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A new computational measurement and optimization approach for DSmT

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***Abstract*—*A great deal of interest has been paid to computation problem of Dezert-Smarandache theory (DSmT). But there are still problems on complex analysis and frequently search. The computational measurement of DSmT is presented in which the computation is generated in the search for focal elements, the combination of focal elements and basic belief assignment, the expression of focal elements. A new DSmT computational optimization approach is presented to solve the problems. The proposed approach optimizes the original evidence and combination of focal elements. The original evidence is reduced to keep the effective focal elements. And the focal element relationship is integrated into evidence code to realize self-adaption for combination of focal elements. Numerical results are provided to validate our approach.*

Keywords*—*Dezert-Smarandache theory; Computational measurement; Computational optimization.

I. INTRODUCTION

The evidence reasoning theory is an important approach for uncertain information fusion. In evidence reasoning theory, Dempster-Shafer (DST) and Dezert-Smarandache theory (DSmT) are approved by the experts for their effective fusion of multi-source evidence [1-3]. But the tremendous computation which is produced by focal element explosion in evidence combination obstructs the application of evidence reasoning theory [4].

For this reason computation problem is studied and analyzed by many experts. And a lot of approximate methods are presented to solve the issue. The methods can be divided into two categories. The first class is evidence and focal element approximate method which simplify the original evidence and focal element to reduce the computation, such as the Tessem's *k-l-x* method [5], energy function method [6] and Bayes approximate method [7]. The second class is combination rule approximate method which simplifies the combination rule for specific application to avoid unnecessary computation, such as hierarchical hypothesis approximate method [8], layering tree approximate method [9-12].

But the existing methods cannot solve the computation problem absolutely. The first problem is the complex analysis and different programming for existing methods with undefined logical relationship. The second problem is the frequently search of focal elements in practical application to produce extra computation. Thirdly computation only involves focal element and basic belief assignment (BBA) combination in existing methods, which cannot measure computation of DSmT comprehensively.

So computation optimization method with information process is presented in the paper to solve the computation problem. The method keeps the merit of existing methods to manage the focal elements effetely. And the idea of evidence coding is added into the method to make computer coding practicable and optimize the computation of focal element logic and focal element search.

For the measurement of computation, the traditional computation estimated method is developed to obtain a new computation measurement system. The computation of focal element search and focal element expression is added into the system based on computation of focal element and BBA combination. So the computation measurement of DSmT evidence fusion process is more perfect than before.

In the end of the paper, the combination result and computation are analyzed and compared in the experiment. The

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simulation result demonstrates the effectiveness of the computation measurement and optimization method.

II. THE PROBLEMS OF EXISTING METHODS

From 1980s the experts started to study the approximate method of DST theoretically [1]. There has been much attention on computation of DSmT when Dezert and Smarandache presented it [13]. These approximate methods mainly involve the computation of evidence combination. The methods can be divided into two categories as the approximate method for original evidence and the approximate method for combination rule.

The first class is evidence and focal element approximate method which classify the original evidence and focal element to obtain the useful part. The main achievements of existing methods belong to this class. Tessem presented the famous *k-l-x* method [5]. The *k-l-x* method reduce the number of focal element in original evidence ignore the influence of focal element with small value. D1 approximate method [14], energy function method [6] and ranking fusion method [15] are considered as the improved method or specific form. Some scholars unite evidence reasoning theory with other uncertain theory to present Bayes approximate method [7] and hierarchy proportion approximate method [16] and so on. The evidence is converted into other uncertain theory to control focal element for reducing computation. Some experts think that the evidence should be deleted when the evidence is confirmed as unreliable [17].

The second class is combination rule approximate method which simplifies the combination rule for specific application to avoid unnecessary computation. Barnett presented fast algorithm for simple structure evidence [18]. The method only combined single focal element and its complementary set which is suitable for loose classification. Gordon and Shortliffe proposed that the evidence subset can be reduced as a hierarchy tree when the evidence supports single focal element and non-intersection [8]. Li xinde presented three layering tree approximate method for different situation [9-12].

