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Some Generalized Single Valued Neutrosophic Linguistic Operators and Their Application to Multiple Attribute Group Decision Making

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Abstract This paper proposes a group decision making method based on entropy of neutrosophic linguistic sets and generalized single valued neutrosophic linguistic operators. This method is applied to solve the multiple attribute group decision making problems under single valued neutrosophic liguistic environment, in which the attribute weights are completely unknown. First, the attribute weights are obtained by using the entropy of neutrosophic linguistic sets. Then three generalized single valued neutrosophic linguistic operators are introduced, including the generalized single valued neutrosophic linguistic weighted averaging (GSVNLWA) operator, the generalized single valued neutrosophic linguistic ordered weighted averaging (GSVNLOWA) operator and the generalized single valued neutrosophic linguistic hybrid averaging (GSVNLHA) operator, and the GSVNLWA and GSVNLHA operators are used to aggregate information. Furthermore, similarity measure based on single valued neutrosophic linguistic numbers is defined and used to sort the alternatives and obtain the best alternative. Finally, an illustrative example is given to demonstrate the feasibility and effectiveness of the developed method.

Keywords decision making; neutrosophic set; single valued neutrosophic linguistic set; generalized single valued neutrosophic linguistic operators; entropy of neutrosophic linguistic set

1 Introduction

In real decision making problems, the decision information is usually inaccurate uncertain or incomplete. So it is more and more difficult to make scientific and reasonable decisions.

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Under these circumstances, Zadeh[1] proposed the remarkable theory of fuzzy sets (FS), where the membership degree is represented by a real number between zero and one, is regarded as an important tool for solving multiple attribute decision making problems. Subsequently, Atanassov[2] introduced the idea of intuitionistic fuzzy set (IFS) in 1983, in which each member having a membership degree as well as non-membership degree. This is an extension of Zadehs FS. Furthermore, hesitant fuzzy set (HFS) were introduced by Torra[3] to support the decision makers who are hesitant in expressing preference in a decision making. Although these sets are very successfully applied to multiple attribute decision making problems[4–6], FS and IFS cannot describe and deal with the indeterminate and inconsistent information that exits in real world[7]. Take vote as an example, thirty percent vote Yes, twenty percent vote No, ten percent give up, and forty percent are undecided. This is beyond the scope of IFSs, and it cannot distinguish the information between giving up and undecided. Hence further generalizations of fuzzy and intuitionistic fuzzy sets are required.

Smarandache[7] originally proposed the concept of a neutrosophic set (NS) by adding an independent indeterminacy-membership on the basis of IFS which means decision makers use truth-membership, indeterminacy-membership and falsity-membership to describe their judgment on an object respectively[8]. The neutrosophic set is a powerful general formal framework which generalizes the concepts of the classic set, FS, IFS and paraconsistent set etc. Recently, neutrosophic set have become an interesting research topic and made some achievements. Wang, et al.[9, 10] proposed the single valued neutrosophic set (SVNS) and interval neutrosophic set (INS), and provided the set theoretic operators and various properties of them. Wang, et al.[11] defined the multi-valued neutrosophic sets and multi-valued neutrosophic number, and proposed the TODIM method under multi-valued neutrosophic number environment. Majumdar, et al.[12] introduced the entropy of SVNS and three similarity measures. Also, Ye[13] proposed a cross-entropy measure of single valued neutrosophic and applied it to multiple attribute decision making problems. And Ye[14] defined the similarity measure of INS based on Hamming and Euclidean distances, and applied it to multiple attribute decision making problems under interval neutrosophic circumstance. Zhang, et al.[15] developed interval neutrosophic number weighted arithmetic averaging (INNWAA) operator and interval neutrosophic weighted geometric averaging (INNWGA) operator, and their application in multicriteria decision making problems. Ye[16] proposed a multiple attribute decision making method based on the possibility degree ranking method and ordered weighted aggregation (OWA) operators of interval neutrosophic numbers. Liu, et al.[17] introduced the single valued neutrosophic normalized weighted Bonferroni operator and applied it to the multiple attribute decision making method.

