the cycle, so that the process can be reversed at any time. Carnot had therefore invented a theoretical cyclic process that consisted of a reservoir (a steam vessel or a combustion chamber) with a volume V_1 at a high temperature T_h and a reservoir (the cold sink) at a low temperature T_c and a cylinder that was filled with an ideal gas and that had on one side a movable piston that could glide frictionless in the cylinder.
In that way a cyclic motion of the piston can be realized in four consecutive stages:

1.   The cylinder is maintained in close thermal contact with the hot source at a temperature T_h by means of a perfectly conducting wall that allows the momentum flow perpendicular to the wall to pass through, but not the particles themselves. This transfer increases the momentum flow of the gas particles in the cylinder, which pushes the piston in its outward direction. In that way the gas particles in the cylinder, who initially were in isotropic thermal motion, temporarily obtain a unilateral rectification of their motion in the in the direction of the outward moving piston. During this isothermal expansion from a volume V_1 to V_2 at temperature T_h, the heat Q_h delivered by the hot source (to the machine) is transformed into the work -W_{12} :

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[i] Sadi Carnot died in 1832 at the age of 36 and his work was not recognized until after his death.
[ii] In this paper I will use the symbol 'q' for the isotropic speed and 'v' for congruent speed of the particles.

$$Q_h = \int_1^2 dQ = n\,R\,T_h\,\ln\frac{V2}{V1}$$

2. The ideal gas in the cylinder undergoes a reversible adiabatic expansion during which no heat can enter or leave the cylinder and the temperature of the gas drops from T_h to T_c (the temperature of the cold sink), while the piston moves in the outward direction of the cylinder, and the rectification of the isotropic motion of the gas particles continues. During this adiabatic expansion, the temperature of the gas drops from T_h to T_c, so that the work supplied by the piston on the environment is:

$$-\,W_{23} = -\int_2^3 dW = n\,C_v\,(T_h - T_c)$$

3. The ideal gas in the cylinder undergoes a reversible isothermal compression during which the cylinder is in contact with the cold sink at a temperature T_c and an amount of heat Q_c is released to the cold sink while the piston moves in the inward direction of the cylinder. During this isothermal compression at temperature T_c , the compression heat supplied to the cold source is:

$$-\,Q_c = -\int_3^4 dQ = W_{34} = n\,R\,T_c\,\ln\frac{V3}{V4}$$

4. The ideal gas in the cylinder undergoes a reversible adiabatic compression during which no heat enters the system while the piston moves further inwards in the cylinder. During this adiabatic compression, the temperature of the gas increases from T_c to T_h, so that the work supplied on the piston is:

$$W_{41} = \int_4^1 dW = n\,C_v\,(T_h - T_c)$$

This description demonstrates that in a Carnot machine for the cyclic transformation of heat into work, the cylinder acts as a kind of rectifier because it only allows expansion in a specific direction and produces in that way a rectification of the isotropic motion (heat) of the gas particles in that direction under the form of congruent translational motion (work/kinetic energy).

1.1.3 The thermodynamic efficiency of a Carnot cycle

In an ideal, reversible Carnot cycle, the difference between the heat extracted from the hot source during the isothermal expansion (Q_h), and the heat supplied to the cold source during the isothermal compression (Q_c), is completely transferred into work (W).
The intrinsic thermodynamic efficiency 'ε' of a Carnot cycle can therefore be expressed as the proportion of the amount of work that is produced in the cylinder, to the amount of heat that is subtracted from the hot source:

$$\varepsilon = W/Q_h$$

For a reversible Carnot process, where no energy is lost to the environment, the work done by the Carnot engine equals the thermal energy it receives from the hot source minus the thermal energy that is passed unused to the cold source: $W = Q_h - Q_c$

So that the intrinsic thermodynamic efficiency: $\varepsilon = W/Q_h = (Q_h - Q_c)/Q_h = 1 - Q_c/Q_h$

