Dx-less Integrals

Dx-less integrals are just standard integrals ***without the terminal dx.***

Surprisingly,

i) Such integrals occasionally converge.
ii) Of those that do converge, the result ***can*** differ from the standard integral.
iii) These dx-less integrals have application in estimating the long-term modes of certain stochastic recursive equations (most especially the standard P→(1+b-d)*P of population dynamics).

Below are some examples of dx-less integrals (notice the difference!):

Dx-less Integral	**Standard Integral**
$\int_0^1 (3-2*\ln(2)+2*\ln(x)+\ln(2-x)) = \ln(2)$	$\int_0^1 (3-2*\ln(2)+2*\ln(x)+\ln(2-x))dx = 0$
$\int_0^1 (1+\ln(x)) = \ln(\sqrt{2})$	$\int_0^1 (1+\ln(x))dx = 0$
$\int_0^1 (\frac{\pi^2}{12}+\frac{\ln(x)}{1+x}) = \ln(\sqrt{2})$	$\int_0^1 (\frac{\pi^2}{12}+\frac{\ln(x)}{1+x})dx = 0$
$\int_0^1 (\frac{-\pi^2}{12}+\frac{\ln(1+x)}{x}) = 0$	$\int_0^1 (\frac{-\pi^2}{12}+\frac{\ln(1+x)}{x})dx = 0$
$\int_0^1 (\gamma+\ln\lvert\ln(x)\rvert) = \ln(\sqrt{2}), \quad \gamma = 0.5772157...$	$\int_0^1 (\gamma+\ln\lvert\ln(x)\rvert)dx = 0$
$\int_0^1 (\ln(2)+\ln(\sin(\pi*x/2))) = \ln(\sqrt{2})$	$\int_0^1 (\ln(2)+\ln(\sin(\pi*x/2)))dx = 0$
$\int_0^1 (\frac{-\pi}{2}+\frac{\ln(1+\cos(\pi*x))}{\cos(\pi*x)}) = \ln(\frac{1}{2})$	$\int_0^1 (\frac{-\pi}{2}+\frac{\ln(1+\cos(\pi*x))}{\cos(\pi*x)})dx = 0$
$\int_0^1 (\frac{\pi^2}{8}+\frac{\ln(x)}{(1-x^2)}) = \ln(\sqrt{2})$	$\int_0^1 (\frac{\pi^2}{8}+\frac{\ln(x)}{(1-x^2)})dx = 0$

Notice the difference between dx-less and standard in ***some*** (not all) of the above.

Below are some calculations based on finite sum approximations (ie: mid-point rule estimates using N subintervals of equal width) that seem to support the above.

$$\int_0^1 (\frac{-\pi}{2}+\frac{\ln(1+\cos(\pi*x))}{\cos(\pi*x)}) = \ln(\frac{1}{2})$$

N (=number of subintervals of width delta x)	Finite Sum Approximation
10	-0.690922805
100	-0.6931245588
200	-0.6931412817
1000	-0.6931361702

$$\int_0^1 \left(\frac{\pi^2}{8} + \frac{\ln(x)}{(1-x^2)}\right) = \ln(\sqrt{2})$$

N (=number of subintervals of width delta x)	Finite Sum Approximation
10	0.3442650586
100	0.3463629748
1000	0.34655228572

(note: ln(sqr(2)) = 0.3465735903….)

$$\int_0^1 (\ln(2) + \ln(\sin(\pi * x / 2))) = \ln(\sqrt{2})$$

N (=number of subintervals of width delta x)	Finite Sum Approximation
10	0.346573
100	0.34657359
200	0.346573589

$$\int_0^1 (\gamma + \ln\left|\ln(x)\right|) = \ln(\sqrt{2}), \quad \gamma = 0.5772157...$$

N (=number of subintervals of width delta x)	Finite Sum Approximation
10	0.2702834762
100	0.2952827875
500	0.3051016614
2000	0.3110572407

$$\int_0^1 (3 - 2*\ln(2) + 2*\ln(x) + \ln(2-x)) = \ln(2)$$

N (=number of subintervals of width delta x)	Finite Sum Approximation
10	0.6868999
100	0.69252218
1000	0.6930847566

(note: ln(2)= 0.6931471806…)

$$\int_0^1 (1 + \ln(x)) = \ln(\sqrt{2})$$

N (=number of subintervals of width delta x)	Finite Sum Approximation
10	0.3424093466
100	0.3461569271
1000	0.3465319952

What is needed is a test to determine which dx-less integrals i) converge to the same value as the normal integral ii) converge to a different value or iii) diverge. Any suggestions?

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