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A New Probabilistic Transformation Based on Evolutionary Algorithm for Decision Making

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Abstract—**The study of alternative probabilistic transformation (PT) in DS theory has emerged recently as an interesting topic, especially in decision making applications. These recent studies have mainly focused on investigating various schemes for assigning both the mass of compound focal elements to each singleton in order to obtain Bayesian belief function for real-world decision making problems. In this paper, work by us also takes inspiration from both Bayesian transformation camps, with a novel evolutionary-based probabilistic transformation (EPT) to select the qualified Bayesian belief function with the maximum value of probabilistic information content (PIC) benefiting from the global optimizing capabilities of evolutionary algorithms. Verification of EPT is carried out by testing it on a set of numerical examples on 4D frames. On each problem instance, comparisons are made between the novel method and those existing approaches, which illustrate the superiority of the proposed method in this paper. Moreover, a simple constraint-handling strategy with EPT is proposed to tackle target type tracking (TTT) problem, simulation results of the constrained EPT on TTT problem prove the rationality of this modification.**

Keywords: Evidence Reasoning, Probabilistic Transformation, Evolutionary Algorithm, Target Type Tracking problems, Decision Making.

I. Introduction

Since the pioneering work of Dempster and Shafer [1], [2], which is known as Dempster-Shafer evidence theory (DST), in the late 70's regarding the possibility of distinguishing "unknown" and "imprecision" and fusing different evidences based on associative and commutative Dempster's combination rule, this new area of research (now known as evidence reasoning) has grown considerably as indicated by the notable increment of technical papers in peer-reviewed journals, conference and special sessions. However, the computational complexity of reasoning with DST is one of the major points of criticism this formalism has to face.

To overcome this difficulty, various approximating methods have been suggested that aim at reducing the number of focal elements in the frame of discernment (FoD) so as to maintain the tractability of computation computation. One common strategy is to simplify FoD by removing and/or aggregating focal elements for approximating original belief funcion [3]. Among these methods, probability transformations (PTs) seem particularly desirable for reducing such computation complexity by means of assigning the mass of non-singleton elements to each singleton [4], [5]. The research on this probabilistic measure has received a lot of attentions and accordingly many efficient PTs have been pointed out by scholars in recent years. In them, a classical transformation, denoted as BetP [4], which offers a good compromise between the maximum of credibility (Bel) and the maximum of plausibility (Pl) for decision making. Unfortunately, BetP does not provide the highest probabilistic information content (PIC) [7]; Sudano [8] also proposed series of alternatives and principles of these similar to BetP, which were called PrPl, PrBel and PrHyb; CuzzP [9], which was proposed by Cuzzolin in the framework of DST in 2009, showed its ability of probabilistic transformation; Another novel transformation was proposed by Dezert and Smarandache in the framework of DSmT (free DSm model, hybrid DSm model or Shafer's model), which was called DSmP [7] and comprehensive comparisons have been made in [7] to prove the capabilities of DSmP in probabilistic transformation.

However, most mentioned aforementioned PTs have been always concentrated mainly on two crucial issues: (1) How to implement this operation (or assignment)? (2) How to evaluate the quality of this transformation? In this paper, we suggest a novel PT method based on evolutionary algorithm, namely, evolutionary-based probabilistic transformation (EPT), which alleviates the above two difficulties together based on optimization using a reasonable criteria. A similar idea was proposed by Han et.al [10] and the difference lies in the optimization approaches and objective functions. In the EPT method, the global search replaces the assigning operator used in the classical PTs and the evaluation criteria is embedded into EPT to provide important guidance for the searching procedure. Specifically, the mass of the singletons are randomly generated in evolutionary-based framework, which need to satisfy the constraints of probability distributions in evidence reasoning. Also, a selection operator is presented to assess the best individual in all populations by a special objective function (desirable evaluation criteria). Referring to the previous works [7], the PIC is used in this paper to select the best[1] solution as an objective function in EPT. Simulation results on 4D frames test cases show that the proposed EPT, in these problems, is able to outperform other PTs that pay special attention to some ratio created from the available information

[1]based on the highest PIC value.

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(i.e. Bel or Pl). Moreover, we suggest a simple constraint-handling strategy with EPT that suits well for two target type tracking (TTT) problems. These first appealing results of EPT method encourage its use for more complex and real-world decision making problems.

The rest of this paper is organized as follows. In Section II we briefly summarize the basis of DST and several classical PT formulas. A novel EPT approach is presented in details in Section III. In Section IV several cases and comprehensive comparisons borrowed from previous papers are carried out to demonstrate the superiority of proposed method. Also, target type tracking problem and the pertinent analysis of EPT in TTT are described in detail in this section. Moreover, the limitation of EPT are also discussed in Section. V. The conclusion is drawn in Section. VI.

II. Basis of belief functions

In this section, we introduce the belief functions terminology of DST and the notations used in the sequel of this paper.

A. DST basis

In DST [2], the elements θ_i ($i = 1, \ldots, N$) of the frame of discernment (FoD) $\Theta \triangleq \{\theta_1, \ldots, \theta_N\}$ must be mutually exhaustive and exclusive. The power set of the FoD is denoted 2^Θ and a basic belief assignment (BBA), also called a mass function, is defined by the mapping: $2^\Theta \rightarrow [0,1]$, which satisfies $m(\emptyset) = 0$ and

$$\sum_{A \subseteq 2^\Theta} m(A) = 1 \tag{1}$$

where $m(A)$ is defined as the BBA of A. The element A is called a focal element of $m(.)$ if $m(A) > 0$. The belief and plausibility functions, which are in one-to-one mapping with the BBA $m(.)$, are defined for all $A \subseteq \Theta$ by

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{2}$$

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{A \cap B \neq \emptyset} m(B), \forall A \subseteq \Theta \tag{3}$$

where $\bar{A} \triangleq \Theta \setminus A$ is the complement of A in Θ. The belief interval $[Bel(A), Pl(A)]$ represents the uncertainty committed to A and the bounds of this interval are usually interpreted as lower and upper bounds of the unknown (possibly subjective) probability of A. This interval plays an important role in the implementation of EPT as shown in details in Section III.

