A Step -by- Step Proof of Beal's Conjecture

Zhang Tianshu

Zhanjiang city, Guangdong province, China
Email: chinazhangtianshu@126.com
china.zhangtianshu@tom.com

Abstract

In this article, we first classify A, B and C according to their respective odevity, and thereby get rid of two kinds which belong not to $A^X+B^Y=C^Z$. Then, affirm $A^X+B^Y=C^Z$ in which case A, B and C have at least a common prime factor by several concrete equalities. After that, prove $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by mathematical induction with the aid of the symmetric law of odd numbers whereby even number $2^{W-1}H^Z$ as symmetric center after divide the inequality in four. Finally, reach a conclusion that the Beal's conjecture holds water via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

AMS subject classification: 11D41, 11D85 and 11D61

Keywords: Beal's conjecture; indefinite equation; inequality; odevity; mathematical induction, the symmetric law of odd numbers

Introduction

The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still both unproved and un-negated a conjecture hitherto.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

The Proof

The Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

Let us regard limits of values of aforesaid A, B, C, X, Y and Z as given requirements for hinder indefinite equations or inequalities concerned.

First we classify A, B and C according to their respective odevity, and thereby remove following two kinds which belong not to $A^X+B^Y=C^Z$.

1. A, B and C, all are positive odd numbers.

2. A, B and C are two positive even numbers and a positive odd number.

After that, we merely continue to have following two kinds of $A^X+B^Y=C^Z$ under the given requirements.

1. A, B and C, all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For the indefinite equation $A^X+B^Y=C^Z$ which satisfies aforesaid either set of qualifications, in fact, it has many sets of the solution with A, B and C as positive integers. Let us illustrate with examples as follows.

When A, B and C all are positive even numbers, if let A=B=C=2 and X=Y≥3, then $A^X+B^Y=C^Z$ is changed into $2^X+2^X=2^{X+1}$, where z=x+1. Obviously the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 2, 2 and 2, and that A, B and C have common prime factor 2.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

In addition, if let A=B=162, C=54, X=Y=3 and Z=4, then $A^X+B^Y=C^Z$ is changed into $162^3+162^3=54^4$. So the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3 and Z=5, then $A^X+B^Y=C^Z$ is changed into $3^3+6^3=3^5$. So the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 3, 6 and 3, and that A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7 and Z=3, then $A^X+B^Y=C^Z$ is changed into $7^6+7^7=98^3$. So the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 7, 7 and 98, and that A, B and C have common prime factor 7.

Therefore the indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either set of qualifications is able to hold water, but A, B and C must have at least a common prime factor.

By this token, if we can prove that there is only $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then the conjecture is tenable definitely.

Since A, B and C have the common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

two positive odd numbers and a positive even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, yet another has not it, then it will lead up to $A^X+B^Y\neq C^Z$ according to the unique factorization theorem of natural number.

Unquestionably, following two inequalities add together, then it is able to replace fully $A^X+B^Y\neq C^Z$ under the given requirements and A, B and C are two positive odd numbers and a positive even number without a common prime factor.

1. $A^X+B^Y\neq 2^Z G^Z$ under the given requirements, and A and B are two positive odd numbers, and G is a positive integer, and that they have not a common prime factor.

2. $A^X+2^Y D^Y\neq C^Z$ under the given requirements, and A and C are two positive odd numbers, and D is a positive integer, and that they have not a common prime factor.

For $A^X+B^Y\neq 2^Z G^Z$, it can be divided into two inequalities as follows.

(1) $A^X+B^Y\neq 2^W$, where A and B are positive odd numbers without a common prime factor, and that X, Y and W are integers ≥ 3.

(2) $A^X+B^Y\neq 2^W H^Z$, where A, B and H are positive odd numbers without a common prime factor, and $H\geq 3$, and that X, Y, Z and W are integers ≥ 3.

For $A^X+2^Y D^Y\neq C^Z$, it can be divided into two inequalities as follows.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

(3) $A^X+2^W \neq C^Z$, where A and C are positive odd numbers without a common prime factor, and that X, W and Z are integers ≥3.

(4) $A^X+2^W R^Y \neq C^Z$, where A, R and C are positive odd numbers without a common prime factor, and R ≥3, and that X, Y, Z and W are integers ≥3.

