Bell's theorem refuted for STEM students

Gordon Watson*

Abstract

Here begins a precautionary tale from a creative life in STEM. Bringing an elementary knowledge of vectors to Bell (1964)—en route to refuting Bell's inequality and his theorem—we aim to help STEM students study one of the strangest double-errors in the history of science. To that end we question du Sautoy's (2016) claim that Bell's theorem is as mathematically robust as they come.

1 Preamble

1.1. (i) This essay is to be read in conjunction with Bell (1964). (ii) To aid discussion, improvement, correction: all paragraphs, equations, figures, etc, are numbered; and all key texts are freely available online (see References). (iii) Taking math to be the best logic, ours may flow for several lines before we comment; ▲ = absurd, ■ = fact; Bell-(1) is shorthand for Bell 1964:(1); etc. (iv) After Bell-(14), we identify his unnumbered math-expressions as (14a)-(14c), (15a), (21a)-(21e), (23). (v) We then focus on Bell-(15), Bell's famous inequality: its fame deriving from Bell's claim that—given QM's validity [which we accept] and Bell-(15)—seemingly sensible Bell-(2) cannot equal the QM-based RHS Bell-(3). (vi) Calling this claim Bell's theorem—and retaining Bell-(2) and RHS Bell-(3)—we refute it..

1.2. For newcomers to Bell's theorem, Watson (2017d; ¶¶2.1-2.7) introduces EPR-Bohm (EPRB)—the famous experiment discussed in Bell (1964)—in our notation. The key to our work there is this: rejecting inferences that are false in quantum settings, we posit the principle of true local realism: the union of true locality (after Einstein) and true (non-naive) realism (after Bohr). Since all our results agree with QM, its demystification, and with experiment—ie, in our terms, they are QM-true—we'll be pleased to address any concerns re our approach: for us, critical comments can be most helpful.

1.3. In Bell (1964), P denotes an expectation (an average). We, reserving P for probabilities, often denote expectations via $\langle\cdot\rangle$; see LHS Bell-(3). To make our analysis here easier to follow wrt Bell (1964), we replace Bell's $P(\vec{a},\vec{b})$ with $E(a,b)$—which is no bad thing—our a and b being unit-vectors.

1.4. We now set the EPRB-scene in our terms: and since our comments are based on our QM-true analysis—as will be seen—our work is thus backed by valid math and QM; and thus by experiment.

$$\text{Bell's inequality: } |E(a,b)-E(a,c)| - E(b,c) - 1 \leq 0. \blacktriangle \tag{1}$$

$$\text{Our inequality: } |E(a,b)-E(a,c)| + E(a,b)E(a,c) - 1 \leq 0. \blacksquare \tag{2}$$

$$\text{Bell's theorem: } E(a,b) = -\int d\lambda\, \rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda}) \neq -a\cdot b. \blacktriangle \tag{3}$$

$$\text{Our theorem: } E(a,b) = -\frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i) = -\int d\lambda\, \rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda}) = -a\cdot b. \blacksquare \tag{4}$$

· (i) (1) is Bell-(15), Bell's famous (1964) inequality per ¶1.3: it is doubly absurd and false.

· (ii) (2) corrects (1): QM-true, (2) is the result that Bell should have reached; see (7)-(12).

· (iii) (3) is Bell's theorem: inferred by Bell via (1) and QM, (3) is false because (1) is false.

· (iv) (4) is our refutation of Bell's theorem (3): (4) is QM-true; ie, (4) & (2) are QM-true together.

* eprb@me.com [Ex: BTR24Wv2, 20171116.] Ref: BTR24Av3. Date: 20171123.

1.5. Sharing Bell's indifference (1964:195) as to whether $\boldsymbol{\lambda}$ denotes discrete or continuous variables, we include both possibilities in (4): and though preferring $\boldsymbol{\lambda}$-discrete—since we like to work with individual particles here; see ¶3.5—we can work with both. Using Bell-(13), that variant of Bell-(2) in (3) & (4) is Bell-(14).