The total work produced during a reversible Carnot cycle is then:

$$W = - \oint dW = n R T_h \ln \frac{V2}{V1} - n R T_c \ln \frac{V3}{V4}$$

The equation for an adiabatic process is given by the equation: pV^γ = a constant
For an ideal gas, this equation is equivalent to: $TV^{\gamma - 1}$ = a constant
Applying this to the adiabatic expansion and compression of the Carnot cycle, we obtain:

$$T_h V_2{}^{\gamma - 1} = T_c V_3{}^{\gamma - 1} \quad \text{and} \quad T_h V_1{}^{\gamma - 1} = T_c V_4{}^{\gamma - 1}$$

When we divide these equations, we find:

$$(V_2/V_1)^{\gamma - 1} = (V_3/V_4)^{\gamma - 1} \quad \text{so that:} \quad V_2/V_1 = V_3/V_4$$

And the work can be expressed as:

$$W = n R T_h \ln \frac{V2}{V1} - n R T_c \ln \frac{V2}{V1}$$

$$= n R (T_h - T_c) \ln \frac{V2}{V1}$$

And since: $Q_h = n R T_h \ln \frac{V2}{V1}$

The intrinsic thermodynamic efficiency becomes:

$$\varepsilon = W/Q_h = (Q_h - Q_c)/Q_h = (T_h - T_c)/ T_h$$
or
$$\varepsilon = W/Q_h = 1 - Q_c/Q_h = 1 - T_c/T_h$$

This demonstrates that the intrinsic thermodynamic efficiency of a reversible Carnot cycle solely depends on the proportion of the temperatures of the hot and the cold sources, which varies between:

-        $\varepsilon = 0$ when one disposes of only one heat source ($T_c = T_h$), and

-        $\varepsilon = 1$ when the temperature of the cold sink T_c (theoretically) = 0 Kelvin.

1.1.4 Clausius' thermal entropy

In the former section we have seen that for a reversible Carnot cycle:

$$Q_h/T_h = Q_c /T_c = S = \text{a constant}$$

This allowed Clausius to define the relation 'Q/T' as the thermal (or thermodynamic) 'entropy S' of a Carnot cycle. It is thereby easily demonstrated that for reversible processes, the entropy change is independent of the followed course (Fig. 6.1):
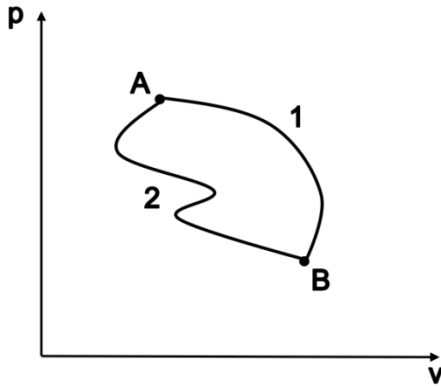
Fig 6.1

$$\int_{A\,way\,1}^{B} dQ/T \ + \ \int_{B\,way\,2}^{A} dQ/T \ = \ \oint dQ/T$$

So that: $(S_B - S_A) + (S_A - S_B) = 0$

Which demonstrates that : $S = \int_{A}^{B} dQ/T$

So that for ideal gases, the entropy is a function of state, and doesn't depend on the followed path.

1.1.5 The thermodynamic expression of Clausius' entropy for an ideal gas[2] [3]

Clausius' entropy is mathematically defined as: $S = \int dQ/T$

Since: $dQ = dE - dW$

$$S \ = \ \int \frac{dE - dW}{T}$$

$$= \ \int \frac{nCvdT + pdV}{T}$$

$$= \ \int \frac{nCvdT + nRTdV/V}{T}$$

$$= \ \int \frac{nCvdT}{T} \ + \ \int \frac{nRdV}{V}$$

$$S \ = \ n\,C_V\,lnT \ + \ n\,R\,lnV$$

In this equation:
-          $n\,C_V\,lnT = S_T$ is the temperature-dependent thermodynamic term, and
-          $n\,R\,lnV = S_V$ is the volume-dependent thermodynamic term,
of Clausius' entropy.

1.1.6 The second law of thermodynamics

In the course of a spontaneous change, the entropy of an isolated system increases:
$\Delta S_{tot} > 0$ (where ΔS_{tot} is the total entropy of the system and its surroundings).
This means that spontaneous processes are irreversible processes, which leads to the formulation of the second law of thermodynamics:
"*It is impossible to devise an engine which, working in a cycle, shall produce no other effect than the extraction of heat from a reservoir and the performance of an equal amount of work*".
The second law comes down to the practical impossibility of an ideal Carnot process.
Clausius later summarized the first and the second law of thermodynamics by stating that the energy of the world remains constant, but that its entropy tends to a maximum.

