# On Dark Energy and the Relativistic Bohm-Poisson Equation

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**Abstract**

Recently, solutions to the *nonlinear* Bohm-Poisson (BP) equation were found with relevant cosmological applications. We were able to obtain an exact analytical expression for the observed vacuum energy density, and explain the origins of its *repulsive* gravitational nature. In this work we considerably improve our prior arguments in support of our findings, and provide further results which include two possible extensions of the Bohm-Poisson equation to the full relativistic regime; explain how Bohm's quantum potential in four-dimensions could be re-interpreted as a gravitational potential in five-dimensions, and which explains why the presence of dark energy/dark matter in our $4D$ spacetime can only be inferred *indirectly*, but not be detected/observed directly. We conclude with some comments about the Dirac-Eddington large numbers coincidences.

## 1 Dark Energy and the Bohm-Poisson Equation

In physical cosmology and astronomy, dark energy is an unknown form of energy which is hypothesized to permeate all of space, tending to accelerate the expansion of the universe [1] . Assuming that the standard model of cosmology is correct, the best current measurements indicate that dark energy contributes 68.3 percent of the total energy in the present-day observable universe. The mass-energy of dark matter and ordinary (baryonic) matter contribute 26.8 and 4.9 percent respectively, and other components such as neutrinos and photons contribute a very small amount. The density of dark energy much less than the density of ordinary matter or dark matter within galaxies. However, it dominates the mass-energy of the universe because it is uniform across space [1]. Two proposed forms for dark energy are the cosmological constant, [2] representing a constant energy density filling space homogeneously, and scalar fields such as quintessence or moduli, dynamic quantities whose energy density can vary in time and space.

The nature of dark energy is more hypothetical than that of dark matter, and many things about the nature of dark energy remain matters of speculation [1]. Dark energy is thought to be very homogeneous, not very dense and is not known to interact through any of the fundamental forces other than gravity. In the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, it can be shown that a strong constant negative pressure in all the universe causes an acceleration in universe expansion if the universe is already expanding, or a deceleration in universe contraction if the universe is already contracting. This accelerating expansion effect is sometimes labeled "gravitational repulsion".

A major outstanding problem is that quantum field theories predict a huge cosmological constant, more than 100 orders of magnitude too large. This would need to be almost, but not exactly, cancelled by an equally large term of the opposite sign. Some supersymmetric theories require a cosmological constant that is exactly zero, which does not help because supersymmetry must be broken. Nonetheless, the cosmological constant is the most economical solution to the problem of cosmic acceleration. Thus, the current standard model of cosmology, the Lambda-CDM (cold dark matter) model, includes the cosmological constant as an essential feature [1].

The evidence for dark energy is heavily dependent on the theory of general relativity. Therefore, it is conceivable that a modification to general relativity also eliminates the need for dark energy. There are very many such theories, and research is ongoing [3], [4]. The measurement of the speed of gravity with the gravitational wave event GW170817 ruled out many modified gravity theories as alternative explanation to dark energy [1].

In quintessence models of dark energy, the observed acceleration of the scale factor is caused by the potential energy of a dynamical field, referred to as quintessence field. Quintessence differs from the cosmological constant in that it can vary in space and time. In order for it not to clump and form structure like matter, the field must be very light so that it has a large Compton wavelength. This class of theories attempts to come up with an all-encompassing theory of both dark matter and dark energy as a single phenomenon that modifies the laws of gravity at various scales. This could for example treat dark energy and dark matter as different facets of the same unknown substance, a "dark fluid" [5], or postulate that cold dark matter decays into dark energy.

The Schrödinger-Newton equation has had a long history since the 1950's [6], [7]. It is the name given to the system coupling the Schrödinger equation to the Poisson equation. In the case of a single particle, this coupling is effected as follows: for the potential energy term in the Schrödinger equation take the gravitational potential energy determined by the Poisson equation from a matter density proportional to the probability density obtained from the wave-function. For a single particle of mass $m$ the coupled system of equations leads to the nonlinear and nonlocal Newton-Schrödinger integro-differential equation

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + V(\vec{r},t)\,\Psi(\vec{r},t) - \left( Gm^2 \int \frac{|\Psi(\vec{r}',t)|^2}{|\vec{r}-\vec{r}'|}\, d^3r' \right) \Psi(\vec{r},t) \tag{1}$$

Bohm's quantum potential $V_Q = -\frac{\hbar^2}{2m}(\nabla^2\sqrt{\rho}/\sqrt{\rho})$ has a *geometrical* description as the Weyl scalar curvature produced by an ensemble density of paths associated with one, and only one particle [8]. This geometrization process of quantum mechanics allowed to derive the Schroedinger, Klein-Gordon [8] and Dirac equations [9]. Most recently, a related geometrization of quantum mechanics was proposed [10] that describes the time evolution of particles as geodesic lines in a curved space, whose curvature is induced by the quantum potential. This formulation allows therefore the incorporation of all quantum effects into the geometry of space-time, as it is the case for gravitation in the general relativity.