However, in real multiple attribute decision making problems, because of the ambiguity of peoples thinking and the complexity of objective things, the attribute values cannot always be expressed by crisp numbers, and it is easier to be expressed by linguistic terms, such as good, general, and poor. In the last decades, a number of linguistic multiple attribute decision making problems and linguistic aggregation operators were developed[18–23]. Herrera, et al.[18] proposed a model of consensus in group decision making under linguistic assessments. Xu[19] introduced some generalized induced linguistic aggregation operators to assemble the linguistic information. Wei[20] proposed a grey relational analysis method for 2-tuple linguistic, and applied it to multiple attribute group decision making with incomplete weight information. Wang, et al.[21] proposed the concept of the intuitionistic linguistic set, the intuitionistic linguistic number, and the intuitionistic two-semantic. Wang, et al.[22, 23] introduced some intuitionistic linguistic aggregation operators, and applied these operators to solve decision making problems.

In order to make full use of the merits of the single valued neutrosophic set and linguistic term set, Ye[24] introduced the concept of a single valued neutrosophic linguistic set, and define basic operational relations. Motived by the above literature, this paper proposed a multiple attribute group decision making method based on generalized single valued neutrosophic linguistic operators. This paper is structured as follows. The preliminaries are provided in Section 2. In Section 3, three generalized single valued neutrosophic linguistic operators are introduced, including the generalized single valued neutrosophic linguistic weighted averaging (GSVNLWA) operator, the generalized single valued neutrosophic linguistic ordered weighted averaging (GSVNLOWA) operator and the generalized single valued neutrosophic linguistic hybrid averaging (GSVNLHA) operator. In Section 4, a multiple attribute group decision making method based on entropy of neutrosophic linguistic sets and generalized single valued neutrosophic linguistic operators is proposed. An illustrative example is presented to demonstrate the application of the proposed method in Section 5. Section 6 is the conclusion.

2 Preliminaries

This section briefly reviewed the concept of the neutrosophic set (NS), the single valued neutrosophic set (SVNS), the linguistic term set (LS) and the single valued neutrosophic linguistic set (SVNLS), then defined the distance between single valued neutrosophic linguistic numbers (SVNLNs) and the entropy of single valued neutrosophic linguistic sets, which will be used in the rest of the paper.

2.1 The Neutrosophic Set and the Single Valued Neutrosophic Set

Definition 1 (see [9]) Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, a falsity-membership function $F_A(x)$. The function $F_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, i.e., $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2 (see [11]) Let X be a space of points (objects) with generic elements in X denoted by x. A simple valued neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, a falsity-membership function $F_A(x)$, where $T_A(x) \in [0, 1]$, $I_A(x) \in [0, 1]$, $F_A(x) \in [0, 1]$ for each point x in X. Then, a simple valued neutrosophic set A can be expressed as:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}.$$

Thus, the simple valued neutrosophic set satisfies the condition $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$.

2.2 The Linguistic Term Set

Definition 3 (see [25, 26]) Let $S = \{s_\theta \,|\theta = 0, 1, \cdots, \tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable. For example, when $\tau = 8$, a set S could be given as follows:

$$\begin{aligned} S &= \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\} \\ &= \{\text{extremely poor, very poor, poor, slightly poor,} \\ &\quad \text{fair, slightly good, good, very good, extremely good}\}. \end{aligned}$$

In these cases, it is usually required that there exist the following[25, 26]:

1) A negation operator: $Neg(s_i) = s_{\tau-i}$,
2) The set is ordered: $s_i \le s_j$ if and only if $i \le j$,
3) Maximum operator: $\max(s_i, s_j) = s_i$, if $i \ge j$,
4) Minimum operator: $\min(s_i, s_j) = s_i$, if $i \le j$.

In order to preserve all the given information, Xu[26] extended the discrete term set S to a continuous term set $\bar{S} = \{s_\theta \,|\theta \in [0, q]\}$, where, if $s_\theta \in S$, then we call s_θ the original term, otherwise, we call s_θ the virtual term. In general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear in the actual calculation[25].

Consider two linguistic terms s_α, $s_\beta \in \bar{S}$, $\mu > 0$, the operations are defined as follows[26]:

1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$,
2) $\mu s_\alpha = s_{\mu\alpha}$,
3) $s_\alpha / s_\beta = s_{\alpha/\beta}$.