1.2 Statistical entropy

1.2.1 Boltzmann's spatial dispersion entropy [4]

Statistical thermodynamics, which is developed by the Austrian physicist Ludwig Boltzmann[(iii)], provides the link between the microscopic properties and the bulk properties of matter. To do so, Boltzmann considered a system of N monatomic molecules that are randomly distributed over W possible places.
Instead of considering the behavior of the individual particles, Boltzmann considered the effect of microscopic changes on the macroscopic appearance, especially the number of microscopic ways in which a same macroscopic pattern could be realized. If there are W_i microscopic ways of building up the i_{th} pattern, then the total number of microscopic ways of building up all the different macroscopic patterns is $W = \Sigma W_i$.
This allowed Boltzmann to conclude that the entropy of a given state was related to the total number of microscopic ways of building up the a given macroscopic pattern: $S_i = F(W_i)$

Suppose a gas in a vessel that is divided into 2 portions, by means of an isolating partition, and that the entropies of the gases in the 2 sections are $S_1 = F(W_1)$ and $S_2 = F(W_2)$.
If this system is regarded as one system with a total entropy S_T, it is clear that any one of the W_1 ways representing state 1 can then be combined with any one of the W_2 ways of representing state 2, so that : $F(W_1) + F(W_2) = F(W_1.W_2)$.
Expressed in the system's entropy, this means that $S_T = S_1 + S_2 = F(W_1.W_2)$
This equation is satisfied by the solution: $S = k \ln W$
This is Boltzmann's expression for the (statistical) entropy of a particle system, in which the factor 'k' is known as Boltzmann's constant (which is a conversion factor to express Boltzmann's entropy in the units of Clausius' thermal entropy).

In that way Boltzmann's entropy of a particle system is a measure for the degree of spatial dispersion of the particles and the factor 'W' in Boltzmann's formula represents the number of different microscopic ways that the individual particles can be arranged, without causing a macroscopic change.

(iii) Ludwig Boltzmann (1844 – 1906) used probability to understand how the atomic constitution of matter determined the laws of thermodynamics. Because he did this long before the existence of atoms was generally accepted, his work was not taken seriously during his lifetime. He committed suicide 3 years before his atomistic viewpoint became accepted. His famous equation $S = k \ln W$ is engraved on his tombstone in Vienna.

1.2.2 The correspondence between Boltzmann's and of Clausius' entropies [5]

For an ideal gas of 'n' moles (or nN_A particles) in a container with volume V, the number of possible rearrangements in this volume depends on the size of the smallest possible volume, which is the unit volume 'V_u' occupied by one particle.

a. The volume dependent term of Boltzmann's statistical entropy
There are $(V/V_u)^{nN_A}$ different ways that the nN_A particles can be distributed over the V/V_u unit volumes. Then, the total number of possible states (whereby the particles can in principle be found anywhere in a gigantic volume 'V_0') is then: $(V_0/V_u)^{nN_A}$, so that:

$$W = \frac{(V/V_u)nNA}{(V0/Vu)nNA} = (V/V_0)^{nNA}$$

Which gives:

$$S = k\,\ln W = k\,n\,N_A\,\ln V - 0$$

Or: $S_V = n\,R\,\ln V + $ a constant

b. The temperature dependent term of Boltzmann's statistical entropy
To obtain the temperature dependent part of the statistical entropy, we have to consider the distribution of the particles over the possible velocities, which is given by the Maxwell-Boltzmann distribution.
The temperature is proportional to the square of the RMS speed: $T = Nm_1 v^2/N_A$, and a unit volume in the velocity space is proportional with v^3, so that one volume in this velocity space is proportional to $T^{3/2}$ and:

$$(V/V_0) = (T/T_0)^{3/2}$$

Which allows us to write the temperature dependent part of the statistical entropy as:

$$S_T = k\,\ln W$$

$$= k\,\ln (V/V_0)^{nNA}$$

$$= k\,\ln(T/T_0)^{3nNA/2}$$

$$= (3/2)\,n\,R\,\ln T$$

Since $(3/2)R = C_V$

this gives us: $S_T = n\,C_V\,\ln T + $ a constant

c. The correspondence between both entropies
This allows us to conclude that Boltzmann's statistical entropy:
$S_B = k\,\ln W + $ a constant
is equal to:
$S_B = n\,R\,\ln V + n\,C_V\,\ln T + $ a constant $= S_T$

Which demonstrates that Boltzmann's statistical dispersion entropy (S_B) is equivalent to Clausius' thermal entropy (S_T).