The existing methods reduce computation by approximating evidence and combination rule. But the three are computation of focal element logic and focal element search besides combination computation in the evidence process. The existing methods store the focal element and BBA in array. There are three problems for existing methods. Firstly, the logic relationship between focal element is undefined, which is different to parse the mix focal element. So the existing methods are hard to apply in the reality. Secondly, the focal elements are searched frequently in the evidence processing to produce extra computation. Thirdly, the computation measurement of existing methods only involves the combination which is not all-inclusive for measurement. The problem 1 and problem 2 belong to computation optimization which is discussed in section 2. The problem 3 belongs to computation measurement which is discussed in section □.

III. COMPUTATIONAL MEASUREMENT OF DSMT

Only the computation of focal elements and BBA combination is considered in the traditional methods. And the information process contains four steps. Every step finish each function with different computation. The analysis the computation of each step is the basis of the research of whole computation for information process of DSmT.

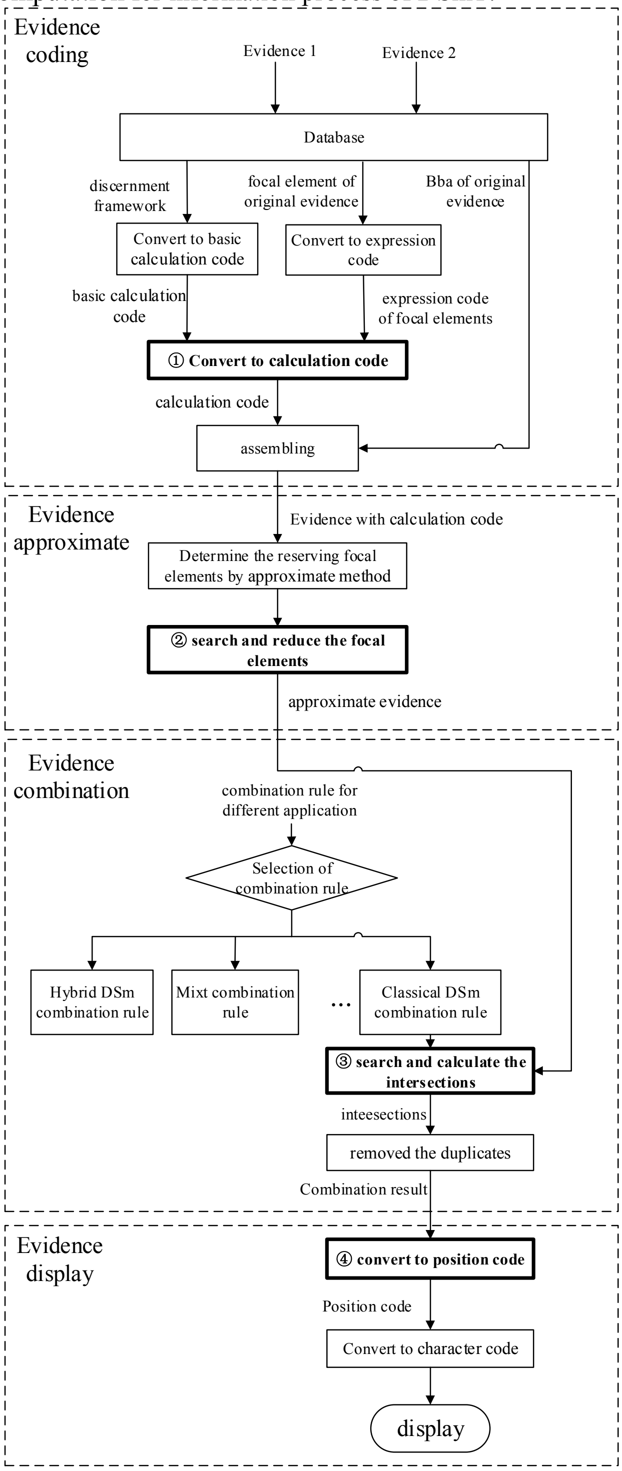

Fig. 1. The information process of DSmT

In the evidence coding step, the main computation is made by the translation from character code to calculation code (block diagram ① in Fig.1) which is called coding computation. In the evidence approximate step, the main computation is made by the search operation of evidence approximate (block diagram ②) which is called approximate search computation. The most computation is caused in evidence combination step. The computation contains the search operation of evidence combination, focal elements combination and BBA combination(block diagram ③) which are called combination search computation, focal element combination computation and BBA combination computation. In the evidence display step, the main computation is made by the translation from calculation code to character code (block diagram ④) which is called decoding computation.