We regard limits of values of A, B, C, H, R, X, Y, Z and W in listed above four inequalities plus their co-prime relation in each inequality as known requirements for hinder inequalities or indefinite equations concerned.

Thus it can be seen, the proof of $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove listed above four inequalities under the known requirements. For this purpose, we are necessary to expound beforehand some circumstances relating to these proofs, ut infra.

First let us classify all positive odd numbers into two kinds, i.e. Φ and Ω. Namely the form of Φ is 1+4n, and the form of Ω is 3+4n, where n ≥ 0.

As thus, positive odd numbers from small to large form infinitely many cycles of Φ plus Ω, to wit Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; …

After that, add even numbers $2^{W-1}H^Z$ among the sequence of positive odd numbers, where H is an odd number ≥1, and W, Z ≥ 3.

Let us regard each of $2^{W-1}H^Z$ as a symmetric center of positive odd numbers concerned, then positive odd numbers on the left side of the symmetric center and positive odd numbers near the symmetric center on

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

the right side of the symmetric center are one-to-one bilateral symmetries at the number axis or in the sequence of natural numbers.

Such symmetric relations of positive odd numbers indicate that for any of $2^{W-1}H^{Z}$ as a center of symmetry, it can only symmetrize one of Φ and one of Ω, yet symmetrize not two of either kind, and that start from any symmetric center, there are both finitely many cycles of Ω plus Φ leftwards until Φ=1, and infinitely many cycles of Φ plus Ω rightwards.

Clearly two distances from a symmetric center to bilateral symmetric Φ and Ω on two sides of the symmetric center are either two equilong segments at the number axis, or two identical differences in the sequence of natural numbers.

Thus the sum of two bilateral symmetric odd numbers is equal to the double of even number as the symmetric center. Yet over the left, a sum of two non-symmetric odd numbers is unequal to the double of even number as the symmetric center.

We term the symmetry between two kind's odd numbers plus the relation proper to the sum of two symmetric odd numbers = the double of even number as the symmetric center "the symmetric law of odd numbers".

In addition, for a positive odd number, it is able to be expressed as one of O^{V} where V≥3, or V=1 and 2, and V expresses the greatest common divisor of exponents of distinct prime factors of the positive odd number, and O is a positive odd number. For O^{V} with V=1 and 2, write it to $O^{1\sim2}$.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

By now, we set to prove aforesaid 4 inequalities orderly by mathematical inductions with the aid of the symmetric law of odd numbers.

Firstly, Let us regard 2^{W-1} as a center of symmetry of positive odd numbers concerned to prove $A^X+B^Y\neq 2^W$ under the known requirements by the mathematical induction, thereinafter.

(1) When W-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric centers 2^{W-1} are listed below successively.

1^6, 3, (2^2), 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

By this token, there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^{W-1} as a symmetric center, where W-1=2, 3, 4, 5 and 6. Namely there are $A^X+B^Y\neq 2^3$, $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements.

(2) When W-1=K with K≥6, suppose that there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^K as a symmetric center. Namely suppose $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) When W-1=K+1, prove that there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center, i.e. prove $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

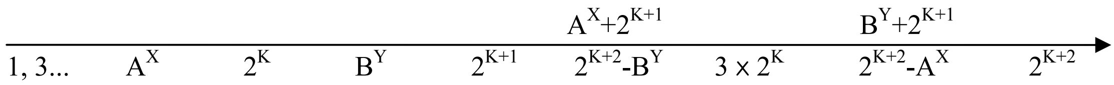
Proof* Since odd numbers whereby 2^{W-1} including 2^K plus 2^{K+1} as a symmetric center are possess of one-to-one symmetric relations, then positive odd numbers whereby 2^K as a symmetric center are exactly positive odd numbers on the left side of symmetric center 2^{K+1}.

Thus, for positive odd numbers whereby 2^{K+1} as a center of symmetry, their a half retains still on original places after move the symmetric center to 2^{K+1} from 2^K, and the half lies on the left side of 2^{K+1}. While, another half is formed from 2^{K+1} plus each of positive odd numbers whereby 2^K as a symmetric center, and that the half lies on the right side of 2^{K+1}.

Suppose that A^X and B^Y are a pair of bilateral symmetric positive odd numbers whereby 2^K as a symmetric center, then there is $A^X+B^Y=2^{K+1}$ according to the symmetric law of odd numbers.