B. DSmT basis

In the framework of Dezert-Smarandache Theory (DSmT) [5], the FoD Θ is considered as a finite set of N exhaustive elements only (without the requirement of exclusivity of the elements). The BBA $m(.)$ is then defined on the hyper-power set of the FoD (i.e. the free Dedekind's lattice D^Θ), taking eventually into account some integrity constraints (if any). The main differences between DST and DSmT frameworks are: (1) the model on which one works with, and (2) the combination rule. In the sequel, we will work with BBA defined only on the classical power-set for simplicity. Instead of distributing equally total conflicting mass onto elements of 2^Θ as within Dempster's rule through the normalization step, or transferring the partial conflicts onto partial uncertainties as within DSmH rule [4], we use the Proportional Conflict Redistribution rules (PCRs) [5] based on the transfer of conflicting masses (total or partial) proportionally to non-empty sets involved in the model according to all integrity constraints. In DSmT, the most effective rule is the PCR6 rule which is defined[2] for the fusion of two BBA's $m_1(.)$ and $m_2(.)$ as $m_{PCR6}(\emptyset) = 0$ and $\forall A \in 2^\Theta \setminus \{\emptyset\}$

$$m_{PCR6}(A) = m_{12}(A) + \sum_{B \in 2^\Theta \setminus \{A\} | A \cap B = \emptyset} [\frac{m_1(A)^2 m_2(B)}{m_1(A) + m_2(B)} + \frac{m_2(A)^2 m_1(B)}{m_2(A) + m_1(B)}] \tag{4}$$

where $m_{12}(A)$ is the conjunctive operator, and each element A and B are expressed in their disjunctive normal form.

C. Classical Probabilistic Transformations

The efficiency of probabilistic transformation (PT) in the field of decision making has been analyzed in deep by Smets [4]. Various PTs have been proposed in the open literature and the main transformations are briefly recalled in this section.

1) BetP: Smets in [4], [6] first proposed pignistic probability to make decision which aims to transfer the mass of belief of each non-specific element onto the singletons. The classical pignistic probability is defined as $BetP(\emptyset) = 0$, and $\forall A \in 2^\Theta \setminus \{\emptyset\}$:

$$BetP(\theta_i) \triangleq \sum_{A \subseteq 2^\Theta, A \neq \emptyset} \frac{|\theta_i \cap A|}{|A|} \frac{m(A)}{1 - m(\emptyset)} \tag{5}$$

Because in Shafer's framework $m(\emptyset) = 0$, the formula (5) can simply be rewritten for any singleton $\theta_i \in \Theta$ as

$$\begin{aligned} BetP(\theta_i) &= \sum_{B \in 2^\Theta, \theta_i \subseteq B} \frac{1}{|B|} m(B) \\ &= m(\theta_i) + \sum_{B \in 2^\Theta, \theta_i \subset B} \frac{1}{|B|} m(B) \end{aligned} \tag{6}$$

2) CuzzP: An intersection probability denoted as CuzzP [9] was proposed using the proportional repartition of the total non-specific mass (total non-specific mass ($TNSM$) $= \sum_{A \in 2^\Theta, |A|} m(A)$) for each contribution of the non-specific masses involved. CuzzP is defined by $CuzzP(\emptyset) = 0$, and for any singleton $\theta_i \in \Theta$ by

$$CuzzP(\theta_i) \triangleq m(\theta_i) + \frac{Pl(\theta_i) - m(\theta_i)}{\sum_j (Pl(\theta_i) - m(\theta_j))} \cdot TNSM \tag{7}$$

[2]PCR6 rule coincides with PCR5 rule when combining only two BBA's [5].

3) DSmP: In 2008, Dezert and Smarandache [7] have proposed a new generalized pignistic transformation defined by $DSmP_\varepsilon(\emptyset) = 0$ and for any singleton $\theta_i \in \Theta$ by

$$DSmP_\varepsilon(\theta_i) \triangleq m(\theta_i) + (m(\theta_i) + \varepsilon) \times \left\{ \sum_{A \in 2^\Theta, \theta_N \subset A, |A| \geq 2} \frac{m(A)}{\sum_{B \in 2^\Theta, B \subset A, |B|=1} m(B) + \varepsilon \cdot |A|} \right\} \quad (8)$$

As shown in [7], DSmP makes a remarkable improvement compared with BetP, and CuzzP, since a more judicious redistribution of the ignorance masses to the singletons have been adopted by DSmP.

4) PrBP1 and PrBP2: Two novel pignistic probabilistic transformations were proposed by Pan in [11], which assume that the BBA is proportional to the product of $Bel(\theta_i)$ and $Pl(\theta_i)$ among each singleton element θ_i of $A \subseteq \Theta$.

$$PrBP1(\theta_i) = \sum_{\theta_i \subseteq A} \left(\frac{Bel(\theta_i)Pl(\theta_i)}{\sum_{\theta_j \subseteq A} Bel(\theta_j)Pl(\theta_j)} \right) \cdot m(A) \quad (9)$$

Also, Pan et.al. assume that the masses are distributed proportionally to some given parameters $s_i = Bel(\theta_i)/(1 - Pl(\theta_i))$ or $s_i = Pl(\theta_i)/(1 - Bel(\theta_i))$:

$$PrBP2(\theta_i) = \sum_{A, \theta_i \subseteq A} \left(\frac{s_i}{\sum_{j, \theta_j \subseteq A} s_j} \right) \cdot m(A) \quad (10)$$

As we can see, a Bayesian mass function which has only singleton focal elements can be obtained by any of these PTs.