For the quantitative analysis of the computation, the computation is classified as search computation, combination computation and expression computation by the function of the operation. The search computation is the number of times in the search process which contains the focal elements search operation in evidence approximate step and evidence combination step. The combination computation contains the addition and multiplication in the evidence combination step. The expression operation means the computation of coding and decoding.

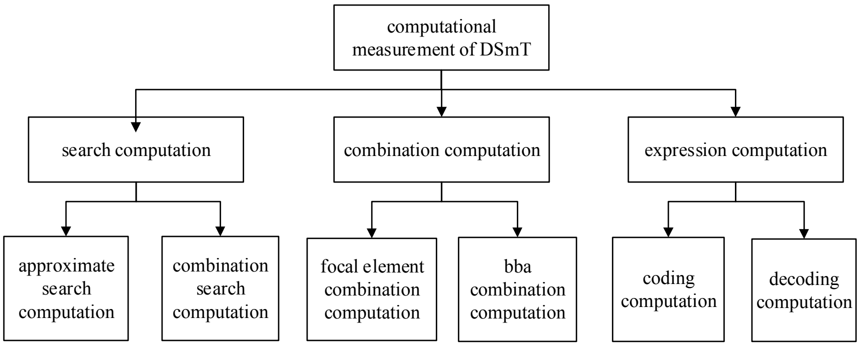

Fig. 2. Computational measurement of DSmT

A. Search computation

The search computation is measured by the time of search operation. The search computation is the value of expectation for compared time between goal focal element and focal elements in the memory.

Definition 1 Let's consider *n* focal elements in the evidence, m focal elements for searching, the search computation is expressed as the average search times, which is $O_{Search}(f(n,m))$.

For the length of an article, the derivation process is not listed here. The approximate search computation and combination search computation are obtained with the corresponding parameters.

B. Combination computation

The combination of evidence includes focal elements combination and BBA combination. So the combination computation consists of focal element combination computation and BBA combination computation. The focal element combination computation is caused by logic relationship simplification process. The BBA combination computation is the fusion of the BBA for every focal element.

In this paper, the computation code contains the logic relationship. So the computation code of evidence fuses directly and chooses the same part to obtain the temporary result which remove duplicates acquire combination result. So the focal element computation is ignored in computation optimization method. Here is an example to explain the focal element combination with calculation code.

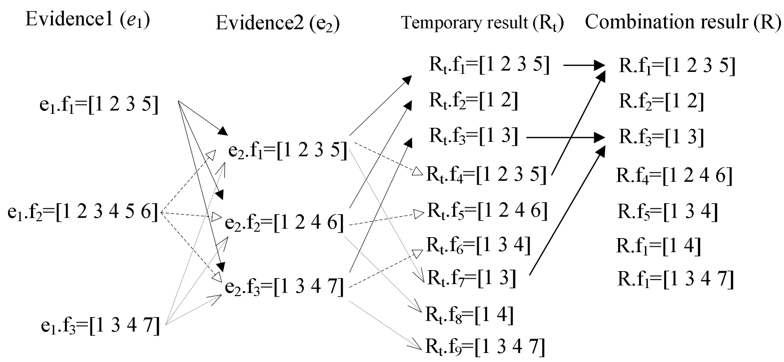

Fig. 3. The combination process of calculation code evidence

Example 1 Let's take a 3D frame of discernment $\Theta=\{\theta_1,\theta_2,\theta_3\}$ and consider two evidence with calculation code e_1 and e_2 .The focal elements of evidence are $e_1.\mathrm{f}=\{\theta_1,\theta_1\cup\theta_2,\theta_3\}$ and $e_2.\mathrm{f}=\{\theta_1,\theta_2,\theta_3\}$. The combination process of computation code evidence is in Fig.3.

In the area of BBA combination, evidence combination can be considered as the composition of multiplication operation (division operation) and addition operation (subtraction operation) which are atom operation. The computation of multiplication operation is bigger than addition operation. So the BBA combination computation is the weighted sum of multiplication operation and addition operation.