Since there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^K as a symmetric center according to second step of the mathematical induction, so tentatively let A^X as one of O^V with $V\geq3$, and B^Y as one of $O^{1\sim2}$, i.e. let $X\geq3$, $Y=1$ and 2.

By now, let B^Y plus 2^{K+1} to make B^Y+2^{K+1}. Please, see also a simple illustration at the number axis as follows.



Since there is only $A^X+B^Y\neq2^{K+1}$ under the known requirements according to second step of the mathematical induction, therefore there is inevitably $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 and 2.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

As thus, we deduce $B^Y+2^{K+1}=A^X+2B^Y=2^{K+2}-A^X$ from $A^X+B^Y=2^{K+1}$.

Also A^X and $2^{K+2}-A^X$ are bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center due to $A^X+(2^{K+2}-A^X)=2^{K+2}$ according to the symmetric law of odd numbers.

So A^X and A^X+2B^Y are two bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center, and there are $A^X+(2^{K+2}-A^X)=A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y, and Y=1 and 2.

Of course, A^X and A^X+2B^Y in the case are still a pair of bilateral symmetric Φ and Ω whereby 2^{K+1} as a symmetric center.

But then, there is only $A^X+B^Y\neq 2^{K+1}$ under the known requirements, thus it has $A^X+ [A^X+2B^Y] =2[A^X+B^Y] \neq 2^{K+2}$ in that case.

In any case, A^X+2B^Y are only a positive odd number. So let $A^X+2B^Y=D^E$, where E expresses the greatest common divisor of exponents of distinct prime factors of the positive odd number, and D is a positive odd number, then we get $A^X+ [A^X+2B^Y] =A^X+D^E\neq 2^{K+2}$ under the known requirements.

That is to say, no matter what positive integer which E equals and no matter what positive odd number which D equals from $A^X+2B^Y=D^E$, there is $A^X+[A^X+2B^Y]=A^X+D^E\neq 2^{K+2}$ under the known requirements invariably. Namely A^X and D^E under the known requirements are not two bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center.

Whereas A^X and A^X+2B^Y, i.e. A^X and D^E under the known requirements except for Y, and Y=1 and 2, two are indeed a pair of bilateral symmetric

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

odd numbers whereby 2^{K+1} as a symmetric center due to $A^X+[A^X+2B^Y]=A^X+D^E=2^{K+2}$ according to the symmetric law odd numbers.

Such being the case, provided you slightly change a bit of valuation of any letter of A^X+2B^Y, then it at once is not original that A^X+2B^Y under the known requirements except for Y, and Y=1 and 2.

Naturally, now it lies not on the place of the symmetry with A^X either. Namely A^X and A^X+2B^Y under the known requirements are not two bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center, because the value of Y is changed into $Y\geq3$ from Y=1 and 2.

Thus there is $A^X+[A^X+2B^Y]=A^X+D^E\neq2^{K+2}$ under the known requirements according to the symmetric law of odd numbers.

Moreover, A^X has been supposed as one of O^V with $V\geq3$ on the left side of symmetric center 2^{K+1}. Also there is $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 and 2. So it has $A^X+2B^Y=2^{K+1}+B^Y$.

As thus, A^X+2B^Y i.e. D^E lies on the right side of symmetric center 2^{K+1}.

For inequality $A^X+D^E\neq2^{K+2}$ under the known requirements, let us substitute D by B, since B and D can express any positive odd number; additionally substitute Y for E where $E\geq3$, since $Y\geq3$.

Consequently, we obtain $A^X+B^Y\neq2^{K+2}$ under the known requirements.

In this proof, if B^Y is one of O^V with $V\geq3$, then A^X is surely one of $O^{1\sim2}$, yet a conclusion concluded on the premise is really one and the same with $A^X+B^Y\neq2^{K+2}$ under the known requirements.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

If A^X and B^Y are bilateral symmetric two of $O^{1\sim2}$ whereby 2^K as a symmetric center, then whether A^X and A^X+2B^Y, or B^Y and B^Y+2A^X, they are uniformly a pair of bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center. But, no matter what positive odd number which A^X+2B^Y or B^Y+2A^X equal, it can not change the pair of bilateral symmetric odd numbers into two of O^V with $V\geq3$, because A^X or B^Y in the pair is not one of O^V with $V\geq3$ originally.

Overall, we have proven that when W-1=K+1 with K ≥6, there is only $A^X+B^Y\neq2^{K+2}$ under the known requirements. In other words, there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^{K+1} as a symmetric center.