D. Probabilistic Information Content (PIC)

The PIC criterion [12] is classically adopted to evaluate the performances of a probabilistic transformation of a BBA. If $m(.)$ is a Bayesian BBA defined on a discrete finite FoD $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$, its PIC value is defined as[3]

$$PIC(m) \triangleq 1 + \frac{1}{\log_2 N} \sum_{i=1}^{N} m(\theta_i) \log_2 m(\theta_i) \quad (11)$$

The PIC metric actually measures the information content of a (probabilistic) source characterized by a Bayesian BBA $m(.)$, which the value of this metric always belong to $[0; 1]$. It corresponds to the (normalized) dual of Shannon's entropy measure. When the Bayesian BBA is uniform over the FoD Θ, one has $m(\theta_i) = 1/N$ for $i = 1, 2, \ldots, N$ and the PIC metric is minimum, i.e. $PIC(m) = PIC_{\min} = 0$. The PIC metric is maximum, i.e. $PIC(m) = PIC_{\max} = 1$ if the Bayesian BBA $m(.)$ is deterministic, that is if there exists an element θ_i of Θ such that $m(\theta_i) = 1$. While simple, appealing and generally adopted by the community, the PIC criteria is however not always sufficient to evaluate the efficiency of a PT as discussed in [14]. This point will be addressed in Section V.

[3] where $0 \log_2(0) = 0$ by convention.

III. EVOLUTIONARY-BASED PROBABILISTIC TRANSFORMATION (EPT)

The idea to approximate any BBA into a Bayesian BBA (i.e. a subjective probability measure) using the minimization of the Shannon entropy under compatibility constraints has been proposed recently by Han et al. in [10], [14] using "on-the-shelf" optimization techniques. In this paper, we present in details a new optimization method to achieve this PT based on a random evolutionary algorithm to acquire maximization of the PIC value. That is why we call it the Evolutionary-based Probabilistic Transformation (EPT) method.

Let's assume that the FoD of the original BBA $m(.)$ to approximate by a Bayesian BBA is $\Theta \triangleq \{\theta_1, \theta_2, \ldots, \theta_N\}$. The EPT method consists of the following steps:

- Step 0 (setting parameters): $t_{\max}$ is the max number of iterations; $n_{\max}$ is the population size in each iteration; P_c is the crossover probability, and P_m is the mutation probability.
- Step 1 (population generation): A set $\mathbf{P}_t$ of $j = 1, 2, \ldots, n_{\max}$ random probability values $P_t^j = \{P^j(\theta_1), \ldots, P^j(\theta_N)\}$ is generated such that the constraints (12)–(14) for $j = 1, 2, \ldots, n_{\max}$ are satisfied in order to make each random set of probabilities P_t^j compatible with the original BBA $m(.)$

$$P^j(\theta_i) \in [0; 1], \quad i = 1, 2, \ldots, N \quad (12)$$

$$\sum_{i=1}^{N} P^j(\theta_i) = 1 \quad (13)$$

$$Bel(\theta_i) \leq P^j(\theta_i) \leq Pl(\theta_i), \quad i = 1, 2, \ldots, N \quad (14)$$

- Step 2 (fitness assignment): To each probability set P_t^j, ($j = 1, 2, \ldots, n_{\max}$), we compute its PIC value and use it as its fitness factor F. More precisely, one takes $F(P_t^j) = PIC(P_t^j)$.
- Step 3 (best approximation of $m(.)$): the best set of probability $P_t^{j_{\text{best}}}$ with highest PIC value is sought, and its associated index j_{best} are stored respectively in "Best-Individual" and "Index-of-BestIndividual".
- Step 4 (selection, crossover and mutation): The tournament selection, crossover and mutation operators drawn from evolutionary theory framework [13] are implemented to create the associated offspring population $\mathbf{P}'_t$ based on the parent population $\mathbf{P}_t$. If $F(P_t^{j_{\text{best}}}) \geq F(P_t'^{j_{\text{best}}})$, then the "Best-Individual" remains unchanged; otherwise, Best-Individual $= P_t'^{j_{\text{best}}}$.
- Step 5 (Stopping EPT): The steps 1–4 illustrate the t-th iteration of EPT method. If $t \geq t_{\max}$ then EPT method is completed, otherwise another iteration must be done by taking $t + 1 = t$ and going back to step 1.

The scheme of EPT method is shown in Fig.1 and its pseudo-code is given in Algorithm 1.

IV. SIMULATION RESULTS

In this section we compare EPT's results to other mentioned PTs. In particular, we show the important gain of PIC we

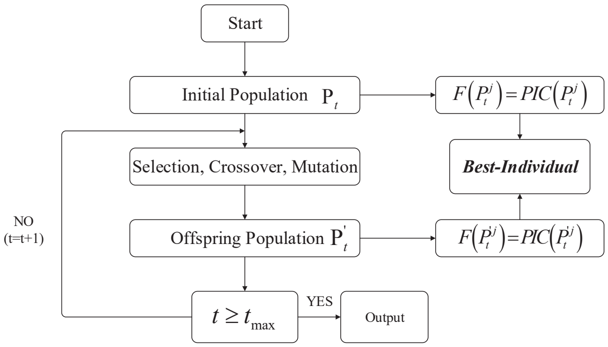

Figure 1: Scheme of EPT algorithm.