Based on these results we proposed in [11] the following nonlinear quantum-like Bohm-Poisson equation for *static* solutions $\rho = \rho(\vec{r})$

$$\nabla^2 V_Q \ = \ 4\pi G m \rho \ \Rightarrow \ - \ \frac{\hbar^2}{2m} \ \nabla^2 \ (\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}) \ = \ 4\pi G m \rho \tag{2}$$

such that one could *replace* the nonlinear Newton-Schrödinger equation for the above non-linear quantum-like Bohm-Poisson equation (2) where the fundamental quantity is *no* longer the wave-function $\Psi$ (complex-valued in general) but the real-valued probability density $\rho = \Psi^* \Psi$. Eq-(2) is based on Bohm's quantum potential

$$V_Q \ \equiv - \frac{\hbar^2}{2m} \ \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \tag{3}$$

If one wishes to introduce a temporal evolution to $\rho$ via a Linblad-like equation, for instance, this would lead to an overdetermined system of differential equations for $\rho(\vec{r}, t)$. Replacing $\nabla^2$ in eqs-(1,2) for the D'Alambertian operator $\Box = \nabla_\mu \nabla^\mu, \mu = 0, 1, 2, 3$ has the caveat that in QFT $\rho(x^\mu) = \rho(\vec{r}, t)$ *no* longer has the interpretation of a probability density but is now related to the particle number current. Despite this caveat we will propose an eq-(10) below involving the D'Alambertian $\Box$ operator and a proper mass density (mass per proper four-volume).

For the time being we shall just focus on static solutions $\rho(\vec{r})$. The de Sitter space metric can be written in static coordinates in the form $g_{tt}(r) = -(1 - \frac{\Lambda}{3} r^2); g_{rr}(r) = -(g_{tt})^{-1}, \cdots$, and given in terms of the cosmological constant $\Lambda = (3/R_H^2)$, where $R_H$ is the Hubble radius. Hence, there is no inconsistency in focusing for now on static solutions $\rho_m(\vec{r})$ for the probability density.

Since almost 95 percent of the energy/mass content of the Universe is comprised of dark energy/dark matter, in a recent manuscript we envisioned the Universe's dark energy/dark matter (dark-fluid) density distribution as being proportional to a QM probability density obeying the Bohm-Poisson (BP) equation [12], in the same vain that one can view an electron orbiting the Hydrogen nucleus as an "electron probability cloud" surrounding the nucleus, permeating all of space, and whose mass density distribution is $\rho = m_e \Psi^* \Psi$, where $\Psi(\vec{r})$ are the stationary wave-function solutions to the Schrödinger equation, and $m_e$ is the electron's mass.

The density $\rho_m = m\rho$ of dark energy/dark matter (dark-fluid) permeating all of space was postulated to be a solution to the nonlinear quantum-like Bohm-Poisson (BP) equation. It is straightforward to verify that a spherically symmetric solution to eq-(2) in a

$3D$ spatially $flat$ background [1] is given by

$$\rho_m(r) \;=\; \frac{A}{r^4}, \;\; A \;=\; -\; \frac{\hbar^2}{2\pi G m^2} \;<\; 0 \tag{4}$$

At first glance, since $\rho_m(r) \leq 0$ one would be inclined to dismiss such solution as being unphysical. Nevertheless, we can bypass this problem by focusing instead on the *shifted* density $\tilde{\rho}_m(r) \equiv \rho_m(r) - \rho_0$ obeying the Bohm-Poisson equation

$$-\; \frac{\hbar^2}{2m}\, \nabla^2\, (\frac{\nabla^2\sqrt{\tilde{\rho}_m}}{\sqrt{\tilde{\rho}_m}}) \;=\; 4\pi G m \tilde{\rho}_m, \;\;\; \nabla^2 f(r) \equiv r^{-2}\partial_r(r^2\partial_r f(r)). \tag{5}$$

and whose solution for the *shifted* mass density is given by

$$\tilde{\rho}_m \;=\; A/r^4 \;=\; \rho_m(r) \;-\; \rho_o \;\leq 0, \;\;\; \Rightarrow \; \rho_m(r) \;=\; \frac{A}{r^4} \;+\; \rho_0, \;\; A \;=\; -\; \frac{\hbar^2}{2\pi G m^2} \tag{6}$$

