The Proton Charge Radius of muonic-hydrogen

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January 21, 2026

Abstract

This study proposes a geometric solution to the proton radius puzzle based on the theory of a dense ether and Minkowski metrics. It is demonstrated that the discrepancy between the radius measured with the electron (0,875fm) and that measured with the muon (0,84fm) is not caused by experimental error, but by a real metric contraction induced by the mass of the probe particle. Through the radial differential identity dR/dR = 1, the Minkowski volume is derived as a relativistic invariant, explaining why the scattering angle remains unchanged despite the reduction in radius. Finally, MUSE experiment at PSI is identified as the " Michelson-Morley experiment of the 21st century", capable of detecting the variation in the muon's velocity induced by the metric resistance of the ether. The ether was a fundamental physics problem that Albert Eistein and Hendrik Lorentz were unable to fully resolve throughout their lives.

Introduction

The proton charge radius represents one of the most profound dilemmas in modern nuclear physics, know as the "Proton Radius Puzzle". The discrepancy between measurements obtained with electronic hydrogen (0,875fm) and muonic hydrogen (0,84fm) has challenged the completeness of the Standard Model. In this study, a new geometric perspective is introduced: it is hypothesized that the radius reduction is not an experimental anomaly, but a real metric contraction of space-time induced by the mass of the probe particle. Utilizing the formalism of the Minkowski metric and the concept of the dence ether, we will demonstrate how the volumetric variation is linked to the relativistic invariant ds^2, providing a theoretical prediction that will find its validation in the data from the MUSE experiment at PSI.

Theory

Let us consider a spherical shell, where $R_{sup}(R)$ represents the upper bound of the radius R and $R_{inf}(R) > 0$ represents the lower bound, ($R \geq 0$ being the radial coordinate) and we shall demonstrate how the volumetric variation translates into the geometric distance within Minkowski space-time.

The volume is given by:

$$V = \int 4\pi[R_{sup}(R)^2 - R_{inf}(R)^2]dR = (4/3)\pi\, R_{sup}(R)^3 - (4/3)\pi\, R_{inf}(R)^3 \qquad 1)$$

Therefore, differentiating (1) respect to R, we obtain:

$$dV/dR = [4\pi R_{sup}(R)^2(dR_{sup}/dR)] - [R_{inf}(R)^2(dR_{inf}/dR)] \qquad 1')$$

Since any potential variation of $R_{sup}(R)$ and $R_{inf}(R)$ in 1'), can only occur along the radial direction R, we must have:

$$dR_{sup}(R)/dR = 1 \text{ e } dR_{inf}(R)/dR = 1, \qquad 2)$$

substituting 2) into 1'), we obtain:

$$dV/dR = 4\pi(R_{sup}^2 - R_{inf}^2) \quad 3)$$

and by applying the substitutions $R_{sup}^2 = (ct)^2$ and $R_{inf}^2 = -(x^2 + y^2 + z^2)$ and removing the geometric factor 4π, in the 3), we derive the exact equation for the invariant four-dimensional interval of Minkowski space:

$$ds^2 = (ct)^2 - (x^2 + y^2 + z^2) \quad 4)$$

It must hold that $ds^2 > 0$ since $V > 0$ (timelike interval).

Consequently, the volume V > 0 (the spherical shell), assumes the properties of pseudo-euclidan space-time, as it constitutes the spatial representation of the four-dimensional interval ds^2, which is a relativistic invariant.
In the General Relativity, the equation obtained from the curvature of a light ray undergoes in the presence of gravitational field, generated by material point of mass M, to distance Δ from the material point, is:

$$C = 2\alpha/\Delta = 8\pi GM/(2\pi c^2)\Delta \quad 5)$$

Then, from 5) we obtain the equation:

$$C(2\pi c^2)\Delta = 4\pi c^2\alpha = 8\pi J_G M, \text{ with } |J_G| = |G| \quad 6)$$

In which we have eliminated the time.

Given the Cartesian coordinate system of origin $O(x_1, x_2, x_3, x_4 = ct)$ the volume $4\pi c^2\alpha$, can be interpreted as the scalar triple product $4\pi \mathbf{c}\times\mathbf{c}\cdot\boldsymbol{\alpha}$, in which the vector $\boldsymbol{\alpha}$ is perpendicular at each point to the spherical surface $4\pi c^2$ of a sphere S of radius R = c, with the direction parallel to the x_4 axis. Since this volume $4\pi c^2\alpha$ is smaller than the volume $4\pi c^3/3$ of the sphere S, having demonstrated that the volume V of the spherical shell assumes the properties of pseudo-euclidean space-time, as it constitutes the spatial representation of the four-dimensional interval ds^2, which is a relativistic invariant; then putting:

$$4\pi c^3/3 = (4/3)\pi\, R_{sup}(R)^3 \text{ and } 4\pi c^2\alpha = (4/3)\pi\, R_{inf}(R)^3 \quad 7)$$

by the condition $ds^2 > 0$, and substituting 7) in 1), must be:

$$V_f = (4\pi c^3/3) - 4\pi c^2\alpha < 4\pi c^3/3 \quad 8)$$

(in the space-time cone we are in the region of absolute past).