Definition 2 Let's consider n focal elements $\Theta=\{\theta_1,\theta_2,...,\theta_n\}$ in the k evidence, the set of focal elements is G^Θ (G^Θ can be Shafer model or free DSm model), the number of focal elements is $|G^\Theta|$, for the evidence $m_j(A_{jl})$, $j=1,2,\cdots,k$, $l=1,2,\cdots,|G^\Theta|$, $A_{jl}\subseteq\Theta$. The weighted sum of multiplication operation times (division operation) and addition operation times (subtraction operation) for certain combination rule is the BBA combination computation which is expressed as $O_{bba\,combination}(f(n,k))$.

C. Expression computation

The expression computation contains coding computation and decoding computation. Coding computation is generated in the process of converting character code to calculation code. The calculation code is assembling by basic calculation code according to the relationship between focal elements (the relevant definition is declared in section □).

Definition 3 Let's consider l focal elements $\Theta=\{\theta_1,\theta_2,...,\theta_n\}$ evidence with n focal elements, the number of elements in discernment frame is $|\Theta|$, the number of operations (intersect operation and union operation) for focal elements which are not in the discernment frame is a_k (k=1,…,n-$|\Theta|$, $a_k=l$, l is positive integer). The coding computation is $O_{Coding}(f(n,\Theta))=\sum_{k=1}^{n-|\Theta|}(a_k+1)$.

Decoding computation is generated in the process of converting calculation code to character code. The main computation is caused by converting position code. The essence of decoding computation is the times of search. The specific process is explained in section □ step 4.

Definition 4 Let's consider n focal elements in the combination result, the decoding calculation is $O_{Decoding}(f(n))=n$.

D. Analysis of computation

The three classes of computation are defined and analyzed in above sections. But it is unreasonable to define total computation as the simply sum of three classes of computation. The units of search computation and expression computation are the times of searching. The unit of combination computation is the times of multiplication operation (division operation) and addition operation (subtraction operation). The artificial operation time should be taken account in the method without evidence coding. So the computation can divide into two kinds by the unit of computation. First kind is combination computation. And the second kind is search computation and expression computation. The two kinds of computation are defined as a two-tuples in definition 5.

Definition 5 In the information process, the combination computation, search computation and expression is measured as $O_{combination}$, O_{search} and $O_{expression}$ respectively, the total is defined as O_{total}=[$O_{combination}$, O_{search} +$O_{expression}$].

The coding operation is not in the existing method whose computation is composed by approximate search computation, combination search computation and combination computation. The approximate search computation is ignored if the approximate operation is not made. The computation of existing method shows in formula(1).

$$O_{Existing}=\left[O_{combination},O_{search}\right]=\begin{bmatrix}O_{focal\ element\ combination}+O_{bba\ \ combination},\\ O_{approximate\ search}+O_{combination\ search}\end{bmatrix} \tag{1}$$

The computation of computation optimization method in this paper is composed by approximate search computation, combination and computation. The computation of optimization method shows in formula(2).

$$O_{optimization}=\left[O_{combination},O_{search}+O_{expression}\right]=\begin{bmatrix}O_{bba\ combination},\\ O_{coding}+O_{approximate\ search}+O_{decoding}\end{bmatrix} \tag{2}$$

IV. COMPUTATIONAL OPTIMIZATION BASED ON INFORMATION PROCESS

The existing methods have difficulties in focal element logic and focal element search, whose core issue is the undefined relationship between the focal elements. Coding the original evidence can obtain the good features of existing methods and reduce the extra computation. So two key issue should be solved. The first issue is the selection of code. The logic relationship should be contained in the code for the effective processing. The second issue is structuring an integrated information process to reduce computation in theoretical and engineered aspects.

A. The relationship between focal elements

The evidence is expressed in character in existing methods, such as the target 1 is expressed as θ_1, target 1 or target 2 is expressed as $\theta_1 \cup \theta_2$. The original expression method of focal element is called character code. The character code is easy for understanding. But the relationship between focal elements need be handled by users which block the application of DST and DSmT in computer system. Meanwhile the focal element and its BBA must be searched in every processing operation. So the expression code, Smarandache code [19] and calculation code [20] are proposed by experts to solve the problem of character code.

A perceptual intuition improved method is the digitization of character code to make it easy for the computer processing. Expression code is character code expressed in numbers, such as $\theta_1 \cap \theta_2$ is expressed in [1 -1 2], 1 and 2 corresponding to θ_1 and θ_2, -1 express symbol $\cap$. And other symbols have corresponding numbers. Expression code is the initial digitization of character code. But the logic relationship between focal elements is not contained in the expression code.