One may notice that by shifting the density $\rho_m \to \rho_m - \rho_o = \tilde{\rho}_m$, and the (radial) pressure $p \to p + p_o = \tilde{p}$, one can eliminate the cosmological constant in the Einstein's field equations with a cosmological constant, when $\rho_o = p_o$. For example, given a Friedman-Lemaitre-Robertson-Walker (FLRW) model with a metric $ds^2 = -(dt)^2 + a^2(t)(d\Sigma)^2$, the resulting Einstein's field equations (in units $c = 1$) lead to the Friedmann equations

$$(\frac{\dot{a}}{a})^2 \;+\; \frac{k}{a^2} \;-\; \frac{\Lambda}{3} \;=\; \frac{8\pi G}{3}\; \rho_m \tag{7a}$$

$$\frac{2\ddot{a}}{a} \;+\; (\frac{\dot{a}}{a})^2 \;+\; \frac{k}{a^2} \;-\; \Lambda \;=\; -\; 8\pi G\; p \tag{7b}$$

where $k = 1, 0, -1$ is the spatial curvature index. A simple inspection of eqs-(7a,7b) reveals that that the cosmological constant can be *reabsorbed* into a mere redefinition of $\rho_m$ and $p$ as follows

$$\rho_m \;\to\; \rho_m \;-\; \frac{\Lambda}{8\pi G} \;=\; \tilde{\rho}_m, \;\;\; p \;\to\; p \;+\; \frac{\Lambda}{8\pi G} \;=\; \tilde{p} \tag{8}$$

consequently, if $\rho_m, p$ appear in the Friedmann equations (7a,7b) with a cosmological constant $\Lambda$, then $\tilde{\rho}_m = \rho_m - \rho_o$, and $\tilde{p} = p + p_o$ ($\rho_o = p_o = \Lambda/8\pi G$), will appear in eqs-(7a, 7b) without the cosmological constant $\Lambda$. Furthermore, the cosmological equation of state associated to the dark energy permeating all of space

$$\tilde{\rho}_m \;=\; -\; \tilde{p} \;\Leftrightarrow\; \rho_m \;=\; -\; p \tag{9}$$

remains invariant.

To sum up, given an actual solution $\tilde{\rho}_m$ of the BP equation, the shifting provided in eq-(8) will allow us to focus on the *domain* of values where $\rho_m(r) \geq 0$. And, in doing so, it will permit us to show that the value of $\rho_0$ can be made to coincide exactly with the

[1] For the time being we shall not discuss solutions in *curved* backgrounds

(extremely small) observed vacuum energy density, by simply introducing an ultraviolet length scale $l$ that is *very close* to the Planck scale, and infrared length scale $L$ equal to Hubble scale $R_H$.

A covariant extension of the BP equation, for signature $(-,+,+,+)$, may be defined in terms of the D'Alambertian operator, and a proper mass density $\sigma(\vec{r},t)$ of physical dimensions $(length)^{-5}$, such that $m = \int \sigma(\vec{r},t)\ \sqrt{|g|}\ d^4x$, as follows

$$-\ \Box\ \left(\frac{\Box\sqrt{\sigma(\vec{r},t)}}{\sqrt{\sigma(\vec{r},t)}}\right)\ =\ 4\pi G m\ \sigma(\vec{r},t),\quad \Box \equiv \frac{1}{\sqrt{|g|}}\partial_\mu(\sqrt{|g|}\ g^{\mu\nu}\ \partial_\nu),\quad \hbar = c = 1 \tag{10}$$

Focusing for now on the static solutions (6) of the BP equation, the ultraviolet scale $l$ is chosen at the node of $\rho_m(r)$ such that

$$\rho_m(r=l)\ =\ -\ \frac{\hbar^2}{2\pi G m^2}\ \frac{1}{l^4}\ +\ \rho_o\ =\ 0\ \Rightarrow\ \rho_o\ =\ \frac{\hbar^2}{2\pi G m^2}\ \frac{1}{l^4} \tag{11}$$

The domain of physical values of $r$ must be $r \geq l$ in order to ensure a positive-definite density $\rho_m(r) \geq 0$. One could include all the values of $r$ from 0 to $\infty$. The density diverges at $r=0$, while the integral $\int_0^\infty \rho(r) 4\pi r^2 dr = \infty - \infty$. The $+\infty$ contribution stems from the region $r \geq l$, while the $-\infty$ contribution stems from the region $r < l$. Therefore one needs to introduce a suitable and judicious regularization involving an ultraviolet and infrared scale.