The Muon-Proton Interaction
Since $8\pi J_G$ has the dimension of a volumetric density, we can ascribe to $8\pi J_G M$ the physical meaning of that volume, which the mass M occupies in a substance with volumetric density $8\pi J_G$; then the theory of the general relativity through the equation $4\pi c^2\alpha = 8\pi J_G M$, should lead us assume the existence of an ether with the inverse of the volumetric density is equal to $8\pi J_G$ that fills all space. From this perspective, we must address the gravitational effects of a material point of mass M within an ether whose inverse volumetric density is $8\pi J_G$. Thus, we can hypothesize that the inverse volumetric density of the proton $1/\rho_p$, is nothing more than a contraction of the inverse volumetric density of the ether $8\pi J_G$. Therefore we can deduce that the physical phenomenon in which an elementary particle such as the muon, penetrates inside the mass of the proton (a phenomenon predicted by quantum mechanics), generates a gravitational field inside the sphere enclosing the proton mass, that will reduce its volume by a $(1/\rho_p)\, M_P$ quantity and so the proton mass radius. Since there is no evidence that the gravitational field alters the constants of physics, the internal structure of the proton cannot be altered; therefore, when the muon penetrates inside the

volume of proton radius mass, the measure of the ratio α between the proton charge radius and the proton mass radius, it must necessarily remain constant. Thus, calculated the proton mass radius at the istant the muon is inside it, we obtain the proton charge radius by multiplying its mass radius by α. Since the reduced volume V'_P of the proton, due to the presence of the muon inside it, the 8) is equal to:

$$V'_P = V_P - (1/\rho_p)M_\mu \qquad 9)$$

This description would imply the creation of a black hole inside the proton by the muon; this is due to the fact that, in the formulation of the Theory of General Relativity, mass is treated as a material point. In this case, the subtracted spherical volume is distributed throughout the entire volume of the proton. By dividing the volume to be subtracted, $V_{sub} = M_\mu(1/\rho_p)$ by the volume of the proton V_P we obtain $k = M_\mu/M_P$; then, from 9) it follows that:

$$V'_P = V_P(1 - M_\mu/M_P) = (1 - k) \qquad 10)$$

that is, for each unit of volume of the proton, a volume equal to k is missing.

Then, from 10), its reduced proton mass radius must be:

$$r'_p = r_p (1 - M_\mu/M_P)^{1/3} = (1 - k)^{1/3} \qquad 11)$$

We obtain the proton radius mass and we obtain the proton charge radius multiplying the formula by α, and assuming αr_p = 0,877fm we obtain $\alpha r'_p$ = 0,8409fm; the fact that this formula leads exactly to the experimental value of 0,84087fm (measured using the muon by the scientists Aldo Antognini e Randolf Pohl) demonstrates that the 8π factor appearing in the equation $4\pi c^2 \alpha = 8\pi J_G M$ (where $8\pi J_G$ is defined as the inverse of the volumetric density of the ether), is intrinsically distributed within the volumetric density of the proton.

Experimental Predictions for MUSE

Consequently, the proton space-time (the contracted ether) undergoes a contraction due to the presence of the muon within it. According to the condition dR/dR = 1, this contraction is radial; specifically, the geodesics undergo a conformal transformation, approaching each other radially. Since the transformation is conformal, the tangent vector to the geodesic (considered as a circumference) remains unaltered. In the specific case of the MUSE experiment, a particle within the proton does not perceive this self-induced contraction, as the transformation is conformal due to dR/dR = 1. Therefore, the muon and the electron will emerge with the same scattering angle, even if the proton were to have an elliptical shape. To provide a simple conceptualization of this phenomenon, I employ the analogy of a sheet of paper being rolled up: the more the paper is rolled, the more the applied force **F** must be increased to continue the process, due to the resulting increased to continue the process, due to the resulting increase in its density. This is must occur with the muon, as its penetration into the proton increase the density of the proton's ether. Consequently, given that $m\mathbf{a} = m(d\mathbf{v}/dt) = \mathbf{F} = q(\mathbf{v}\times\mathbf{B})$, as the muon traverses the proton, it should undergo a variation in its momentum (a variation that is negligible for the electron). Compared to the electron, the muon could undergo a significant change only in its velocity, detectable by the Time-of-Flight (ToF) detectors and the Magnetic Spectrometer.

Conclusion

This study demonstrates that the discrepancy in the proton charge radius is a direct manifestation of the metric contraction of space-time induced by the mass of the probe particle. From this perspective, the MUSE experiment could detect the exitence of the ether; a quest pursued by, Albert Einstein and Hendrik

Lorentz throughout their lives following the publication of General Relativity. MUSE is the Michelson-Morley experiment of the 21st century. The MUSE experiment at PSI thus stands as the Michelson-Morley experiment of the 21st century; its 2026 data not only resolve a decade-long enigma but also pave the ways for a reconciliation between Lorentz's vision and Einstein's relativity, identifying gravity as a property of the volumetric density of the ether. Then I hypothesize that the extreme precision measurements conducted via laser spectroscopy induce a metric contraction of the proton equivalent to that produced by the muonic mass; this phenomenon suggests that the internal Minkowski metric is sensitive to the energy stress to the probe, driving the charge radius towards the equilibrium value of 0,84fm.

Reference

1) Antognini, A., Pohl R., et al. (2013)

2) Albert Einstein, Die Grundlage der allgemeinen Relativitstheorie.

3) V. A. Ugarov, Special Theory of Relativity

4) G. Piscedda, Gravity from Electricity