According to the different processing of focal element logic, Smarandache and Arnaud Martin presented Smarandache code and calculation code separately. Smarandache code takes the separate part of set as a unit. Absolutely separate parts are composed by the tuples of number. Compound separate parts are composed multiple absolutely separate parts which are the collection of one or more tuples of number. The logic relationship between focal elements shows in Venn diagram. The Venn diagram and corresponding Smarandache code are shown in Fig.4(a) as *n=3*. In Fig.4(a), θ_1 is expressed as {[1] [12] [13] [123]}, θ_2 is expressed as {[1] [12] [23] [123]}, $\theta_1 \cap \theta_2$ is the intersection of θ_1 and θ_2 expressed as {[12] [13]}, θ_1 and θ_2 are compound separate parts, [1] [12] are absolutely separate parts.

The calculation code is also composed by the separate parts of Venn diagram. But the corresponding number is integers in $[1:2^n-1]$ for calculation code. The Venn diagram and corresponding calculation code are shown in Fig.4(b) as *n=3*. In Fig.4(b), θ_1 is expressed as [1 2 3 5], θ_2 is expressed as [1 2 4 6], $\theta_1 \cap \theta_2$ is the intersection of θ_1 and θ_2 expressed as [1 2]. Calculation code can reach the result by adding and subtracting two sets.

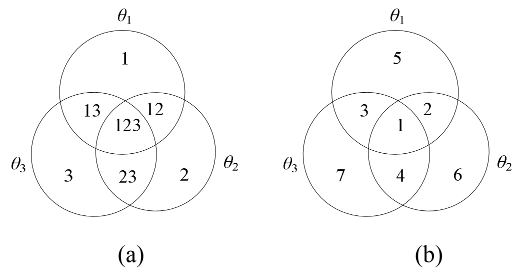

Fig. 4. The Venn diagram and corresponding two code (n=3)

TABLE I. ANALYSIS OF THE DIFFERENT CODE

Expression method	Computation	Code format (n=3)	Range of application
Character code	approximate searching computation, combination searching computation, focal element combination computation, BBA combination computation	$\theta_1 \cap \theta_2$	Display of original evidence and combination result
Expression code	middle process quantity in which computation problem is not involved	[1 -1 2]	Preparing data for the convection to Smarandache code and calculation code
Smarandache code	coding computation, approximate searching computation, BBA combination computation, decoding computation	{[12] [123]}	Combination of focal elements and BBA
Calculation code	coding computation, approximate searching computation, BBA combination computation, decoding computation	[1 2]	Combination of focal elements and BBA

The codes have own range of application. The computation is decided by the characteristic of the code. The codes are analyzed in the TABLE I to conclude that the Smarandache code and calculation code could be the code for evidence combination. Smarandache code can be understand easily but with more logic add and multiply operations which increasing the computation. The calculation code solves the problem of Smarandache code. The calculation code has less total computation with the consideration of coding computation and decoding computation which is illustrated in the next section.

B. *The steps of optimization*

The Coding solves the code logic relationship and focal elements searching issue. The coding method and existing method are two different directions. The incorporation of the two kinds of methods can reduce the computation comprehensively. Based on this idea, the information process of evidence fusion should be analyzed to clear and definite the position of the two methods.

The information process of evidence fusion can subdivide as seven steps. The information process is simplified into four steps according to the computation issue as evidence coding, evidence approximate, evidence combination and evidence display in Fig.1.

Step1: evidence coding. First of all, the basic calculation code is obtained according to the discernment framework. The character code of input evidence is converting to expression code and to calculation code at last.

The calculation code of the elements of discernment framework is the foundation of calculation code of evidence. The calculation code of the elements of discernment framework is fixed in free DSm model and Shafer model. The calculation code of the elements of discernment framework is called basic calculation code which stores in database. For example, the calculation code of θ_1 , θ_2 and θ_3 are basic calculation code in discernment framework n=3. The discernment framework can be obtained by original evidence then to convert into basic calculation code. If the focal element is the union set of discernment framework elements, calculation code is the disjunction of basic calculation code. If the focal element is the intersection set of discernment framework elements, calculation code is the conjunction of basic calculation code. The union set, intersection set and their mixed set are called group focal elements. θ_1 is expressed as [1 2 3 5], θ_2 is expressed as [1 2 4 6], $\theta_1 \cup \theta_2$ is expressed as [1 2 3 4 5 6], $\theta_1 \cap \theta_2$ is expressed as [1 2]. Only the generation of calculation code in free DSm model and Shafer model is studied in the paper. The generation and coding of calculation code in hybrid DSm model involves the restraint condition, which is our following research area.