2. The physical meaning of entropy

2.1 The physical meaning of Clausius' thermal entropy

In section 1.1.3 I have demonstrated that during an ideal, reversible Carnot cycle, the thermal entropy, which is defined as the proportion $S_T = Q/T$, remains constant and that in all other non-ideal thermodynamic cycles, the thermal entropy must inevitably increase, which means that: $dS_T = dQ/T > 0$

When we take into account that:
- The thermal kinetic energy of the particles of a monatomic ideal gas is a mathematical expression of its reversibly transferrable momentum flow: $K_T = Nm_1q^2/2 = mv^2/2$
- The temperature of an ideal monatomic gas is a mathematical expression of the average two-sided momentum flow per unit area in each direction, for a unit number density (one molecule or one mole) of its molecules.:
  $kT = pV/N = m_1q^2/3$ or $RT = pV/n = N_Am_1q^2/3$

then the mathematical expression of the equation of the thermodynamic entropy of the Carnot cycle becomes:

$$S_T = Q/T = K_T/T = \frac{Nm1q^2/2}{NA\,m1q^2/3R} = (3/2)RN/N_A = (3/2)nR = (3/2)kN$$

This means that Clausius' thermal entropy of a monatomic particle system is in fact a mathematical expression of the proportion of isotropic moving particles of a given particle system.

- For a reversible Carnot cycle, this proportion of isotropic moving particles remains constant during the process: $S_h = Q_h/T_h = Q_c/T_c = S_c = (3/2)nR$
  Only their thermal speed has decreased from q_h to q_c.

- In a real cycle however, due to imperfections and irreversible technological processes, such as non-equilibrium changes, thermal conduction, leakages and friction, a part of the high temperature momentum flow leaves the hot reservoir as such and doesn't participate in the rectification process, and is passed unused to the (cold) sink.
  This means that the thermal momentum flow (Q_h') that effectively participates in the production of work has decreased ($Q_h'/T_h < Q_h/T_h$) and that the amount of particles with thermal motion at the cold sink has increased ($Q_c/T_c > Q_c'/T_c$)
  so that in real processes, the overall entropy has inevitably increased:
  $Q_h'/T_h < Q_h/T_h = Q_c/T_c > Q_c'/T_c$

2.2 The physical meaning of Boltzmann 's statistical entropy

In section 1.2.1, I have demonstrated that in Boltzmann's statistical entropy of a particle system $S = k \ln W$, the W represents the total number of the different spacial distributions in which the individual particles can be arranged, without causing a macroscopic change [6].

This can be illustrated by an example of an initial situation where 100 gas particles are confined in a small box in a way that each atom is stuck in a unit volume, like this is the case in solids. Since this is the only possible arrangement, its probability W is equal to one: W = 1, and its entropy is consequently equal to $S = k \ln 1 = 0$.

When we open this small box and allow the gas particles to expand into a bigger box, with a total of 1000 unit volumes (including the small box), then the number of possible places for each supplementary particle is multiplied by 10, so that for 2 particles the total number of possible arrangements is multiplied by 10^2, for 3 particles by 10^3, etc..
For all 100 particles, the number of possible arrangements is $W = 10^{100}$, so that its entropy has increased from $S = 0$ to $S = k \ln 10^{100} = 100\, k \ln 10 = 230{,}26\, k$.

Boltzmann's entropy has to do with the effects of temperature on the spatial distribution of the particles, because it incites the particles to occupy a maximum space and impedes in that way the coherence of the system.