Apply the preceding way of doing, we can continue to prove that when W-1=K+2, K+3…up to every integer ≥3, there are merely $A^X+B^Y\neq2^{K+3}$, $A^X+B^Y\neq2^{K+4}$ … up to $A^X+B^Y\neq2^W$ under the known requirements.

Secondly, Let us prove $A^X+B^Y\neq2^WH^Z$ under the known requirements by the mathematical induction successively, and point out $H\geq3$ emphatically.

(1) When H=1, $2^{W-1}H^Z$ i.e. 2^{W-1}, we have proven $A^X+B^Y\neq2^W$ under the known requirements in the preceding section. Namely there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^{W-1} as a symmetric center.

(2) When H=J and J is an odd number ≥1, $2^{W-1}H^Z$ i.e. $2^{W-1}J^Z$, suppose $A^X+B^Y\neq2^WJ^Z$ under the known requirements. Namely suppose that there

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}J^Z$ as a symmetric center.

(3) When H=K with K=J+2, $2^{W-1}H^Z$ i.e. $2^{W-1}K^Z$, prove $A^X+B^Y\neq2^WK^Z$ under the known requirements. Namely prove that there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center.

Proof* Since after regard $2^{W-1}H^Z$ as a symmetric center, the sum of every pair of bilateral symmetric odd numbers is equal to 2^WH^Z, while a sum of two odd numbers of no symmetry is unequal to 2^WH^Z.

In addition, there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}J^Z$ as a symmetric center. Namely there is $A^X+B^Y\neq2^WJ^Z$ under the known requirements according to second step of the mathematical induction.

Such being the case, thus we suppose that A^X and B^Y are a pair of bilateral symmetric odd numbers whereby $2^{W-1}J^Z$ as a symmetric center, also tentatively let $Y\geq3$ and X=1 and 2. So there is $A^X+B^Y=2^WJ^Z$ undoubtedly.

On the other, after regard $2^{W-1}K^Z$ as a symmetric center, B^Y and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers due to $B^Y+(2^WK^Z-B^Y)=2^WK^Z$ according to the symmetric law of odd numbers.

By now, let A^X plus $2^W(K^Z-J^Z)$ to make $A^X+2^W(K^Z-J^Z)$. Due to $A^X+B^Y=2^WJ^Z$, then there are $A^X+2^W(K^Z-J^Z)=A^X+2^WK^Z-2^WJ^Z=2^WK^Z-(2^WJ^Z-A^X)=2^WK^Z-B^Y$ under the known requirements except for X, and X=1 and 2.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

Now that there is $A^X+2^W(K^Z-J^Z)=2^WK^Z-B^Y$ under the known requirements except for X, and X=1 and 2; additionally B^Y and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center, therefore B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center.

Thus we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X, and X=1 and 2.

Of course, B^Y and $A^X+2^W(K^Z-J^Z)$ in the case are still a pair of bilateral symmetric Φ and Ω whereby $2^{W-1}K^Z$ as a symmetric center.

From $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$ and beforehand supposed $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, we get $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^WK^Z-2^WJ^Z\neq 2^WK^Z$ under the known requirements.

Thus it can be seen, B^Y and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center due to $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^WK^Z$ according to the symmetric law of odd numbers.

It is obvious that $A^X+2^W(K^Z-J^Z)$ in aforesaid two cases expresses two disparate odd numbers due to $X\geq 3$ in one, and X=1 and 2 in another.

From $A^X+2^W(K^Z-J^Z)=2^WK^Z-(2^WJ^Z-A^X)$ and $2^WJ^Z-A^X\neq B^Y$ under the known requirements, we get $A^X+2^W(K^Z-J^Z)\neq 2^WK^Z-B^Y$.

In any case, $A^X+2^W(K^Z-J^Z)$ are only a positive odd number, thus we let $A^X+2^W(K^Z-J^Z)=F^V$, where V expresses the greatest common divisor of

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

exponents of distinct prime factors of the positive odd number, and F is a positive odd number. Thus there is $F^V\neq 2^W K^Z-B^Y$ due to $A^X+2^W(K^Z-J^Z)\neq 2^W K^Z-B^Y$ under the known requirements.

Namely there is $B^Y+F^V\neq 2^W K^Z$ under the known requirements.