2 Introduction

2.1. A separate Introduction is planned to discuss Bell's nomination for a Nobel Prize in physics, Bell's theorem being widely regarded as one of the most profound discoveries in science and one of the few essential achievements of 20th Century physics. [nb: our work raises questions re that nomination.]

2.2. From Bell (2004, cover): 'John Bell ... is particularly famous for his discovery of the crucial difference between the predictions of conventional quantum theory and the implications of local causality, a concept insisted on by Einstein.' [nb: and insisted on by us; our point being that there is no difference.]

2.3. We believe a wider knowledge of Bell's efforts to resolve his 'action-at-a-distance' (AAD) dilemma (Bell 1990:7)—and of his 'don't be a sissy' have-a-go attitude (Mermin 2001:1)—will bring many students to life, and to a life, in STEM. [nb: accepting locality, contra Bell (1990:13), we reject AAD.]

3 Analysis

3.1. Encouraging students to study Bell (1964) deeply—and enjoy discovery—some small steps are not included here. [Depending on responses, such steps may be added as endnotes in later updates.]

3.2. En route to showing that (1) is doubly absurd—and thus (3) also—we first confirm an oft-neglected fact: (1) is absurd under QM. Thus, using RHS Bell-(3): (1) under QM becomes

$$|a \cdot c - a \cdot b| + b \cdot c - 1 \quad \leq \quad 0. \tag{5}$$

3.3. We now test (1), hence (5), using RHS (4). If $a \cdot b = b \cdot c = \frac{1}{2}$ & $a \cdot c = -\frac{1}{2}$,

$$\text{then LHS (1) and LHS (5)} = |-\tfrac{1}{2} - \tfrac{1}{2}| + \tfrac{1}{2} - 1 = \tfrac{1}{2} \nleq 0. \blacktriangle \tag{6}$$

3.4. Thus, via RHS (4), (6) being QM-true: Bell's (1) and Bell's (3) are refuted; the latter via our use of its contradiction. Why is it so? Well: Bell's derivation of that famous (1) is itself absurd. ■

3.5. To see this, we apply LHS (4) to LHS Bell-(14a): ie, wrt Bell's derivation of Bell-(15)—(1)—we (and Bell) start at the same place. But we randomly distribute $3n$ particle-pairs—using up to $3n$ detector-pairs—over randomized detector-settings $(a, b), (a, c), (b, c)$. [nb: (a, b) denotes the angle between a and b; etc.] And—avoiding any possibility of false inference—we allow each particle-pair to be unique: and thus uniquely indexed $[i = 1, 2, ..., 3n]$ for identification purposes. We then allow n to be such that—to an adequate accuracy; ie, satisfying serious critics—and for convenience in presentation:

$$\begin{aligned}
\text{LHS Bell}(14a) \quad &= \quad E(a,b) - E(a,c) &(7)\\
&= \quad -\frac{1}{n}\sum_{i=1}^{n}[A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i) - A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i})] &(8)\\
&= \quad \frac{1}{n}\sum_{i=1}^{n}A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)[A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i}) - 1]. &(9)\\
\text{But } A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i) \quad &= \quad \pm 1: \text{ from Bell-(1).} &(10)\\
\therefore |\, E(a,b) - E(a,c)\, | \quad &\leq \quad 1 - \frac{1}{n}\sum_{i=1}^{n}A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i}) &(11)\\
&\leq \quad 1 - E(a,b)E(a,c)\text{: which is (2); and QM-true,} &(12)\\
\because \text{ for any } a, b, c \quad &: \quad |a \cdot c - a \cdot b| + (a \cdot b)(a \cdot c) - 1 \leq 0\text{; using (4).} \blacksquare &(13)
\end{aligned}$$

3.6. Thus, using a not-too-difficult STEM-level particle-by-particle analysis of EPRB, we refute both (1) & (3) via (7)-(13). For, using the discrete version of LHS (4), we refute Bell's famous inequality (1)—and thus its consequents (5)-(6)—via (7)-(12). And we refute Bell's famous theorem (3) by using its contradiction—RHS (4)—in (13). [nb: all our results are QM-true; see Appendix A for (11)-(12).]