2.3 A new dynamic entropy concept

2.3.1 The degree of coherence of particle systems

In section 3 of my paper Part 1 on the physical nature of 'linear momentum', I have demonstrated that the velocities of the elements of a monatomic particle system can be split up into their natural components:

-       Their common, congruent translational velocity '$v_c = \Sigma m_j v_j / m$' with which all unit particles move in a coherent way, that is with the same speed in the same direction '$v = v_c$'. This means that the congruent velocity 'v' of a particle system can be represented on a real x-axis as a real vector with a given length and a give direction.

-       Their internal translational/rotational velocities with witch the individual particles move in regard of the center of mass of the particle system and which are isotropicly distributed over all possible directions. Since these isotropic velocities do not produce a resultant velocity and don't in that way affect the congruent velocity of the particle

    system, they can be represented by their average RMS-speeds $q = \sqrt{\dfrac{3kT}{m1}}$

    on an 'isotropic' axis that stands perpendicular on the real x-axis. Mathematically this means that we can represent the RMS-speed of the internal motion of a particle system on an imaginary axis 'i', with a given magnitude 'q' but with no specific direction.

Both considerations allow us to represent the total velocity of an individual element of a moving particle system, composed of identical basic unit particles, as a complex number: $z = v + iq$ (Fig. 6.2).
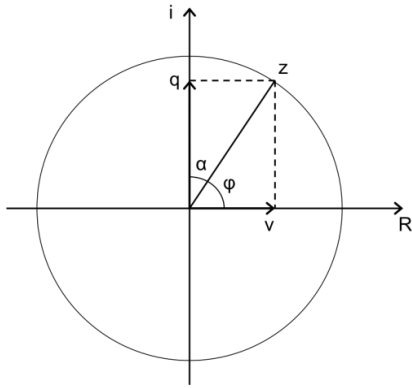
FIG 6.2

The magnitude (modulus) of this complex number is then equal to: $z = \sqrt{v^2 + q^2}$
So that: $v^2 + q^2 = z^2$
or: $v^2/z^2 + q^2/z^2 = 1$
and: $\sin^2\alpha + \cos^2\alpha = 1$

This representation of the velocities of a particle system allows us to define:

- an angle of 'coherence', 'congruence or 'rectification'(α), which is the angle between the isotropic velocity axis 'i' and the total speed 'z'.

- a (complementary) angle of 'divergence', 'dispersion' or 'distortion' (φ) between the real velocity axis 'x' and the total speed 'z'.

From this we can define:

- The degree of 'coherence', 'congruence or 'rectification' of the velocities of the particles, as the sine of the angular rectification angle 'α':
  $\sin\alpha = v/z$ or $v = z \sin\alpha$
  If α is 0°, $\sin\alpha = 0$ and there is no congruent motion
  If α is 90°, $\sin\alpha = 1$ and all motion will be present as congruent motion.

- In the same way we can define the degree of 'dispersion', 'divergence' or 'distortion' of the velocities of the particles, as the cosine of the angular rectification 'α':
  $\cos\alpha = q/z$ or $q = z \cos\alpha$
  If α is 0°, $\cos\alpha = 1$ and all motion will be present as isotropic motion
  If α is 90°, $\cos\alpha = 0$ and there is no isotropic motion

2.3.2 The intrinsic efficiency of the generation of work in particle systems

In the former sections we have demonstrated that the generation of work in particle systems consist in the (partial) rectification of the isotropic motion of the particles, into congruent motion, by means of an adiabatic expansion in a given (x-) direction [iv].

(iv) For classic Earth bound applications, where one disposes of a fixed, immovable point. In the case where there is no stationary point (e.g. in space) the adiabatic expansion will cause a bilateral rectification in both opposite directions of the x-axis, but the principle of the rectification of the thermal motion remains the same.

During that adiabatic expansion, a part of the thermal isotropic motion is transformed into congruent translational motion, which means that their thermal speed has decreased from q_h to q_c so that the frequency of the collisions between the molecules, and consequently their thermal radiation will equally have been decreased.