Since B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center due to $B^Y+[A^X+2^W(K^Z-J^Z)]=2^W K^Z$ under the known requirements except for X, and X=1 and 2 at here, according to the symmetric law of odd numbers.

Such being the case, provided you slightly change a bit of valuation of any letter of $A^X+2^W(K^Z-J^Z)$, then it at once is not original that $A^X+2^W(K^Z-J^Z)$ under the known requirements except for X, and X=1 and 2. Naturally, now it lies not on the place of the symmetry with B^Y either.

Namely B^Y and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center because the value of X is changed into $X\geq 3$ from X=1 and 2.

Thereby there is $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^W K^Z$ under the known requirements according to the symmetry law of odd numbers. Namely there is only $B^Y+F^V\neq 2^W K^Z$ under the known requirements due to $A^X+2^W(K^Z-J^Z)=F^V$.

For inequality $B^Y+F^V\neq 2^W K^Z$, let us substitute F by A, since A and F express any positive odd number, and substitute X for V where $V\geq 3$, since $X\geq 3$.

Consequently we obtain $A^X+B^Y\neq 2^W K^Z$ under the known requirements.

In this proof, if A^X is one of O^V with $V\geq 3$, then B^Y is surely one of $O^{1\sim 2}$,

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

yet a conclusion concluded on the premise is really one and the same with $A^X+B^Y\neq 2^WK^Z$ under the known requirements.

If A^X and B^Y are bilateral symmetric two of $O^{1\sim2}$ whereby $2^{W-1}J^Z$ as a symmetric center, then whether B^Y and $A^X+2^W(K^Z-J^Z)$, or A^X and $B^Y+2^W(K^Z-J^Z)$, they are uniformly a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center. But, no matter what positive odd number which $A^X+2^W(K^Z-J^Z)$ or $B^Y+2^W(K^Z-J^Z)$ equal, it can not change the pair of bilateral symmetric odd numbers into two of O^V with $V\geq3$, since B^Y or A^X in the pair is not one of O^V with $V\geq3$ originally.

On balance, we have proven $A^X+B^Y\neq2^WK^Z$ with K=J+2 under the known requirements. Namely when H=J+2, there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}(J+2)^Z$ as a symmetric center.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, J+6… up to every positive odd number, there are merely $A^X+B^Y\neq2^W(J+4)^Z$, $A^X+B^Y\neq2^W(J+6)^Z$… up to $A^X+B^Y\neq2^WH^Z$ under the known requirements.

Thirdly, On the basis of the anterior conclusion got, we continue to prove $A^X+2^W\neq C^Z$ under the known requirements by the mathematical induction.

(1) When W=3, 4, 5, 6 and 7, bilateral symmetric odd numbers on two sides of symmetric centers 2^3, 2^4, 2^5, 2^6 and 2^7 are listed below successively.

1^7, 3, 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35,

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127, (2^7), 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225,227, 229, 231, 233, 235, 237, 239, 241, 3^5, 245, 247, 249, 251, 253, 255.

Ut supra, there is only higher power's 1^7 on the left side of the symmetric center 2^3;

There is only higher power's 1^7 on the left side of the symmetric center 2^4;

There are higher power's 1^7 and 3^3 on the left side of the symmetric center 2^5 altogether;

There are higher power's $1^7$and 3^3 on the left side of the symmetric center 2^6 altogether;

There are higher power's 1^7, 3^3, 3^4 and 5^3 on the left side of symmetric center 2^7 altogether.

We observe that they only have $1^7+2^3\neq C^Z$; $1^7+2^4\neq C^Z$; $1^7+2^5\neq C^Z$, $3^3+2^5\neq C^Z$; $1^7+2^6\neq C^Z$, $3^3+2^6\neq C^Z$; $1^7+2^7\neq C^Z$, $3^3+2^7\neq C^Z$, $3^4+2^7\neq C^Z$ and $5^3+2^7\neq C^Z$.

Therefore there are $A^X+2^3\neq C^Z$, $A^X+2^4\neq C^Z$, $A^X+2^5\neq C^Z$, $A^X+2^6\neq C^Z$ and $A^X+2^7\neq C^Z$ under the known requirements.

(2) When W=N with N≥7, suppose that there is $A^X+2^N\neq C^Z$ under the known requirements, where $A^X < 2^N < C^Z$.

