BBP – High-Precision Arithmetic

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abstract

This note presents two BBP- type formulas

The BBP (Bailey- Borwein- Plouffe) formula for Pi

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \tag{1}$$

This formula can generate the nth base-16 digit of π without computing any prior digits. for details see: [1,2,5,6].

In this note we present two formulas for Pi.

Formulas

- Formula 1:

$$\begin{aligned}\frac{\pi}{5\sqrt{2+\phi}} = \sum_{n=0}^{\infty} \left(\frac{1}{2\phi}\right)^{15n} \Bigg(& \frac{1}{(2\phi)(15n+1)} + \frac{\phi^{-1}}{(2\phi)^2(15n+2)} - \frac{\phi^{-1}}{(2\phi)^3(15n+3)} \\ & - \frac{1}{(2\phi)^4(15n+4)} + \frac{1}{(2\phi)^6(15n+6)} + \frac{\phi^{-1}}{(2\phi)^7(15n+7)} \\ & - \frac{\phi^{-1}}{(2\phi)^8(15n+8)} - \frac{1}{(2\phi)^9(15n+9)} + \frac{1}{(2\phi)^{11}(15n+11)} \\ & + \frac{\phi^{-1}}{(2\phi)^{12}(15n+12)} - \frac{\phi^{-1}}{(2\phi)^{13}(15n+13)} - \frac{1}{(2\phi)^{14}(15n+14)} \Bigg)\end{aligned} \tag{2}$$

❖ Formula 2:

$$\frac{\pi}{5\sqrt{2+\phi}}=\sum_{n=0}^{\infty}\left(\frac{1}{2\phi}\right)^{30n}\left(\frac{1}{(2\phi)(30n+1)}+\frac{\phi^{-1}}{(2\phi)^{2}(30n+2)}-\frac{\phi^{-1}}{(2\phi)^{3}(30n+3)}\right.$$
$$-\frac{1}{(2\phi)^{4}(30n+4)}+\frac{1}{(2\phi)^{6}(30n+6)}+\frac{\phi^{-1}}{(2\phi)^{7}(30n+7)}$$
$$-\frac{\phi^{-1}}{(2\phi)^{8}(30n+8)}-\frac{1}{(2\phi)^{9}(30n+9)}+\frac{1}{(2\phi)^{11}(30n+11)}$$
$$+\frac{\phi^{-1}}{(2\phi)^{12}(30n+12)}-\frac{\phi^{-1}}{(2\phi)^{13}(30n+13)}-\frac{1}{(2\phi)^{14}(30n+14)}$$
$$+\frac{1}{(2\phi)^{16}(30n+16)}+\frac{\phi^{-1}}{(2\phi)^{17}(30n+17)}-\frac{\phi^{-1}}{(2\phi)^{18}(30n+18)}$$
$$-\frac{1}{(2\phi)^{19}(30n+19)}+\frac{1}{(2\phi)^{21}(30n+21)}+\frac{\phi^{-1}}{(2\phi)^{22}(30n+22)}$$
$$-\frac{\phi^{-1}}{(2\phi)^{23}(30n+23)}-\frac{1}{(2\phi)^{24}(30n+24)}+\frac{1}{(2\phi)^{26}(30n+26)}$$
$$\left.+\frac{\phi^{-1}}{(2\phi)^{27}(30n+27)}-\frac{\phi^{-1}}{(2\phi)^{28}(30n+28)}-\frac{1}{(2\phi)^{29}(30n+29)}\right) \tag{3}$$

Remark: $\phi=\frac{1+\sqrt{5}}{2}$.

Some Evaluations

$$e1_k=\frac{\pi}{5\sqrt{2+\phi}}-\sum_{n=0}^{k}\left(\frac{1}{2\phi}\right)^{15n}a_n \quad ,k=0,1,2,3,\ldots \tag{4}$$

$$e2_k=\frac{\pi}{5\sqrt{2+\phi}}-\sum_{n=0}^{k}\left(\frac{1}{2\phi}\right)^{30n}b_n \quad ,k=0,1,2,3,\ldots \tag{5}$$

k	$e1_k$	$e2_k$
0	$4.774 \cdot 10^{-10}$	$5.515 \cdot 10^{-18}$
1	$5.515 \cdot 10^{-18}$	$1.403 \cdot 10^{-33}$
2	$8.317 \cdot 10^{-26}$	$4.709 \cdot 10^{-49}$
3	$1.403 \cdot 10^{-33}$	$1.772 \cdot 10^{-64}$
4	$2.520 \cdot 10^{-41}$	$7.111 \cdot 10^{-80}$
5	$4.709 \cdot 10^{-49}$	$2.969 \cdot 10^{-95}$
10	$1.447 \cdot 10^{-87}$	$5.104 \cdot 10^{-172}$
15	$5.588 \cdot 10^{-126}$	$1.103 \cdot 10^{-248}$
20	$2.389 \cdot 10^{-164}$	$2.645 \cdot 10^{-325}$

Table 1.

References

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