Step2: evidence approximate. The evidence is approximated by evidence and focal element approximate method to reduce the focal elements and obtain approximate evidence.

The classics *k-l-x* method is chosen in this paper. Other methods can also be chosen for different application. There are *k* focal elements at least, l focal elements at most and minimum BBA 1-*x*, $x \in [0,1]$. The BBA of approximate evidence should be normalization.

Setp3: Evidence combination. The combination rule is chosen with the consideration of practical application. The classical is chosen in this article. Firstly, the intersection of focal elements is obtained. Then the BBA of intersections are calculated. Lastly, the intersections are removed the duplicates to get the combination result.

When the fusion model is free DSm model, two evidence are $m_1(.)$ and $m_2(.)$ the classical DSm combination rule is $m_{M^f(\theta)}(.) \equiv m(.) \triangleq [m_1 \oplus m_2](.)$: $C \in D^{\Theta}$

$$m_{M^f(\theta)}(C) \equiv m(C) = \sum_{\substack{A,B \in D^{\Theta} \\ A \cap B = C}} m_1(A) m_2(B) \tag{3}$$

Step4: evidence display. The combination result is decoded in this step to obtain character code to display for users.

All calculation code of elements stores in the database. But it will bring huge computation for the direct searching. So the position code is added corresponding to every calculation code. The position code is obtained before the application of the system. The every number of calculation codes does a decimal-to-binary conversion. The position code is obtained by the calculation code of combination result. The character code is achieved by the position code directly.

V. Comparison and Analysis

In order to show the advantage of computation measurement and optimization method, we compare and analyze three methods from 2 views, i.e. validity and computation. First method is classical DSm combination rule to combine evidence directly, which is called direct method. Second method approximates the evidence with *k-l-x* method and combines approximate evidence with classical DSm combination rule, which is called *k-l-x* method. Third method is computation measurement and optimization method which is presented in this paper.

Example 1. Let's consider discernment frame $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$ in free DSm model, and two evidence E_1 and E_2 obtained by sensor at one moment, the BBA of evidence is listed in TABLE II, the result is supposed to be element D in two evidence. In *k-l-x* method, *k*=3, *l*=5, *x*=0.95. $\alpha = 0.5$ in the measurement of BBA combination computation.

TABLE II. BBA AT CERTAIN MOMENT

Focal element	BBA	
	m_1	m_2
D	0.7	0.6
Z	0.1	0.05
Y	0.04	0.01
Q	0.005	0.006
D∪Z	0.005	0.1
D∪Y	0.01	0.02
D∪Q	0.02	0.04
Z∪Y	0.02	0.01
Y∪Q		0.08
D∪Y∪Z	0.05	0.02
D∪Y∪Q	0.02	0.01
Z∪Y∪Q	0.03	0.054

The preliminary approximate evidence and normalization approximate evidence in *k-l-x* method are shown in the TABLE

Ⅲ. The number of focal element is reduced obviously. The focal elements which do not much affect is filtering.

TABLE III. PRELIMINARY APPROXIMATE EVIDENCE AND NORMALIZATION APPROXIMATE EVIDENCE IN K-L-X METHOD

Focal element	Preliminary approximate evidence		Normalization approximate evidence	
	m_1	m_2	m_1	m_2
D	0.7	0.6	0.822	0.664
Z	0.1	0.05	0.12	0.055
D∪Z		0.1		0.111
Y∪Q		0.08		0.088
D∪Y∪Z	0.05	0.02	0.058	0.022
Z∪Y∪Q		0.054		0.060

The combination result and computation is compared and analyzed in TABLE Ⅳ and TABLE Ⅴ.