3.7. For the record, the math-truth (and hence the QM-truth here) for (1)—and thus for (5)—is,

$$\text{for any } a, b, c \quad : \quad |a \cdot c - a \cdot b| + (b \cdot c) - 1 \leq \tfrac{1}{2}\, ; \blacksquare \tag{14}$$

which agrees with our result at (6). [Proving the limits of (13) & (14) is left as a STEM exercise.]

3.8. The above brings us to Bell's famous claim: ie, the basis for (3), his theorem.

> 'Thus, the quantum mechanical expectation value $-a \cdot b$ cannot [sic] be represented, either accurately or arbitrarily, in the form of Bell-(2),' after Bell (1964:199).

3.9. But before refuting it, let us be clear: this is Bell's theorem—(3)—in our terms; aka Bell's impossibility theorem, and supposedly so profound that Stapp (1975:271) claimed it to be 'the most profound discovery' of science. nb: CHSH (1969) imply that (1) is Bell's theorem. But from the line below Bell-(3), and from "This is the theorem" (Bell 2004: 65), Bell supports (3).

3.10. Either way: whether Bell's theorem be (1) or (3), it is refuted; see ¶3.6.

3.11. In sum, under the same technique that Bell used to derive his QM-false (1) & (3)—ie, Bell's technique is the one that we have followed here—Bell's inequality (1) and Bell's theorem (3) are refuted.

3.12. Then—noting the note below Bell-(14b)—here is Bell's key error;

$$\text{Bell 1964:(14a)} \neq \text{Bell 1964:(14b); cf (13) with (6), etc.} \tag{15}$$

3.13. In other words: the source of $\neq$ in (15) is Bell's move from Bell-(14a) to Bell-(14b) using Bell-(1). And here, from (1) & (2), is the physical significance of that move:

$$E(a, b)E(a, c) + E(b, c) = 0. \tag{16}$$

$$\text{ie: } (a \cdot b)(a \cdot c) - (b \cdot c) = 0. \blacktriangle \tag{17}$$

3.14. Absurdities (1), (3), (6), (17) thus arise from Bell's use of Bell-(1) after Bell-(14a). QED. ■

3.15. So: (i) given the scenario in ¶3.5—given $3n$ particle-pairs, with from one to $3n$ detector-pairs available; at different sites—over varied settings equivalent to $(a, b), (a, c), (b, c)$; (ii) given our provision of the more complete specification that Bell (1964:195) sought to be effected by means of $\boldsymbol{\lambda}$; (iii) given the Bellian absurdities; (iv) given our QM-truths; (v) and given du Sautoy (2016:170)—'Bell's theorem is as mathematically robust as they come'—we ask: Is Bell's theorem as mathematically robust as they come? Or have we missed something?

4 Conclusions

> 'This was our dilemma: our analysis of EPRB led us to admit that, somehow, distant things are subtly connected, or at least not disconnected,' after Bell (1990:7). But there was hope: 'This action-at-a-distance business will pass. ... If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly. ... But I believe the questions will be resolved,' after Bell (1990:9).

4.1. Eliminating 22 math-expressions—between Bell-(3)-(12) and after Bell-(15)—Bell's ideas would be clearer still if our (3) is declared to be Bell's theorem.

4.2. Students should then be encouraged to understand ¶3.5—nb: its subtle particle-by-particle analysis is more elementary (and much less daunting) than it sounds—and thus find the many interesting errors in Bell-(14b)-(22). They might then join us and refute Bell's theorem on their own terms.