In natural units of $\hbar = c = 1$, after introducing the ultraviolet scale $l$ and infrared scale $L = R_H$ in the normalization condition (otherwise the mass would diverge) it yields the integral

$$m\ =\ \int_l^{R_H}\ \rho(r)\ 4\pi r^2\ dr\ =\ \int_l^{R_H}\ (\frac{A}{r^4} + \rho_0)\ 4\pi r^2\ dr\ =\ \int_l^{R_H}\ \left(-\frac{1}{2\pi G m^2}\ \frac{1}{r^4}\ +\ \rho_0\right)\ 4\pi r^2\ dr \tag{12}$$

Upon performing the integral in eq-(12), after plugging in the value of $\rho_0$ derived from eq-(11), with the provision that when $R_H >> l$ the dominant contribution to the integral stems solely from $\rho_o$, one ends up with the following relationship

$$\frac{4\pi R_H^3}{3}\ \rho_o\ =\ \frac{4\pi R_H^3}{3}\ \frac{1}{2\pi G m^2 l^4}\ =\ m\ \ \Rightarrow\ m^3\ =\ \frac{2}{3}\ \frac{R_H^3}{G l^4} \tag{13}$$

solving for $m$ one gets

$$m\ =\ (\frac{2}{3 G l^4})^{1/3}\ R_H \tag{14}$$

One can verify that when the ultraviolet scale $l$ is chosen to be *very* close to the Planck scale, and given by

$$l^4\ =\ \frac{4}{3}\ L_P^4\ \Rightarrow\ l\ =\ (\frac{4}{3})^{1/4}\ L_p\ =\ 1.0745\ L_p \tag{15}$$

then upon inserting the values for $m$ and $l$ obtained in eqs-(14,15) into the expression for $\rho_o$ *derived* in eq-(11), after setting $L_p^2 = 2G$, [2] it gives in natural units of $\hbar = c = 1$

$$\rho_o \;=\; \frac{1}{2\pi G m^2}\,\frac{1}{l^4} \;=\; \frac{1}{2\pi G}\,(\frac{3\;G\;l^4}{2})^{2/3}\,\frac{1}{R_H^2\;l^4} \;=\; \frac{3}{8\pi G}\,\frac{L_p^4}{R_H^2\;L_p^4} \;=\; \frac{3}{8\pi G R_H^2} \tag{16}$$

which is precisely *equal* to the observed vacuum energy density $\rho = (2\Lambda/16\pi G)$ associated with a cosmological constant $\Lambda = (3/R_H^2)$ and corresponding to a de Sitter expanding universe whose throat size is the Hubble radius $R_H$ ( $= c/H_o$, $H_o$ is today's value of the Hubble parameter).

The physical reason behind the choice of the ultraviolet scale $l$ in eq-(15) is based on re-interpreting $\rho_o$ as the uniform energy (mass) density inside a black hole region of Schwarzschild radius $R = 2Gm$

$$\rho_{bh} \;=\; \frac{m}{(4\pi/3)R^3} \;=\; \frac{3}{8\pi G R^2},\quad L_P^2 = 2G,\quad \hbar = c = 1 \tag{17}$$

In the regime $R = 2Gm >> l$, when the dominant contribution to the integral (12) stems from the $\rho_o$ term, we may equate the expression for $\rho_o$ in eq-(11) to $\rho_{bh}$ in eq-(17) giving

$$\frac{1}{2\pi G m^2 l^4} \;=\; \frac{1}{2\pi l^4}\,\frac{(2G)^2}{GR^2} \;=\; \frac{1}{2\pi l^4}\,\frac{L_p^4}{GR^2} \;=\; \frac{3}{8\pi G R^2} \;\Rightarrow\; l \;=\; (\frac{4}{3})^{1/4}\,L_p,\quad \hbar = c = 1 \tag{18}$$

and leading once again to the value of $l = 1.0745 L_p$ in eq-(15). Therefore, when $R = 2Gm >> l$, the value of $l$ is always *very* close to the Planck scale, and *independent* of $R = 2Gm$, because the scale $R$ has *decoupled* in eq-(18).

In this way, one can effectively view the observable universe as a "black-hole" whose Hubble radius $R_H$ encloses a mass $M_U$ given by $2GM_U = R_H$. From eq-(14) it follows that when $R = R_H$, the black hole density $\rho_{bh} = \rho_o = \rho_{obs}$ coincides with the observed vacuum energy density. It is well known that inside the black hole horizon region the roles of $t$ and $r$ are exchanged due to the switch in the signature of the $g_{tt}, g_{rr}$ metric components. Cosmological solutions based on this $t \leftrightarrow r$ exchange were provided by the Kantowski-Sachs metric.