The combination results of three methods are shown in TABLE Ⅳ. The result of three methods support element D, which is the right decision. The *k-l-x* method and optimization method differ from the middle code. So the two methods have the same combination result to be 0.6935 which is higher 12.6% than direct method. In free DSm model, the approximate method deletes the focal element with small BBA to concentrate the BBA value, which remit the BBA decentralizing problem of DSm combination rule. So the optimization method is validity in evidence fusion.

TABLE IV. COMPARISON OF COMBINATION RESULT

Comparison of combination result	Direct method		*k-l-x* method		Optimization method	
Combination result	D	0.616	D	0.6935	D	0.6935
	D∩Z	0.095	D∩Z	0.125	D∩Z	0.125
	D∩Y	0.031	D∩(Y∪Q)	0.072	D∩(Y∪Q)	0.072
	(D∩Z)∪(D∩Y)	0.019	D∩(Z∪Y∪Q)	0.049	D∩(Z∪Y∪Q)	0.049
	(D∩Y)∪(D∩Q)	0.056	Z	0.0329	Z	0.0329
	(D∩Z)∪(D∩Y)∪(D∩Q)	0.0558	Z∩(Y∪Q)	0.011	Z∩(Y∪Q)	0.011
	Z	0.02865	D∪Z	0.0065	D∪Z	0.0065
	·		Q∩(D∪Y∪Z)	0.0087	Q∩(D∪Y∪Z)	0.0087
	· ·		D∪Y∪Z	0.0014	D∪Y∪Z	0.0014
Num. of focal elements	51		9		9	

TABLE V. COMPARISON OF COMPUTATION RESULT

Comparison of computation result	Search computation		Combination computation		Expression computation		Total computation
	Approximate search computation	*Combination search computation*	*Focal elements combination computation*	*BBA combination computation*	*Coding computation*	*Decoding computation*	
direct method	0	3/4×(11+1)×11 +3/4×(12+1)×12=216	artificial operation time $\mathbf{t_1}$	11×12+0.5×(11×12-1)=197.5	0	0	[197.5+$\mathbf{t_1}$, 216]
k-l-x method	3/4×(11+1)×3 +3/4×(12+1)×6=85.5	3/4×(3+1)×3+3/4×(6+1)×6=40.5	artificial operation time $\mathbf{t_2}$	3×6+0.5×(3×6-1)=26.5	0	0	[26.5+$\mathbf{t_2}$, 125.5]
optimization method	3/4×(11+1)×3 +3/4×(12+1)×6=85.5	0	0	3×6+0.5×(3×6-1) =26.5	14	9	[26.5, 108.5]

The computation of three methods is shown in TABLE Ⅳ. The direct method has the biggest computation, the *k-l-x* method takes the second place, and optimization method has the least computation. In the aspect of search computation, combination search computation of direct method reaches 216 for its large quantity of focal elements. The other two methods approximate original evidence to bring approximate search computation which is 85.5. The combination search computation of *k-l-x* method reduces to 40.5. The optimization method contains focal elements relationship to ignore the combination search. In the aspect of combination computation, the BBA combination computation of *k-l-x* method and optimization method is much smaller than that of direct method. For focal element combination computation, direct method and *k-l-x* method need artificial simplifying operation for the character code expression with artificial operation time t_1 (132 focal elements simplify to 51 focal elements) and artificial operation time t_2 (18 focal elements simplify to 9 focal elements). The optimization method does not have focal element combination computation for calculation code.

Overall, the direct method combines the original evidence forthright to have more focal elements and more computation. The artificial operation is needed in direct method to be difficult in application. The *k-l-x* method reduces the focal elements by approximate algorithm. But *k-l-x* method also obtains artificial operation to disturb its application. The optimization method presented in this paper structure the code for the relationship and combines the evidence via code pattern to reduce and abbreviate computation. Although expression computation is added, the total is smaller than other method to suitable for computer programming and realizes automatic fusion.

VI. CONCLUSIONS

The computation measurement is analyzed to value the computation issue of DSmT evidence fusion totally. A new computation optimization approach of DSmT is presented in this paper. The approach unites the existing method and computation code to reduce computation in multilevel. The proposed approach is suitable for the computer programming. There are three potential issues to research. Firstly, the evidence structure and evidence decision should be added to the information process in the following research. Secondly, the storage structure of code should be studied to facilitate the search, storing and modification of code. Thirdly, the generation and analysis of computation code in hybrid DSm model should be involved based on the different constraint conditions.

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