This demonstrates that the classic transformation of heat into work, can be represented as the rotation over an angle 'α' of the thermal speed 'q_h' which results in a lower thermal speed 'q_c' and a congruent velocity '**v**' (fig. 6.3).
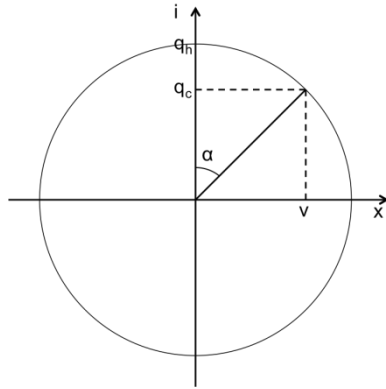

FIG 6.3

In section 2.3.1 we have seen that the degree of 'coherence', 'congruence or 'rectification of the particles' motion can be represented as the sinus of the angular rectification: $\sin\alpha = v/q_h$.

In section 1.1.2 we have seen that the degree of intrinsic thermodynamic efficiency 'ε' of the rectification process during a Carnot cycle is generally expressed as the proportion of the work that is produced, to the heat that is delivered by the hot source:

$$\varepsilon = W/Q_h = (Nm_1v^2/2)/(Nm_1q_h^2/2) = v^2/q_h^2 = \sin\alpha^2$$

- For $\alpha = 0°$: $\varepsilon = 0$
  In this case all motion is present under the form of thermal motion.
- For $\alpha = 90°$: $\varepsilon = 1$
  In this theoretical case all thermal motion has vanished and all the particles of the particle system move as a whole with the same congruent velocity '**v**'.

In the same way: $Q_c/Q_h = (Nm_1q_c^2/2)/(Nm_1q_h^2/2) = q_c^2/q_h^2$

So that the intrinsic efficiency of an ideal Carnot cycle can be expressed as:

$$\varepsilon = W/Q_h = (Q_h - Q_c)/Q_h = 1 - Q_c/Q_h = 1 - T_c/T_h = 1 - q_c^2/q_h^2 = 1 - \cos\alpha^2 = \sin\alpha^2$$

Which demonstrates that this thermodynamic efficiency 'ε' represent in fact the square of the angular rectification of the particles' motion, which is : $\sin\alpha = \sqrt{\varepsilon} = \eta_D = v/q_h$.

2.3.3 A new dynamic angular dispersion entropy

Boltzmann's statistical entropy of a particle system is a mathematical expression of the static spatial dispersion of the particles, of which the exact locations are not known and can only be estimated statistically.

Clausius' thermodynamic entropy of particle systems is a quantitative expression of the total number of particles with thermal, isotropic motion of which neither the exact velocities nor the exact locations are known, but can only be estimated statistically.

In section 1.1.2, I have demonstrated that in an ideal Carnot cycle, the isotropic velocities 'q_h' of the particles of the hot reservoir are partially rectified into a congruent velocity '$\mathbf{v}$', and that the remaining thermal kinetic energy ($K_c = Nm_1q_c^2/2$) is released in the (cold) environment at a temperature T_c.

In the former section we have seen that this wasted thermal energy has a negative influence on the intrinsic degree of efficiency 'ε' of the Carnot cycle:

$$\varepsilon = W/Q_h = (Q_h - Q_c)/Q_h = 1 - Q_c/Q_h$$

This negative influence is expressed by 'Q_c/Q_h' that represents the fraction of the isotropic thermal kinetic energy of the hot source that is transferred unused to the cold sink, and that can therefore be used as a standard indication of the intrinsic degree of inefficiency of the Carnot cycle:

$$1 - \varepsilon = Q_c/Q_h = (Nm_1q_c^2/2)/(Nm_1q_h^2/2) = q_c^2/q_h^2 = \cos\alpha^2$$

This intrinsic inefficiency can be used as a fractional, thermodynamic "angular dispersion entropy 'S_d', which expresses the degree of isotropy of the particles' velocities.

$$S_d = q_c/q_h = \cos\alpha = \sqrt{1 - \frac{v^2}{qh^2}}$$

This fractional dispersion entropy (S_d) varies between 0 and 1:

-        For $\alpha = 0$: $S_d = \cos\alpha = 1$ and all the kinetic energy will be present under the form of thermal kinetic energy;
-        For $\alpha = 90°$: $S_d = \cos\alpha = 0$ and all the kinetic energy will be present under the form of kinetic energy of bulk motion.