To sum up our main result : By postulating that dark energy/dark matter is a dark-fluid permeating all of space, whose mass density is proportional to the probability density, and after finding a particular solution to the Bohm-Poisson equation, while introducing an ultraviolet (very close to the Planck scale) and an infrared (Hubble) scale to regularize the mass, one can naturally obtain a value for the vacuum energy density which coincides *exactly* with the extremely small observed vacuum energy density. It is remarkable that the Bohm-Poisson equation chooses for us a lower scale to be basically equal to the Planck scale. It was not put it in by hand, but is a direct result of the solutions to the Bohm-Poisson equation. The only assumption made was to choose the Hubble scale $R_H$ for the

[2] Some authors absorb the factor of 2 inside the definition of $L_p$, we define the Planck scale such that the Compton wavelength coincides with the Schwarzschild radius

infrared cutoff, and which makes physical sense since $R_H$ is the cosmological horizon. Is it a numerical coincidence or design ? Because Bohm's formulation of QM is by construction non-local, it is this non-locality which casts light into the crucial ultraviolet/infrared entanglement of the Planck/Hubble scales which was required in order to obtain the observed values of the vacuum energy density.

Furthermore, one can also explain the origins of its *repulsive* gravitational nature. The Bohm-Poisson's (BP) equation is invariant under $\tilde{\rho}_m \to -\tilde{\rho}_m$, and $G \to -G$. Consequently $-\tilde{\rho}_m \geq 0$ is a solution to a BP equation associated to a *negative* gravitational coupling $-G < 0$ which is tantamount to *repulsive* gravity. This is perhaps the most salient feature of the results in [12].

To finalize this section we should remark that we found solutions to the BP equation in $flat$ spatial $3D$ backgrounds. The operator $\nabla^2$ is metric-dependent, thus solutions of the BP equation in *curved* backgrounds will differ from those found above. In general, one must have a *coupled* system of equations involving the relativistic analog of the BP equation *plus* Einstein's field equations. Matter affects the geometry (metric), and the latter metric determines the form of the $\nabla^2, \Box$ operators, which in turn will affect the solutions for $\rho_m$, and which in turn will have an affect on the metric, and so forth, $\cdots$ In eqs-(33, 34) we show how this *coupled* system of equations can be lumped into a single equation.

For consistency checks, given $\rho_m = \rho_o + A/r^4$, and a radial pressure $p = -\rho_m$, one can find solutions to the Einstein field equations if one introduces a cutoff $\epsilon$ to avoid the singularity at $r = 0$, such that $M_U\ \epsilon \sim 1$, and $\rho_m \equiv 0$ in the region $r < \epsilon$. The temporal and radial metric components solutions to the field equations are then given by

$$g_{tt} = -(\ 1\ -\ \frac{\Lambda}{3}\ r^2\ +\ \frac{C_1}{r}\ +\ \frac{C_2}{r^2}\ ),\quad g_{rr} = -(g_{tt})^{-1},\quad \Lambda = 8\pi G\rho_o \tag{19a}$$

where

$$C_1\ =\ 8\pi G(\frac{\rho_o\epsilon^3}{3} - \frac{A}{\epsilon})\ >\ 0,\quad C_2\ =\ 8\pi GA\ <\ 0,\quad A\ =\ -\ \frac{1}{2\pi Gm^2},\quad \hbar = c = 1 \tag{19b}$$

when the mass parameter is $m \sim M_U = (R_H/2G) \Rightarrow \epsilon \sim L_p(L_p/R_H) \sim 10^{-61}L_p$, one can verify that the terms defining the coefficients $C_1, C_2$ are infinitesimals of orders $\epsilon, \epsilon^2, \epsilon^3$. Therefore, one recovers the de Sitter metric as expected without having to go to the asymptotic region $r \to \infty$.

## 2 Asymptotic Safety and Covariant Extensions of the BP Equation

In the previous section we studied solutions to the BP equation involving a large mass $m$ of the order of $M_U = (R_H/2G)$ and which followed directly from eqs-(13, 14). In this section we shall be focusing on a particle with a very small mass (large Compton wavelength) of the order of $m \sim (1/R_H)$, and on the effects of the Renormalization Group. This scalar particle might be related to quintessence [5].

The Renormalization Group (RG) improvement of Einstein's equations is based on the possibility that Quantum Einstein Gravity might be non-perturbatively renormalizable and asymptotically safe due to the presence of interacting ultraviolet fixed points [19]. In this program one has $k$ (energy) dependent modifications to the Newtonian coupling $G(k)$, the cosmological constant $\Lambda(k)$ and energy-dependent spacetime metrics $g_{ij,(k)}(x)$.

In $D = 4$ there is a nontrivial interacting (non-Gaussian) ultraviolet fixed point $G_* = G(k)k^2 \neq 0$. The fixed point $G_*$ by definition is *dimensionless* and the running gravitational coupling has the form [20], [19]

$$G(k) \;=\; G_N \; \frac{1}{1 + [G_N k^2 / G_*]} \tag{20a}$$

The scale dependence of $\Lambda(k)$ in the de Sitter case was found to be [20]

$$\Lambda(k) \;=\; \Lambda_0 \;+\; \frac{b\; G(k)}{4}\; k^4, \quad \Lambda_0 > 0 \tag{20b}$$

where $b$ is positive numerical constant.