4.3. 'Britain's most famous mathematician'—du Sautoy (2016, cover)—may well say (p.170), 'Bell's theorem is as mathematically robust as they come.' But bound, as we are—by EPRB, QM, and true local realism—we find no basis for that conclusion here.

4.4. Indeed, in closing, we launch our own impossibility theorem. Given EPRB, QM, and true local realism (the union of true locality and true realism)—ie, given the locality that Bell (1990:12-13) rejects—it is impossible that Bell-(14a) = Bell-(14b). Or have we missed something?

5 Acknowledgments

27 October 2017—for David Bohm (1917-1992)—the 25th anniversary of his passing.

I again thank those cited in Watson (2017d). I thank Phil Watson (Kariong) for suggestions to improve my writing, and Roger Mc Murtrie (again) for his collaboration. Also, John Bell (1928-1990)—my could-have-been collaborator—for his work on local-causality. [Could have been? In June 1989, seeking to contact John to gift him my simple refutation of his theorem—reading Mermin (1988)—I phoned two physics-professors. The first hung up on me a second time; the second convinced me that John was dead.] Reading Bell (1990:9) again, maybe some of us were being rather silly (at times): but so goes many a creative life in STEM.

6 Appendix: Proof that RHS (11) = RHS (12)

6.1. We first reduce the expectation on RHS (11) to a conventional (probability-based) arithmetic mean over all test outcomes; the most basic definition of an expectation. Thus, using Bell-(13) to convert our A-style functions to relevant B- and C-style functions:

$$A(b, \boldsymbol{\lambda}_i) = -B_i; A(c, \boldsymbol{\lambda}_{n+i}) = -C_{n+i}. \tag{18}$$

6.2. Next (eliminating the identifiers $n, n+i$), we need to identify which A result is correlated with the B result, and which with the C result. So—with $A_B^+ = 1$, $A_B^- = -1$, etc—let $A_B^\pm$ ($A_C^\pm$) be correlated with $B^\pm$ ($C^\pm$); with $A_B^\pm$ and $B^\pm$ independent of ($A_C^\pm$) and $C^\pm$, and vice versa.

6.3. Then, since $\boldsymbol{\lambda}_i$ and $\boldsymbol{\lambda}_{n+i}$ are independent random variables, here are our progressive simplifications:

$$P[A(a, \boldsymbol{\lambda}_i) = 1] \equiv P(A_i = 1) = P(A_B^+) = P(A_B^-) = P(A_C^+) = P(A_C^-) = \tfrac{1}{2}. \tag{19}$$

$$\text{So: } P(A_B^+ A_C^+) = P(A_B^+ A_C^-) = P(A_B^- A_C^+) = P(A_B^- A_C^-) = \tfrac{1}{4}. \tag{20}$$

6.4. Thus, via similar progressive simplifications:

$$\frac{1}{n}\sum_{i=1}^{n} A(a, \boldsymbol{\lambda}_i) A(b, \boldsymbol{\lambda}_i) A(a, \boldsymbol{\lambda}_{n+i}) A(c, \boldsymbol{\lambda}_{n+i}) \tag{21}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A_i B_i A_{n+i} C_{n+i}) \tag{22}$$