Algorithm 1 Evolutionary-Based PT (EPT)

1: Define Stopping Criteria, ($t \leq t_{\max}$); population Size $n_{\max}$ for each iteration; crossover probability P_c, and mutation probability P_m.
2: Generate an initial random population $\mathbf{P}_t$ of consistent probabilities P_t^j with $m(.)$.
3: For each individual P_t^j in $\mathbf{P}_t$ do
4: Calculate Fitness $F(P_t^j) = PIC(P_t^j)$ of P_t^j
5: Store the best individual $P_t^{j_{\text{best}}}$
6: End
7: Repeat:
8: Selection: Select 2 individuals based on fitness
9: Crossover: exchange parts of 2 individuals with probability P_c
10: Mutation: mutate the child individuals with probability P_m
11: After these three sub-steps, the updated population $\mathbf{P}'_t$ is obtained
12: Calculate the fitness of individuals of $\mathbf{P}'_t$, and store the best individual $P_t^{\prime j_{best}}$
13: If $F(P_t^{j_{\text{best}}}) \geq F(P_t^{\prime j_{\text{best}}})$
14: Best-Individual remains unchanged
15: else
16: Best-Individual $= P_t^{\prime j_{\text{best}}}$
17: If $t \geq t_{\max}$ then stops, otherwise $t+1 \rightarrow t$ and go back to line 7

can obtain, and the capability of EPT to improve target type tracking.

A. Examples and comparisons

In order to compare different PTs with EPT, two cases borrowed from [11] and [12] are considered, where PIC is used for evaluation. In all the following cases, the parameters $t_{\max}$, $n_{\max}$, P_c and P_m necessary to EPT method have been set to $t_{\max} = 50$, $n_{\max} = 1000$, $P_c = 0.9$ and $P_m = 0.1$ respectively.

Example 1: $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$

The BBA $m(.)$ to approximate by a Bayesian BBA (probability measure) is

$$
\begin{aligned}
&m(\theta_1) = 0.16,\ m(\theta_2) = 0.14,\ m(\theta_3) = 0.01,\ m(\theta_4) = 0.02\\
&m(\theta_1 \cup \theta_2) = 0.20,\ m(\theta_1 \cup \theta_3) = 0.09,\ m(\theta_1 \cup \theta_4) = 0.04\\
&m(\theta_2 \cup \theta_3) = 0.04,\ m(\theta_2 \cup \theta_4) = 0.02,\ m(\theta_3 \cup \theta_4) = 0.01\\
&m(\theta_1 \cup \theta_2 \cup \theta_3) = 0.10,\ m(\theta_1 \cup \theta_2 \cup \theta_4) = 0.03\\
&m(\theta_1 \cup \theta_3 \cup \theta_4) = 0.03,\ m(\theta_2 \cup \theta_3 \cup \theta_4) = 0.03\\
&m(\Theta) = 0.08
\end{aligned}
$$

The Bayesian BBA obtained by classical PT (5)–(10) and EPT with their corresponding PIC values calculated by (11) are given in Table I. As expected, the EPT provides the maximum PIC .

Table I: Results of Different PTs in Example 1.

	θ_1	θ_2	θ_3	θ_4	PIC
$CuzzP$	0.3860	0.3382	0.1607	0.1151	0.0790
$BetP$	0.3983	0.3433	0.1533	0.1051	0.0926
$DSmP_0$	0.5176	0.4051	0.0303	0.0470	0.3100
$DSmP_{0.001}$	0.5162	0.4043	0.0319	0.0476	0.3058
$PrBP1$	0.5419	0.3998	0.0243	0.0340	0.3480
$PrBP2$	0.5578	0.3842	0.0226	0.0354	0.3529
EPT	**0.7246**	**0.2218**	**0.0266**	**0.0270**	**0.4508**

Example 2: $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$

In this case, we randomly generate BBAs and compare EPT with classical PTs (CuzzP, BetP, DSmP, PrBP1 and PrBP2 given by (5)–(10)). The original BBAs to approximate are generated according to Algorithm 2 of [15].

Algorithm 2 Random generation of BBA

Input: Frame of Discernment $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$
N_{max} :Maximum number of focal element
Output : BBA-m
Generate $K(\Theta)$, which is the power set of Θ
Generate a random permutation of $K(\Theta) \rightarrow R(\Theta)$
Generate an integer between 1 and $N_{max} \rightarrow l$
For each First k elements of $R(\Theta)$ do
Generate a value within $[0, 1] \rightarrow m_i, i = 1, \cdots, l$
End
Normalize the vector $m = [m_1, m_2, \cdots, m_l] \rightarrow m'$
$m(\theta_i) = m'_i$

In our test, we have set the cardinality of the FoD to 4 and fixed the number of focal elements to $l = N_{max} = 15$. We randomly generate $L = 30$ BBA's. Six PT methods are tested and PIC is used to evaluate the quality of their corresponding results are shown in Fig.2. As we can see, EPT outperforms significantly other methods based on maximum of PIC criterion, which is normal.

B. Evaluation of EPT in Target Type Tracking problem

Target Type Tracking (TTT) problem can be briefly described below [16]:

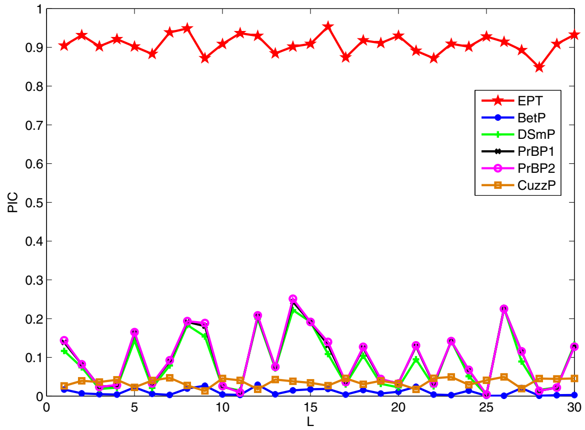

Figure 2: PIC values obtained with EPTs and classical PTs.