In $D = 4$, the dimensionless gravitational coupling has a nontrivial fixed point $G = G(k)k^2 \to G_*$ in the $k \to \infty$ limit, and the dimensionless variable $\Lambda = \Lambda(k)k^{-2}$ has also a nontrivial ultraviolet fixed point $\Lambda_* \neq 0$ [20]. The infrared limits are $\Lambda(k \to 0) = \Lambda_0 > 0$, $G(k \to 0) = G_N$. Whereas the ultraviolet limit is $\Lambda(k = \infty) = \infty; G(k = \infty) = 0$.

Let us choose now an actual positive-definite solution $\hat{\rho}_m \equiv -\tilde{\rho}_m = |A|/r^4 \geq 0$; $|A| = \hbar^2/2\pi G m^2$, of the BP equation associated to *repulsive* gravity $-G < 0$, as explained earlier

$$-\;\frac{\hbar^2}{2m}\;\nabla^2\;(\frac{\nabla^2\sqrt{-\tilde{\rho}_m}}{\sqrt{-\tilde{\rho}_m}}) \;=\; 4\pi\;(-G)\;m\;(-\tilde{\rho}_m) \tag{21}$$

The mass density solution of (21) to focus on (in $\hbar = c = 1$ units) is

$$\hat{\rho}_m(r) \;=\; -\;\tilde{\rho}_m(r) \;=\; \frac{1}{2\pi G m^2 r^4} \;\geq 0 \tag{22}$$

If one selects $m = (1/R_H)$ to coincide with the Compton mass of a particle corresponding to the Hubble scale $R_H$, then at the Hubble scale $r = R_H$ one has

$$\hat{\rho}_m(r = R_H) \;=\; \frac{1}{2\pi G m^2 R_H^4} \;=\; \frac{1}{2\pi G\;(R_H)^{-2}\;R_H^4} \;\sim\; (L_P R_H)^{-2} \sim 10^{-122} M^4_{Planck} \tag{23a}$$

and which agrees with the observed vacuum energy density. It is well known (to the experts) that such extremely small value is of the same order of magnitude as $(m_{neutrino})^4$.

The problem arises when one evaluates $\hat{\rho}_m(r)$ at $L_p$, given $m = 1/R_H$. One gets a huge value

$$\hat{\rho}_m(r = L_p) \ \sim \ \frac{1}{G m^2 L_p^4} \ = \ (\frac{R_H}{L_p})^2 \ L_p^{-4} \ \sim \ 10^{122} \ M_p^4 \tag{23b}$$

We will see how the Asymptotic Safety scenario comes to our rescue by realizing that a Renormalization Group flow of $G$ and $m^2$ solves the problem. The key idea, based on dimensional grounds, is simply to postulate that the flow of $m^2(k)$ has the *same* functional form as the flow of $\Lambda(k)$ in eq-(20)

$$m^2(k) \ = \ m_o^2 \ + \ \frac{b \ G(k)}{4} \ k^4, \quad m_o^2 > 0 \tag{24}$$

The only thing remaining is to related the scale $r$ in eq-(17) with the energy (momentum) scale $k$. The authors [20] expressed $k$ as the inverse of $d(r)$ where $d(r)$ was a proper distance derived from the Schwarzschild metric. If one opts for the simplest choice $k = 1/r$, eq-(22) can be rewritten as

$$\hat{\rho}_m(r) \ = \ \frac{1}{2\pi} \ \frac{1}{G(k) \ m^2(k) \ r^4} \ = \ \frac{1}{2\pi} \ \frac{k^4}{G(k) m^2(k)} \tag{25}$$

note that strictly speaking eq-(25) is *not* a solution to the BP equation, because if it were one must have that $G(k)m^2(k) = constant$, for all values of $k$, which is *not* the case. Similarly, the renormalization-group-improved black hole solutions of [20] are *not* solutions to the Einstein vacuum field equations [17]. Nevertheless, from eqs-(20,24) one learns that

$$lim_{k\to 0} \ (G(k) \ m^2(k)) \ = \ G_N \ m_o^2 \tag{26a}$$

$$lim_{k\to \infty} \ (G(k) \ m^2(k)) \ = \ \frac{b}{4} \ (G_*)^2 \tag{26b}$$

whereas at the Planck scale $k = M_p$

$$lim_{k\to M_p} \ (G(k) \ m^2(k)) \ \sim \ \frac{b}{4} \ (G_*)^2 \tag{27}$$

Consequently, eqs-(25,27) lead to

$$\hat{\rho}_m(r = L_p) \ \sim \ \frac{1}{2\pi \ G(k = M_p) \ m^2(k = M_p) \ L_p^4} \ \sim \ \frac{2}{b\pi} \ (G_*)^{-2} \ L_p^{-4} \ \sim \ M_p^4 \tag{28}$$

which is the expected result for the vacuum energy density at the Planck scale.