$$= \langle A_B^\pm B^\pm A_C^\pm C^\pm \rangle \tag{23}$$

$$
\begin{aligned}
&= \tfrac{1}{4}(A_B^+A_C^+)[P(B^+ \mid A_B^+A_C^+) - P(B^- \mid A_B^+A_C^+)][P(C^+ \mid A_B^+A_C^+) - P(C^- \mid A_B^+A_C^+)] \\
&+ \tfrac{1}{4}(A_B^+A_C^-)[P(B^+ \mid A_B^+A_C^-) - P(B^- \mid A_B^+A_C^-)][P(C^+ \mid A_B^+A_C^-) - P(C^- \mid A_B^+A_C^-)] \\
&+ \tfrac{1}{4}(A_B^-A_C^+)[P(B^+ \mid A_B^-A_C^+) - P(B^- \mid A_B^-A_C^+)][P(C^+ \mid A_B^-A_C^+) - P(C^- \mid A_B^-A_C^+)] \\
&+ \tfrac{1}{4}(A_B^-A_C^-)[P(B^+ \mid A_B^-A_C^-) - P(B^- \mid A_B^-A_C^-)][P(C^+ \mid A_B^-A_C^-) - P(C^- \mid A_B^-A_C^-)] \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
&= \tfrac{1}{4}[\sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b)][\sin^2\tfrac{1}{2}(a,c) - \cos^2\tfrac{1}{2}(a,c)] \\
&- \tfrac{1}{4}[\sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b)][\cos^2\tfrac{1}{2}(a,c) - \sin^2\tfrac{1}{2}(a,c)] \\
&- \tfrac{1}{4}[\cos^2\tfrac{1}{2}(a,b) - \sin^2\tfrac{1}{2}(a,b)][\sin^2\tfrac{1}{2}(a,c) - \cos^2\tfrac{1}{2}(a,c)] \\
&+ \tfrac{1}{4}[\cos^2\tfrac{1}{2}(a,b) - \sin^2\tfrac{1}{2}(a,b)][\cos^2\tfrac{1}{2}(a,c) - \sin^2\tfrac{1}{2}(a,c)] \qquad (25) \\
&= \tfrac{1}{4}[-\cos(a,b)][-\cos(a,c)] - \tfrac{1}{4}[-\cos(a,b)][\cos(a,c)] \\
&- \tfrac{1}{4}[\cos(a,b)][-\cos(a,b)] + \tfrac{1}{4}[\cos(a,b)][\cos(a,c)] \qquad (26) \\
&= \cos(a,b)\cos(a,c) = (a\cdot b)(a\cdot c). \blacksquare \qquad (27) \\
\therefore \text{RHS (11)} &= \text{RHS (12). QED.} \qquad (28)
\end{aligned}
$$

6.5. Using the independence relations identified in ¶6.2 (under true locality; Watson 2017d), the key probability relations in (25) —see Watson 2017d:(20)—are typified by:

$$P(B^+ \mid A_B^+A_C^+) = P(B^+ \mid A_B^+) = \sin^2\tfrac{1}{2}(a,b); P(B^- \mid A_B^+A_C^+) = P(B^- \mid A_B^+) = \cos^2\tfrac{1}{2}(a,b). \qquad (29)$$

7 References [DA = date accessed]

1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200. http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf [DA20170328]

2. Bell, J. S. (1990). "Indeterminism and nonlocality." Transcript of 22 January 1990, CERN Geneva. Driessen, A. & A. Suarez (1997). Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God. A. 83-100. http://www.quantumphil.org./Bell-indeterminism-and-nonlocality.pdf [DA20170328]

3. Bell, J. S. (2004). Speakable and Unspeakable in Quantum Mechanics. Cambridge, Cambridge University.

4. CHSH (1969). "Proposed experiment to test local hidden-variable theories." Physical Review Letters 23 (15): 880-884.

5. du Sautoy, M. (2016). What We Cannot Know. London, 4th Estate.

6. Mermin, N. D. (1988). Spooky actions at a distance: Mysteries of the quantum theory. The Great Ideas Today 1988. M. J. Adler. Chicago, Encyclopædia Britannica Inc: 2-53.

7. Mermin, N. D. (2001). "Whose knowledge?" https://arxiv.org/pdf/quant-ph/0107151.pdf [DA20171116]

8. Stapp, H. P. (1975). "Bell's theorem and world process." Il Nuovo Cimento, 29B (2):270-276.

9. Watson, G. (2017d). "Bell's dilemma resolved, nonlocality negated, QM demystified, etc." http://vixra.org/abs/1707.0322 [DA20171107]