1) Target Type Tracking Problem (TTT):

1. Considering $\zeta = 1, 2, \cdots, \zeta_{max}$ be the time index and let N possible target types $Tar_{\zeta} \in \Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$ in the surveillance area; For instance, in the normal air target surveillance systems the FoD could be $\Theta = \{Fighter, Cargo\}$. That is, $Tar_1 = \theta_1 \triangleq Fighter$, $Tar_2 = \theta_2 \triangleq Cargo$. Similarly, the FoD in a ground target surveillance systems could be $\Theta_{ground} = \{Tank, Truck, Car, Bus\}$. In this paper, we just consider the air target surveillance systems to prove the practicability of EPT.
2. At every time ζ, the true type of the target $Tar(\zeta) \in \Theta$ is immediately observed by an attribute-sensor (here, we assume a possible target probability).
3. A defined classifier is applied to process the attribute measurement of the sensor which provides the probability $Tar_d(\zeta)$ on the type of the observed target at each instant ζ.
4. The sensor is in general not totally reliable and is characterized by a $N \times N$ confusion matrix:

$$\mathbf{M} = [M_{ij} = P(Tar_d = Tar_j | TrueType = Tar_i)] \tag{15}$$

where $0 \le i \le N; 0 \le j \le N$.

Here, we briefly summarize the main steps of TTT using EPT.

1. Initialization. Determine the target type frame $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$ and set the initial BBA $m^{initial}(\theta_1 \cup \theta_2 \cup \cdots \cup \theta_N) = 1$ since there is no information about the first target type that will be observed;
2. Updating BBA. An observed BBA $m_{obs}(.)$ on types of unknown observed target is defined from current target type declaration and confusion matrix $\mathbf{M}$;
3. Combination. We combine the current BBA $m_{obs}(\cdot)$ with initial BBA $m^{initial}(\cdot)$ according to PCR6 combination rule: $m_{PCR6}(\cdot) = m_{obs}(\cdot) \oplus m^{initial}(\cdot)$;
4. Approximation. Using $EPT(\cdot)$ to approximate $m_{PCR6}(\cdot)$ into a Bayesian BBA;
5. Decision Making. Taking a final decision about the type of the target at current observation time based on the obtained Bayesian BBA;
6. Updating BBA. Set $m^{initial}(\cdot) = m_{PCR6}(\cdot)$, and increase time index $\zeta = \zeta + 1$ and go back to step 2.

2) Raw Dataset of TTT: We have tested our EPT-based TTT on a very simple scenario for a 2D TTT, namely $\Theta = \{Fighter, Cargo\}$ for two types of classifiers. The matrix $\mathbf{M}_1$ corresponds to the confusion matrix of the good classifier, and $\mathbf{M}_2$ corresponds to the confusion matrix of the poor classifier.

$$\mathbf{M}_1 = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}; \mathbf{M}_2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix} \tag{16}$$

In our scenario, a true Target Type sequence over 120 scans is generated according to Fig. 3. We can observe clearly from Fig. 3 that Cargo (which is denoted as Type 2) appears at first in the sequence, and then the observation of the Target Type switches three times onto Fighter Type (Type 1) during different time duration (namely, 20s, 10s, 5s).

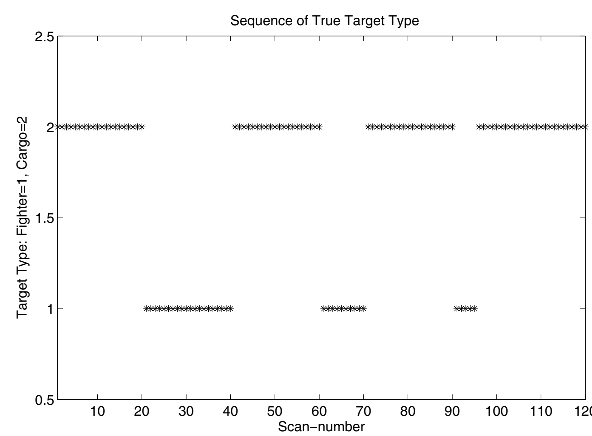

Figure 3: Raw Sequence of True Target Type.

A pathological case for TTT: Our analysis has shown that EPT can nevertheless be in troubles for tracking two target types as proved in this simple particular example (when $0 \le m(\theta_1 \cup \theta_2) \le 0.1$). Let's consider the following BBA

$$m_{target}(.) = [\theta_1, \theta_2, \theta_1 \cup \theta_2] = [0, 1, 0]$$

According to the compatibility constraints (12)–(14), the population $\mathbf{P}'_t$ is obtained from $\mathbf{P}_t$ through a selection procedure. Next, individual P'^j_t in $\mathbf{P}'_t$ which is denoted as $P'^j_t = [m'(\theta_1), m'(\theta_2)]$ is subject to initial constraint (1) and (17):

$$\begin{aligned} m'(\theta_1) &\ge (Bel(\theta_1) = m(\theta_1) = 0) \\ m'(\theta_1) &\le (Pl(\theta_1) = m(\theta_1) + m(\theta_1 \cup \theta_2) = 0 + 0 = 0); \\ m'(\theta_2) &\ge (Bel(\theta_2) = m(\theta_2) = 1) \\ m'(\theta_2) &\le (Pl(\theta_2) = m(\theta_2) + m(\theta_2 \cup \theta_1) = 1 + 0 = 1); \end{aligned} \tag{17}$$

From the above inequalities, one sees that only one probability measure $P_t^S = [m(\theta_1), m(\theta_2)] = [0, 1]$ (where the superscript index S means *Single*) satisfies this constraint. However because of mechanism of EPT and real-coded generic algorithm (RCGA), the probabilities P_t^j in population $\mathbf{P}_t$ which are randomly generated in the interval $[0, 1]$, will have a very little chance to be equal to the suitable measure $[0, 1]$ satisfying the constraints. That is why EPT becomes inefficient in this case which occurs with a probability of $1/n_{\max}$, where $n_{\max}$ denotes the size of population[4] $\mathbf{P}_t$. Unfortunately, in TTT decision making problems, such case cannot be avoided because it can really happens.