To sum up, the renormalization group machinery (Asymptotic Safety) can be implemented such that eq-(23a) furnishes the observed vacuum energy density at the Hubble scale, while eq-(28) is the expected vacuum energy density at Planck scale. Naturally, one needs to generalize the BP equation to the fully relativistic regime as described by eq-(10). The key question is what is the "particle" represented by the mass $m$ in the BP equation (21) ? i.e. a mass that experiences a renormalization group flow (24) similar to the flow experienced by $\Lambda$ (20). Is this a scalar particle related to quintessence ? [5].

We emphasized earlier the key role that $-G < 0$ plays in all of this and which *stems* directly from the invariance of the BP equation under $\rho \rightarrow -\rho; G \rightarrow -G$. Our solutions for $\hat{\rho}_m(r) \geq 0$ correspond to $-G < 0$, thus the "particle" in question exerts a *repulsive* gravitational force which mimics "dark energy". The RG flow behavior of $G$ displayed in eq-(19) shows that $G$ grows as $k$ decreases. Meaning that $G$ *increases* with distance, so that the magnitude of the repulsive force exemplified by $-G < 0$ becomes larger, and larger, as the universe expands. This is what is observed. Next we shall provide a different view of our findings so far.

## Matter Creation from the Vacuum

The second interpretation of the solution (22) to the BP equation (21) is that involving matter creation from the vacuum, as advocated by Hoyle long ago. Imagine one pumps matter out of the vacuum in lumps/units of Planck masses. Let us assume that the Universe expands in such a way that matter is being replenish from the vacuum so that the mass at any moment is linearly proportional to the size of the Universe. As the mass of the universe grows the vacuum energy density decreases since the vacuum is being depleted. In this scenario, at the Hubble scale $R_H$, one has $M_U \sim R_H$.

This result is also compatible with Mach's principle. By equating $GmM_U/R_H$ to the rest mass $m$ of a particle one arrives at $GM_U = R_H$, which once again is very close to the Schwarzschild radius $2GM_U$. Hence, one arrives at the scaling relation

$$\frac{M_p}{L_P} = \frac{M_U}{R_H} \tag{29}$$

which we interpreted long ago [18] as equating the proper forces (after re-introducing $c$) $M_U c^2/R_H = M_p c^2/L_p$ and leading to some sort of maximal/minimal acceleration duality.

Inserting the values of $M_P, M_U$, and $r = L_P$, into the solution (22) of the BP equation gives

$$\hat{\rho}(r = L_p; m = M_p) = \frac{1}{2\pi G M_P^2 \ L_p^4} \sim L_p^{-4} = M_p^4, \quad L_p^2 = 2G \tag{30$a$}$$

which is compatible with the large density at the Planck scale, and

$$\hat{\rho}(r = L_p; m = M_U) = \frac{1}{2\pi G M_U^2 \ L_p^4} \sim \frac{G}{2\pi R_H^2 \ L_p^4} \sim (L_p \ R_H)^{-2} \tag{30$b$}$$

which agrees with the observed vacuum energy density at the Hubble scale and obtained above in eq-(11). Before concluding we add some important remarks.

## Does Dark Energy Resides in the Bulk of a 5D spacetime ?

Evaluating Bohm's quantum potential (3) for $\rho(r)$ given by eq-(4) yields

$$V_Q = - \frac{\hbar^2}{mr^2} \tag{31}$$

and which is reminiscent of an effective gravitational potential in 4 spatial dimensions (a $5D$ spacetime). The "quantum" force $F_Q = -\partial_r V_Q$ corresponding to $V_Q$ in eq-(31) scales as $F_Q \sim -r^{-3}$ which has the same behavior as the gravitational force between two masses $m_1, m_2$ in $5D$

$$F \ = \ - \ G_5 \ \frac{m_1 \ m_2}{r^3} \tag{32}$$

since the $5D$ gravitational constant $G_5$ has dimensions of $(length)^3$. Despite the possibility, we are not going to speculate at the moment as to whether or not the "quantum" force originating from Bohm's quantum potential is the "fifth" force. The main point is that one should consider the possibility that Bohm's quantum potential in 3 spatial dimensions ($4D$ spacetime) mimics classical gravity in 4 spatial dimensions ($5D$ spacetime), and for this reason one can only *indirectly* infer the gravitational effects of dark energy/dark matter in our $4D$ universe without directly detecting it because such dark energy/dark matter resides in $5D$, which is reminiscent of the brane-world scenarios.