2.3 The Single Valued Neutrosophic Linguistic Set

Definition 4 (see [24]) Let X be a finite universal set. A single valued neutrosophic linguistic set (SVNLS) in X is defined as follows:

$$A = \{< x, [s_{\theta(x)}, (T_A(x), I_A(x), F_A(x))] > |x \in X\},$$

where $s_{\theta(x)} \in S$, $T_A(x) \in [0, 1]$, $I_A(x) \in [0, 1]$, and $F_A(x) \in [0, 1]$, with the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for any $x \in X$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent, respectively, the truth-membership degree, the indeterminacy-membership degree, and the falsity-membership degree of the element x in X to the linguistic variable $s_{\theta(x)}$.

For convenience, we can use $a_1 =< s_{\theta(a_1)}(T(a_1), I(a_1), F(a_1)) >$ to represent an element in a single valued neutrosophic linguistic set (SVNLS) and call it a single valued neutrosophic linguistic number (SVNLN).

Definition 5 (see [24]) Let $a_1 =< s_{\theta(a_1)}, (T(a_1), I(a_1), F(a1)) >$ and $a_2 =< s_{\theta(a_2)}, (T(a_2), I(a_2), F(a_2)) >$ be two SVNLNs and $\lambda \ge 0$, then the operations of SVNLNs are defined as follows:

1) $\lambda a_1 =< s_{\lambda\theta(a_1)}, (1 - (1 - T(a_1))^\lambda, I^\lambda(a_1), F^\lambda(a_1)) >$,
2) $a_1^\lambda =< s_{\theta^\lambda(a_1)}, (T^\lambda(a_1), 1 - (1 - I(a_1))^\lambda, 1 - (1 - F(a_1))^\lambda) >$,
3) $a_1 \oplus a_2 =< s_{\theta(a_1)+\theta(a_2)}, (T(a_1) + T(a_2) - T(a_1)T(a_2), I(a_1)I(a_2), F(a_1)F(a_2) >$,
4) $a_1 \otimes a_2 =< s_{\theta(a_1)\times\theta(a_2)}, (T(a_1)T(a_2), I(a_1)+I(a_2)-I(a_1)I(a_2), F(a_1)+F(a_2)-F(a_1)F(a_2) >$.

Furthermore, for any two SVNLNs

$$a_1 =< s_{\theta(a_1)}, (T(a_1), I(a_1), F(a_1)) >, \quad a_2 =< s_{\theta(a_2)}(T(a_2), I(a_2), F(a_2)) >,$$

and any real numbers λ, $\lambda_1 \geq 0, \lambda_2 \geq 0$, then, there are the following properties:

1) $a_1 \oplus a_2 = a_2 \oplus a_1$,
2) $a_1 \otimes a_2 = a_2 \otimes a_1$,
3) $\lambda(a_1 \oplus a_2) = \lambda a_1 \oplus \lambda a_2$,
4) $\lambda_1 a_1 \oplus \lambda_2 a_1 = (\lambda_1 + \lambda_2) a_1$,
5) $a_1^{\lambda_1} \otimes a_1^{\lambda_2} = a_1^{\lambda_1+\lambda_2}$,
6) $a_1^{\lambda_1} \otimes a_2^{\lambda_1} = (a_1 \otimes a_2)^{\lambda_1}$.

Definition 6 Let $a_1 =< s_{\theta(a_1)}, (T(a_1), I(a_1), F(a_1)) >$ and $a_2 =< s_{\theta(a_2)}(T(a_2), I(a_2), F(a_2)) >$ be two SVNLNs, then the normalized Hamming distance measure between a_1 and a_2 is define as

$$\begin{aligned} & D(a_1, a_2) \\ = & \frac{1}{3 \times \tau}(|\theta(a_1)T(a_1) - \theta(a_2)T(a_2)| + |\theta(a_1)T(a_1) - \theta(a_2)T(a_2)| + |\theta(a_1)T(a_1) - \theta(a_2)T(a_2)|). \end{aligned}$$

Definition 7 (see [12]) Let $a_1 =< s_{\theta(a_1)}, (T(a_1), I(a_1), F(a_1)) >$ and $a_2 =< s_{\theta(a_2)}, (T(a_2), I(a_2), F(a_2)) >$ be two SVNLNs, then the similarity measure based on Hamming distance between a_1 and a_2 is define as:

$$S(a_1, a_2) = \frac{1}{1 + D(a_1, a_2)}.$$

2.4 Entropy of Single Valued Neutrosophic Linguistic Set

The entropy of SVNLS was defined to determine the attribute weights based on the intuitionistic fuzzy entropy proposed by literature [27].