To circumvent this problem and make EPT approach working in all circumstances, we need to modify a bit the EPT method to generate enough individuals for making selection step efficiently when the bounds of belief interval $[Bel, Pl]$ take their min and max values ($[0.9, 0.05, 0.05]$, $[0.05, 0.9, 0.05]$). For achieving this, we propose to enlarge the interval through a parameter λ, and maintain the property of original interval in some degree at the same time. More precisely, the modified belief interval, denoted $[Bel', Pl']$, is heuristically computed by a simple thresholding technique as follows:

First, we assume that the original BBA we consider here for FoD $\Theta = \{\theta_1, \theta_2\}$ is $[\theta_1, \theta_2, \theta_1 \cup \theta_2] = [a, b, c], (a + b + c) = 1; 0 \leq c \leq 0.1)$

Step 1: Let $m'(\theta_1 \cup \theta_2) = c + \lambda$;

Step 2: if $a > b$

$$m'(\theta_1) = a - \lambda; m'(\theta_2) = b; m'(\theta_1 \cup \theta_2) = c + \lambda; \tag{18}$$

Step 3: if $a \leq b$

$$m'(\theta_1) = a; m'(\theta_2) = b - \lambda; m'(\theta_1 \cup \theta_2) = c + \lambda; \tag{19}$$

So the value of $[Bel'(\theta_1), Pl'(\theta_1)]$ and $[Bel'(\theta_2), Pl'(\theta_2)]$ can be calculated based on Eq.(18),Eq.(19), which are presented as follows:

When $a > b$:

$$\begin{cases} Pl'(\theta_1) = m(\theta_1) + m'(\theta_1 \cup \theta_2) = a - \lambda + c + \lambda = a + c; \\ Bel'(\theta_1) = 1 - Pl'(\bar{\theta}_1) = 1 - (b + c + \lambda) = a - \lambda. \end{cases} \tag{20}$$

$$\begin{cases} Pl'(\theta_2) = m(\theta_2) + m'(\theta_1 \cup \theta_2) = b + c + \lambda = b + c + \lambda; \\ Bel'(\theta_2) = 1 - Pl'(\bar{\theta}_2) \\ \qquad = 1 - (a - \lambda + c + \lambda) = 1 - (a + c) = b. \end{cases} \tag{21}$$

When $a \leq b$:

$$\begin{cases} Pl'(\theta_1) = m(\theta_1) + m'(\theta_1 \cup \theta_2) = a + c + \lambda; \\ Bel'(\theta_1) = 1 - Pl'(\bar{\theta}_1) \\ \qquad = 1 - (b - \lambda + c + \lambda) = 1 - (b + c) = a. \end{cases} \tag{22}$$

[4]In our simulation, we did take $n_{\max} = 1000$.

$$\begin{cases} Pl'(\theta_2) = m(\theta_2) + m'(\theta_1 \cup \theta_2) = b - \lambda + c + \lambda = b + c; \\ Bel'(\theta_2) = 1 - Pl'(\bar{\theta}_2) = 1 - (a + c + \lambda) = b - \lambda. \end{cases} \tag{23}$$

Explanation: Through step 1, one computes the total singleton mass one has in the entire BBA and the threshold value 0.9 allows to evaluate if the percentage of singleton mass is big enough or not. Here, we not only consider the unique extreme case $m_{target}(\cdot) = [\theta_1, \theta_2, \theta_1 \cup \theta_2] = [0, 1, 0]$, but also other possible cases such as $m_{target}(\cdot) = [\theta_1, \theta_2, \theta_1 \cup \theta_2] = [0.0001, 0.9998, 0.0001]$. Why do we consider the concept of percentage? Actually, the higher percentage of singleton mass, the smaller interval for P_t^j, in other words, the higher value of $m(\theta_1 \cup \theta_2)$, the bigger interval for P_t^j which can be shown in Eq.(17); The step 2 and step 3 give the way of calculating the updated upper bound of belief interval $[Bel', Pl']$ and Eq.(20)–Eq.(23) prove that the parameter λ determines the range of the interval; Next, we give two examples to show how the above method works:

The pathological case 1 for TTT (using modified EPT)

$$m_{target}(.) = [\theta_1, \theta_2, \theta_1 \cup \theta_2] = [0.0001, 0.9998, 0.0001] .$$

Here, the parameter λ is arbitrarily[5] set to 0.4. Then one computes in step 2 the modified plausibility bounds $Bel'(\theta_1) = 0.0001$, $Pl'(\theta_1) = 0.0001 + 0.0001 + \lambda = 0.4002$ and $Bel'(\theta_2) = 0.9998 - 0.4 = 0.5998$, $Pl'(\theta_2) = 0.9999$. So we get $[Bel'(\theta_1), Pl'(\theta_1)] = [0.0001, 0.4002]$ and $[Bel'(\theta_2), Pl'(\theta_2)] = [0.5998, 0.9999]$.

Consequently, any Bayesian BBA $P_t^j = [m'(\theta_1), m'(\theta_2)]$ must be generated according the (modified) compatibility constraints

$$m'(\theta_1) \in [Bel'(\theta_1), Pl'(\theta_1)] = [0.0001, 0.4002]$$
$$m'(\theta_2) \in [Bel'(\theta_2), Pl'(\theta_2)] = [0.5998, 0.9999]$$

The pathological case 2 for TTT (using modified EPT)

$$m_{target}(.) = [\theta_1, \theta_2, \theta_1 \cup \theta_2] = [0.45, 0.48, 0.07] .$$

Here, the parameter λ is set to 0.2. Then any Bayesian BBA $P_t^j = [m'(\theta_1), m'(\theta_2)]$ must be generated according the (modified) compatibility constraints

$$m'(\theta_1) \in [Bel'(\theta_1), Pl'(\theta_1)] = [0.45, 0.72]$$
$$m'(\theta_2) \in [Bel'(\theta_2), Pl'(\theta_2)] = [0.28, 0.55]$$

In order to evaluate the influence of the parameter λ, we have reexamined all the pathological cases based on the following procedure:

1) The value of parameter λ is taken to five possible values: 0, 0.1, 0.2, 0.3, 0.4, 0.5;
2) We randomly generate initial population $\mathbf{P}_t$ based on λ, which is also subjected to the constraints (12)–(14).