**Finsler-Relativistic Extension of the Bohm-Poisson equation**

If one wishes to introduce a temporal dependence to $\hat{\rho}_m$ we should extend the BP equation to full the relativistic regime. It is interesting that a simple exchange of $r \leftrightarrow t$ as it occurs in the Schwarzschild metric leading to the Kantowski-Sachs metric, yields $\hat{\rho}_m(t) = |A| t^{-4}$, and similar findings are obtained for the values of the vacuum energy density, simply by exchanging $L_p \leftrightarrow ct_p; R_H \leftrightarrow ct_H$ in the equations. $t_p, t_H$ are the Planck and Hubble times, respectively. Upon doing so it leads to a Big-Bang-like singularity at $t = 0, \hat{\rho}_m(t = 0) = \infty$. This combined with the repulsive gravitational feature of our model, implies naturally that an expansion would follow.

Besides eq-(10), another relativistic generalization of the BP equation can be constructed from the Lagrange-Finsler geometrical formulation of QM recently advocated by [10]. He described the time evolution of particles as geodesic lines in a curved space, whose *curvature* is induced by the quantum potential. This formulation incorporates all quantum effects into the geometry of space-time, as it is the case for gravitation in the general relativity.

The explicit expression of the metric components in terms of the quantum potential $V_Q$ were provided by [10]. This is not the first time where the metric components are expressed in terms of a potential. In General Relativity (GR) we learned that in the linearized weak gravity limit, and for slow moving bodies, the temporal metric component $g_{00} \sim \eta_{00} + 2V_N$ (c = 1), can be expressed in terms of the Newtonian potential $V_N$. Hence, given the explicit expressions [10] of the metric $g_{\mu\nu} = g_{\mu\nu}(V_Q)$ in terms of Bohm's quantum potential, one can write down the curvature tensors, and the Einstein tensor leading to the field equations

$$R_{\mu\nu}(V_Q) \ - \ \frac{1}{2} \ g_{\mu\nu}(V_Q) \ R(V_Q) \ + \ \Lambda \ g_{\mu\nu}(V_Q) \ = \ 8\pi G \ T_{\mu\nu} \tag{33}$$

where the stress energy tensor is the one associated with a dark energy/dark matter fluid permeating all of space and given in terms of $\rho_m, p$. In particular, the BP equation (2) can be generalized to

$$R_{00}(V_Q) \ - \ \frac{1}{2} \ g_{00}(V_Q) \ R(V_Q) \ + \ \Lambda \ g_{00}(V_Q) \ = \ 8\pi G \ T_{00} \tag{34}$$

where $T_{00} = g_{00}\rho_m$. Solutions to eq-(33) will be provided in future investigations as well as the study of these equations in higher dimensions.

# 3 Conclusions

To conclude we add some remarks pertaining the Dirac-Eddington large numbers coincidences. Nottale [16] found long ago a direct relationship between the fine structure constant $\alpha$ and the cosmological constant $\Lambda$. In $\hbar = c = 1$ units, $\alpha = e^2 = 1/137$, the expression is

$$\Lambda \simeq \frac{(L_p)^4}{(r_e)^6} \ = \ \frac{(m_e)^6 \ (L_p)^4}{(\alpha)^6} \ = \ 10^{-56} \ cm^{-2} \tag{35}$$

the classical electron radius $r_e$ is defined in terms of the charge $e$, and electron mass $m_e$, as

$$\frac{e^2}{r_e} \ = \ \frac{\alpha}{r_e} \ = \ m_e \tag{36}$$

This important relation between $\Lambda$ and $\alpha$ [16] warrants further investigation within the context of the Bohm-Poisson equation and the Dirac-Eddington large number coincidences.

We should mention that of the many articles surveyed in the literature pertaining the role of Bohm's quantum potential and cosmology, [13], [14], [15] we did not find any related to the Bohm-Poisson equation proposed in this work.[3] The authors [14], for instance, have shown that replacing classical geodesics with quantal (Bohmian) trajectories gives rise to a quantum corrected Raychaudhuri equation (QRE). They derived the second order Friedmann equations from the QRE, and showed that this also contains a couple of quantum correction terms, the first of which can be interpreted as cosmological constant (and gives a correct estimate of its observed value), while the second as a radiation term in the early universe, which gets rid of the big-bang singularity and predicts an infinite age of our universe.

To finalize, we must say that the most attractive project is to find nontrivial solutions to the relativistic Bohm-Poisson equation (10). A careful inspection reveals that a separation of variables does not work. Solutions to eq-(33) are more difficult to find. We delegate this difficult task for future investigations as well as the study of these equations and solutions in higher dimensions.

[3] A Google Scholar search provided the response "Bohm-Poisson equation and cosmological constant did not match any articles"

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