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

(3) When W=N+1, prove that there is $A^X+2^{N+1}\neq C^Z$ under the known requirements, where $A^X < 2^{N+1} < C^Z$.

Proof* Since it has $(2^{N+1}+A^X)+(2^{N+1}-A^X)=2^{N+2}$, so $2^{N+1}+A^X$ and $2^{N+1}-A^X$ are a pair of bilateral symmetric odd numbers whereby 2^{N+1} as a symmetric center according to the symmetric law of odd numbers.

Also there is the inequality $2^{N+1}-A^X\neq O^V$ i.e. $A^X+O^V\neq 2^{N+1}$ where $V\geq 3$ according to proven $A^X+B^Y\neq 2^W$ under the known requirements, so $2^{N+1}-A^X$ can only be one of $O^{1\sim 2}$.

Now that $2^{N+1}-A^X$ is one of $O^{1\sim 2}$, then $2^{N+1}-A^{1\sim 2}$ contain both some of $O^{1\sim 2}$ and all of O^V with $V\geq 3$ under even number 2^{N+1}.

In addition, $2^{N+1}+A^{1\sim 2}$ and $2^{N+1}-A^{1\sim 2}$ are two bilateral symmetric odd numbers whereby 2^{N+1} as a symmetric center, since $(2^{N+1}+A^{1\sim 2})+(2^{N+1}-A^{1\sim 2})$ $=2^{K+2}$ according to the symmetric law of odd numbers.

Therefore $2^{N+1}+A^{1\sim 2}$ contain both some of $O^{1\sim 2}$ and all of O^V with $V\geq 3$ under even number 2^{N+2}.

Since $2^{N+1}-A^X$ within $(2^{N+1}+A^X)+(2^{N+1}-A^X)=2^{N+2}$ is one of $O^{1\sim 2}$, then $2^{N+1}+A^X$ is either one of $O^{1\sim 2}$ or one of O^V with $V\geq 3$ under even number 2^{N+2}, since there are not two of O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^{W-1} as a symmetric center.

But $2^{N+1}+A^{1\sim 2}$ contained all of O^V with $V\geq 3$ under even number 2^{N+2}, therefore $2^{N+1}+A^X$, i.e. A^X+2^{N+1} can only be one of $O^{1\sim 2}$.

In addition to this, C^Z is one of O^V with $V\geq 3$ according to stipulations of

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

values of C and Z within the known requirements.

Consequently there is $A^X+2^{N+1}\neq C^Z$ under the known requirements.

Apply the preceding way of doing, we can continue to prove that when W=N+2, N+3…up to every integer≥3, there are merely $A^X+2^{N+2}\neq C^Z$, $A^X+2^{N+3}\neq C^Z$…up to $A^X+2^W\neq C^Z$ under the known requirements.

Fourthly, On the basis of the anterior conclusion got, we prove $A^X+2^WR^Y \neq C^Z$ under the known requirements by the mathematical induction by now.

(1) When R=1, 2^WR^Y i.e. 2^W, we have proven $A^X+2^W\neq C^Z$ under the known requirements in the preceding section.

(2) When R=J and J is an odd number ≥1, 2^WR^Y i.e. 2^WJ^Y, suppose that there is $A^X+2^WJ^Y\neq C^Z$ under the known requirements, where $A^X<2^WJ^Y< C^Z$.

(3) When R=K with K=J+2, 2^WR^Y i.e. 2^WK^Y, prove that there is $A^X+2^WK^Y \neq C^Z$ under the known requirements, where $A^X< 2^WK^Y < C^Z$.

Proof* Since $(2^WK^Y+A^X)+(2^WK^Y-A^X)=2^{W+1}K^Y$, then $2^WK^Y+A^X$ and $2^WK^Y-A^X$ are a pair of bilateral symmetric odd numbers whereby 2^WK^Y as a symmetric center according to the symmetric law of odd numbers.

Also there is the inequality $2^WK^Y-A^X\neq O^V$ i.e. $A^X+O^V\neq 2^WK^Y$ where V≥3, according to proven $A^X+B^Y \neq 2^WH^Z$ under the known requirements, so $2^WK^Y-A^X$ can only be one of $O^{1\sim2}$.

Now that $2^WK^Y-A^X$ is one of $O^{1\sim2}$, then $2^WK^Y-A^{1\sim2}$ contain both some of $O^{1\sim2}$ and all of O^V with V≥3 under even number 2^WK^Y.