[5]The value of the parameter λ can be chosen to any value in $[0, 1]$ by the designer for his/her own reason to ensure the alternative interval effectively in modified EPT version.

With this simulation, we can observe in Fig.4 the impact of λ value on the number of P_t^j in $\mathbf{P}_t$. When $\lambda = 0$ happens [6], there exists no suitable P_t^j for case one which demonstrates the necessity to circumvent the pathological case problem. Obviously, the number of P_t^j increases with the increase of λ value, which efficiently proves the advantage of using the modified EPT approach to make selection step of the evolutionary algorithm more efficient. One point we need to clarify is that the intervals i.e. $[Bel'(\theta_1), Pl'(\theta_1)]$, $[Bel'(\theta_2), Pl'(\theta_2)]$ induced from parameter λ above aims at guaranteeing enough number of P_t^j in $\mathbf{P}_t$ in the implementation of EPT.

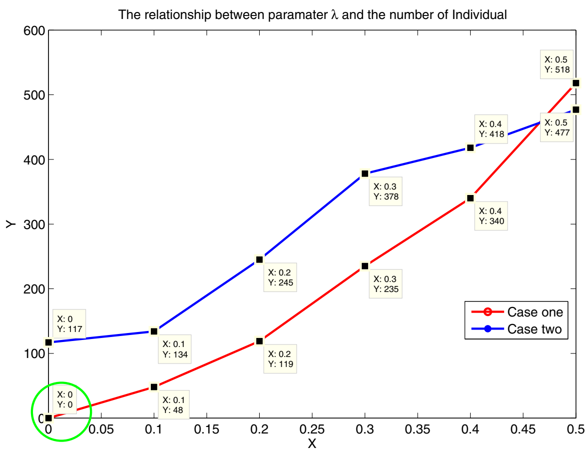

Figure 4: Impact of λ (x-axis) on individuals in $\mathbf{P}_t$ (y-axis).

3) Simulation Results of TTT Based on Modified EPT: Our simulation consists in 100 Monte-Carlo runs and we show in the sequel the averaged performances of EPT and DSmP. The figures 5–8 illustrate the Bayesian BBA's obtained by DSmP [7] -(part a) and our new EPT method-(part b) based on TTT using PCR6 fusion rule. One sees that regardless of the good classifier $\mathbf{M}_1$ and poor classifier $\mathbf{M}_2$, EPT is able to track properly the quick changes of target type.

V. LIMITATION OF EPT

As pointed out by Han et al. in [14], in general it is not enough, nor comprehensive to evaluate the quality of probabilistic transformation of a BBA from only the PIC criterion, even if the chosen PT provides highest PIC value by optimization. Our EPT approach, is not exempt of this problem of course as we can see in the simple example below, where no optimization technique provides useful (robust) solution.

Let's consider the FoD $\Theta = \{\theta_1, \theta_2\}$ with the BBA to approximate chosen as follows:

$$m(\theta_1) = 0.10001,\ m(\theta_2) = 0.10000,\ m(\theta_1 \cup \theta_2) = 0.79999$$

Based on PIC value optimization using EPT (or any other efficient optimization techniques), we will obtain the Bayesian

[6] which actually the original EPT is applied

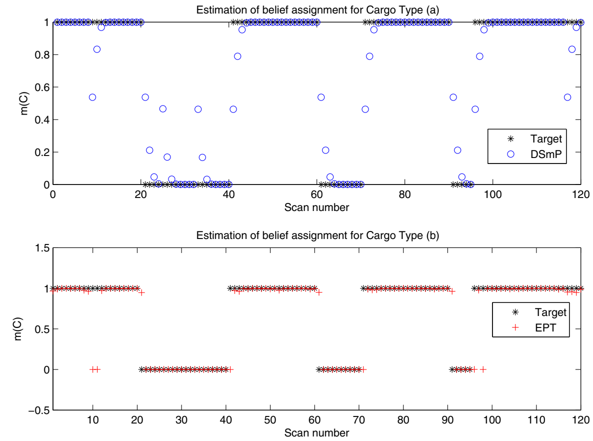

Figure 5: Belief Mass for Cargo Type for $\mathbf{M}_1$.

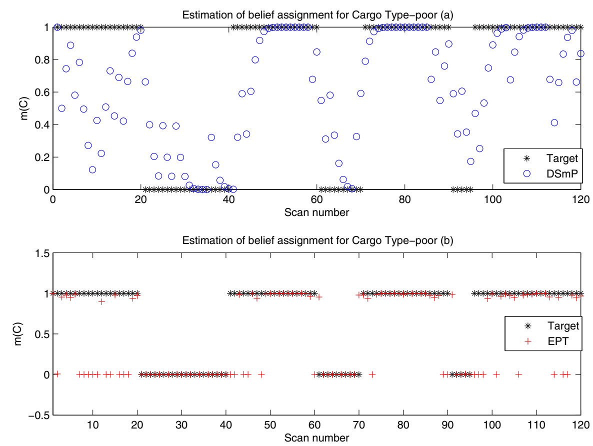

Figure 6: Belief Mass for Cargo Type for $\mathbf{M}_2$.

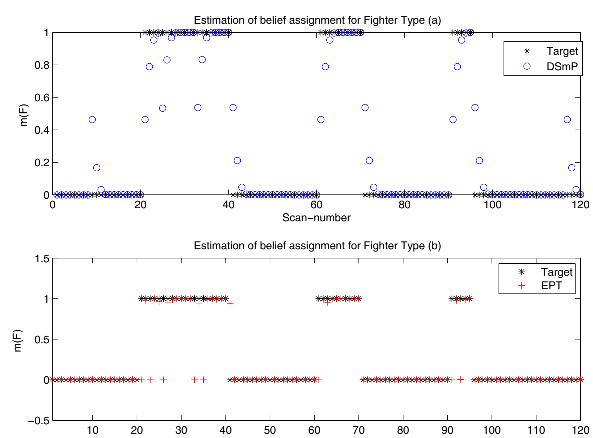

Figure 7: Belief Mass for Fighter Type for $\mathbf{M}_1$.