This means that in an isolated (closed) system that is in an equilibrium state, the transformation of isotropic thermal motion into congruent motion causes a decrease of the directional dispersion entropy, which is however excluded by the second law of thermodynamics. In that way, the directional entropy which expresses the degree of angular dispersion, can be used as a direct mathematical expressions of the second law of thermodynamics, which can be formulated as: "Without external influences, the dispersion entropy of a, isolated particle system cannot diminish"[v].

This 'directional dispersion entropy' is thereby a universal entropy that is valid for mechanical as well as thermodynamic systems and automatically leads to the equivalence between the second law of thermodynamics and Newton's first law of motion for material

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(v) It will allow me, in my paper about the physical nature of "Extent, Velocity and Length contraction", to develop Lorentz' equation for the length contraction of moving particle systems.

"bodies" seen as multi-particle systems. Newton's first law says indeed that [7] "A body at rest remains at rest and a body in motion continues to move at constant velocity, unless acted upon by an external force". In the framework of the second law of thermodynamics, this means that the internal, thermal motion of its components cannot rectify their motions by themselves. This demonstrates that both 'laws' give one and the same particular characteristic of the general law of the conservation of energy for material bodies seen as multi-particle systems.

This equivalence gives the mechanical explanation for the impossibility of perpetual motion machines of the second kind, since this would mean that not only the second law of thermodynamics, but also Newton's first law of motion has to be violated.

2.3.4 The entropy increase in real processes

In the case of real, irreversible processes, due to leakages, friction, thermal conduction and other non-equilibrium processes, a number of high temperature particles that leave the hot reservoir, do not participate in Carnot's rectification process, but are transmitted to the cold sink.

The dispersion entropy allows us to calculate in that case Clausius' thermal entropy increase. Therefore we take the case of a real thermodynamic process, in which a momentum flow
$Q_h = Nm_1q_h^2$ of a hot reservoir at a temperature $T_h = N_Am_1q_h^2/3R$
is cooled down to a temperature $T_c = N_Am_1q_c^2/3R$
while producing a congruent momentum flow $W = 2K = Nm_1v_R^2$.
In other words the RMS-speed of the particles of the hot reservoir is decreased from q_h to q_c (at temperature T_c) while producing a (real) congruent velocity 'v_R'.

In that case: $W/Q_h = Nm_1v_R^2/ Nm_1q_h^2 = v_R^2/q_h^2 = \sin\alpha^2$
And the obtained degree of coherence is: $\sin\alpha = v_R/q_h$

This allows us to calculate the ideal RMS speed 'q_{ci}' necessary to obtain the same congruent velocity v_R by means of an ideal Carnot cycle: $q_{ci}^2 = q_h^2 - v_R^2$ (Fig 6.4).
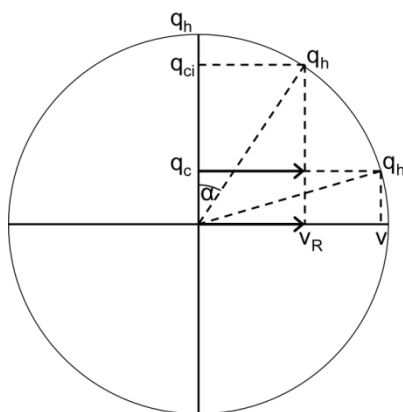

FIG. 6.4

Which gives us: $q_{ci}^2 - q_c^2 = (q_h^2 - v_R^2) - q_c^2 = (q_h^2 - q_c^2) - v_R^2$
In which $(q_h^2 - q_c^2) = v^2$ is the congruent velocity that would be obtained in the case of an ideal Carnot process from q_h to q_c.

So that: $q_{ci}^2 - q_c^2 = v^2 - v_R^2$
Which confirms that the thermal energy that is not rectified is transmitted unused to the cold reservoir.

This allows us to calculate the heat flow that is lost unused to the cold reservoir as:

$\Delta Q = Nm_1(q_h^2 - q_c^2 - v_R^2)$

And since the temperature of the cold reservoir is: $T_c = N_A m_1 q_c^2 / 3R$

The entropy increase is during the process is: $\Delta S = \Delta Q / T_c = n \dfrac{3R(qh^2 - qc^2 - vR^2))}{2qc^2}$

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