In addition, $2^WK^Y+A^{1\sim2}$ and $2^WK^Y-A^{1\sim2}$ are a pair of bilateral symmetric

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

odd numbers whereby $2^W K^Y$ as a symmetric center due to $(2^W K^Y+A^{1\sim2})$ + $(2^W K^Y-A^{1\sim2})$ $=2^{W+1}K^Y$ according to the symmetric law of odd numbers.

Therefore $2^W K^Y+A^{1\sim2}$ contain both some of $O^{1\sim2}$ and all of O^V with $V\geq3$ under even number $2^{W+1}K^Y$.

Since $2^W K^Y-A^X$ within $(2^W K^Y+A^X)+(2^W K^Y-A^X)=2^{W+1}K^Y$ is one of $O^{1\sim2}$, then $2^W K^Y+A^X$ is either one of $O^{1\sim2}$ or one of O^V with $V\geq3$ under even number $2^{W+1}K^Y$, since there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^W H^Z$ as a symmetric center.

Since $2^W K^Y+A^{1\sim2}$ contained all of O^V with $V\geq3$ under even number $2^{W+1}K^Y$, therefore $2^W K^Y+A^X$, i.e. $A^X+2^W K^Y$ can only be one of $O^{1\sim2}$.

In addition to this, C^Z is one of O^V with $V\geq3$ according to stipulations of values of C and Z within the known requirements.

Consequently there is $A^X+2^W K^Y\neq C^Z$, i.e. $A^X+2^W(J+2)^Y\neq C^Z$ under the known requirements.

Apply the preceding way of doing, we can continue to prove that when R=J+4, J+6…up to every positive odd number, there are $A^X+2^W (J+4)^Y \neq C^Z$, $A^X+2^W(J+6)^Y\neq C^Z$…up to $A^X+2^W R^Y\neq C^Z$ under the known requirements.

To sum up, we have proven every kind of $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor.

In addition, we have proven that $A^X+B^Y=C^Z$ under the given requirements

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

plus the qualification that A, B and C have at least a common prime factor in the front of this article.

Such being the case, so long as make a comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements, then inevitably will reach such a conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal's conjecture holds water.

PS. If Beal's conjecture is proved to hold water, then let X=Y=Z, so indefinite equation $A^X+B^Y=C^Z$ is changed into $A^X+B^X=C^X$. In addition, divide three terms of $A^X+B^X=C^X$ by their greatest common divisor, then get a set of the solution of positive integers without a common prime factor. Obviously this conclusion is in contradiction with proven Beal's conjecture, so we proved Fermat's last theorem as easy as the pie extra.

References

[1] Wolfram Mathworld: *http://mathworld.wolfram.com/BealsConjecture.html.*

[2] R. Daniel Mauldin *(1997, "A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem" . Notices of the AMS 44(11): 1436–1439.*

[3] *Beal conjecture.com. Retrieved 2014-03-06,* http://www.bealconjecture.com/

[4] Norvig, Peter. *"Beal's Conjecture: A Search for Counterexamples". Norvig.com.*

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS

Retrieved 2014-03-06.

[5] Darmon, H.; Granville, A. *(1995). "On the equations* $z^m = F(x, y)$ and $Ax^p + By^q = Cz^r$*". Bulletin of the London Mathematical Society* 27: 513–43.

[6] Frits Beukers *(January 20, 2006), "The generalized Fermat equation" , Staff.science.uu.nl, Retrieved 2014-03-06.*

[7] Michel Waldschmidt *(2004). "Open Diophantine Problems". Moscow Mathematics* 4: 245–305.

[8] Castelvecchi, D. "*Mathematics Prize Ups the Ante to $1 Million." June 4, 2013.* http://blogs.nature.com/news/2013/06/mathematics-prize-ups-the-ante-to-1-million.html.

[9] Darmon, H. and Granville, A. "*On the Equations* $z^m=F(x, y)$ and $Ax^p+By^q=Cz^r$. " *Bull, London Math, Soc,* 27, 513-543, 1995.Elkies, N. "The ABCs of Number Theory", *Harvard Math, Rev,* 1, 64-76, 2007.

[10] Mauldin, R. D. "*A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem." Not. Amer. Math, Soc, 44, 1436-1437, 1997.*

Jul 19 2017 06:11:05 EDT
Version 1 - Submitted to JAMS