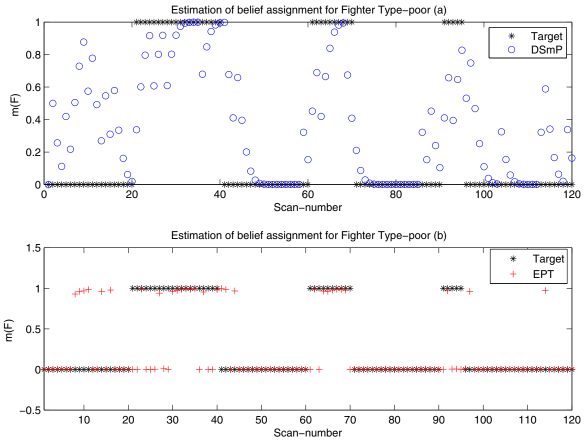

Figure 8: Belief Mass for Fighter Type for $\mathbf{M}_2$.

BBA $m(\theta_1, \theta_2) = [0.0001605, 0.9998394]$ with $PIC = 0.9977$. This simple example shows that in the original BBA $m(\theta_1)$ is almost the same as $m(\theta_2)$ and there is no solid reason to get a very high probability for θ_2 and a small one for θ_1 in the Bayesian BBA, even if a highest PIC is reached. Exaggerated high PIC is not always preferred (unreasonable or directly make wrong decisions), which can be seen in Fig.6 and Fig.8, although the PIC should be as high as possible for decision making problems. Therefore, a reasonable compromise must be found between PIC level and also fidelity level of the transformations to the original BBA, which is a theoretical open challenging problem left for further research works.

VI. Conclusion

An evolutionary algorithm for probabilistic transformation (EPT) has been proposed in this paper. It uses the genetic algorithm to obtain Bayesian belief function with highest PIC value. The utility of EPT was verified on a set of three probabilistic transformation cases borrowed from the literature. On these cases, the performance of EPT has been compared to other existing probabilistic transformations. Our results indicate that EPT performs better than others on all problems from PIC increasing standpoint. The shortcomings of original EPT version have been clearly identified on two type tracking problems, and they have been overcome thanks to a modification of belief interval constraints. As future works, we would like to establish more appropriate evaluation criteria and make more comparisons between performances of this EPT approach with other recent proposed evolutionary algorithms. We would also make more investigations on EPT to extend it to work with more than two targets.

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References

[1] A.P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Ann. Math. Stat., Vol. 38, No. 4, pp. 325–339, 1967.
[2] G. Shafer, A mathematical theory of evidence, Princeton Univ. Press, 1976.
[3] T. Denœux, Inner and outer approximation of belief structures using a hierarchical clustering approach, International Journal of Uncertainty, Fuzziness, and Knowledge-based Systems, Vol.9, No.4, pp. 437-460, 2001.
[4] P. Smets, The Combination of Evidence in the Transferable Belief Model, IEEE Trans. on PAMI, Vol. 5, pp. 29–39, 1990.
[5] F. Smarandache, J. Dezert (Editors), Advances and applications of DSmT for information fusion, American Research Press, Rehoboth, NM, U.S.A., Vol. 1–4, 2004–2015. http://www.onera.fr/staff/jean-dezert?page=2
[6] P. Smets, Decision making in the TBM: the necessity of the pignistic transformation, Int. J. of Approximate Reasoning, Vol. 38, 2005.
[7] J. Dezert, F. Smarandache, A new probabilistic transformation of belief mass assignment, In Proc. of 11th Int. Conf. on Information Fusion, Cologne, Germany, pp. 1–8, June-July 2008.
[8] J. Sudano, Equivalence Between Belief Theories and Na´ive Bayesian Fusion for Systems with Independent Evidential Data-Part I, The theory, In Proc. of Fusion 2003, Cairns, Australia, July 2003.
[9] F. Cuzzolin, The Intersection probability and its properties, in Proc. of ECSQARU 2009, Verona, Italy, July 1–3, 2009.
[10] D. Han, J. Dezert, C.Z. Han. New Basic belief assignment approximations based on optimization, In Proc. of 15th Int. Conf. on Information Fusion, pp. 286–293, July 2012.
[11] W. Pan, J.Y. Hong, New methods of transforming belief functions to pignistic probability functions in evidence theory, 2009 Int. Workshop on Intelligent System and Applications, 2009 (In Chinese).
[12] J. Sudano, The system probability information content (PIC) relationship to contributing components, combining independent multisource beliefs, hybrid and pedigree pignistic probabilities, in Proc. of Int. Conf. on Information Fusion (Fusion 2002), Annapolis, Maryland, U.S.A, Vol. 2, pp. 1277–1283, July 2002.
[13] N. Srinivas, K. Deb, Multiobjective function optimization using non-dominated sorting genetic algorithms, Evolutionary Computation, Vol. 2, No. 3, pp. 221-248, Fall 1995.
[14] D.Q Han, J. Dezert, Z.S. Duan, Evaluation of Probability Transformations of Belief Functions for Decision Making. IEEE Transactions on Systems, Man, And Cybernetics: Systems, Vol. PP, No. 99, 2015.
[15] A.-L. Jousselme, P. Maupin, On some properties of distances in evidence theory, in Workshop on Theory of Belief Functions, pp.1–6, Brest, France, March 31st–April 2nd, 2010.
[16] J. Dezert, A. Tchamova, F. Smarandache, P. Konstantinova, Target Type Tracking with PCR5 and Dempster's rules: A Comparative Analysis, in Proc. of Int. Conf. on Information Fusion (Fusion 2006), Florence, Italy, July 2006.