Definition 8 Let $X = (x_1, x_2, \cdots x_n)$ be a finite universal set. A SVNLS in X is $A = \{< x, [s_{\theta(x)}, (T_A(x), I_A(x), F_A(x))] > | x \in X\}$, then the entropy of SVNLS is define as follows:

$$E(A) = 1 - \frac{1}{n} \sum_{x_i \in X} ((T_A(x_i) + F_A(x_i)) \cdot |I_A(x_i) - I_{A^c}(x_i)|) \frac{\theta(x_i)}{\tau}.$$

In the evaluation matrix of SVNLS $F(a_{ij})$, here a_{ij} represents the evaluation of alternative x_1 with respect to an attribute c_j, the entropy of attribute weights can be calculated based on formula $E(A)$. The entropy value represents the uncertainty of attribute value, with the greater of entropy value, the attribute value will be more uncertainty. Then weights can be derived from the following formula:

$$\omega_j = \frac{1 - E(x_j)}{\sum_j^n (1 - E(x_j))}.$$

3 The Generalized Single Valued Neutrosophic Linguistic Operators

Based on the generalized intuitionistic fuzzy operators proposed by the literature [28] and Definition 5, three generalized single valued neutrosophic linguistic operators were introduced in the following section.

Definition 9 Generalized single valued neutrosophic linguistic weighted averaging (GSVNLWA) operator

Let $a_j = \langle s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j))\rangle$ $(j = 1, 2, \cdots, n)$ be a collection of SVNLNs, and GSVNLWA: $\Omega^n \to \Omega$, if

$$\text{GSVNLWA}(a_1, a_2, \cdots, a_n) = (\omega_1 a_1^\lambda + \omega_2 a_2^\lambda + \cdots + \omega_n a_n^\lambda)^{1/\lambda} = \left(\sum_{j=1}^n \omega_j a_j^\lambda\right)^{1/\lambda}. \tag{1}$$

Then the function GSVNLWA is called a GSVNLWA operator, where $s_\theta \in S$ is a linguistic variable, $\lambda > 0$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is a weight vector associated with the GSVNLWA operator, with $\omega_j \in [0, 1]$, and $\sum_{j=1}^n \omega_j$=1.

Theorem 1 *Let $a_j = \langle s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j))\rangle$ $(j = 1, 2, \cdots, n)$ be a collection of SVNLNs, then their aggregated value by using the $GSVNLWA$ also an SVNLN, and*

$$\begin{aligned}&\text{GSVNLWA}(a_1, a_2, \cdots, a_n)\\ =&\left\langle s_{(\sum_{j=1}^n \omega_j \theta^\lambda(a_j))^{1/\lambda}}, \left(\left(1 - \prod_{j=1}^n (1 - T^\lambda(a_j))^{\omega_j}\right)^{1/\lambda},\right.\right.\\ &\left.\left. 1 - \left(1 - \prod_{j=1}^n (1 - (1 - I(a_j))^\lambda)^{\omega_j}\right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^n (1 - (1 - F(a_j))^\lambda)^{\omega_j}\right)^{1/\lambda}\right)\right\rangle, \end{aligned}\tag{2}$$

where $\lambda > 0$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is a weight vector associated with the GSVNLWA operator, with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j$=1.

The parameter λ plays a regulatory role during the information aggregation process. When the parameter λ was set to a special number, the GSVNLWA operator can be reduced.

For example, when $\lambda = 1$, then,

$$\begin{aligned}&\text{GSVNLWA}(a_1, a_2, \cdots a_n)\\ =&\left\langle s_{\sum_{j=1}^n \omega_j \theta(a_j)}, \left(1 - \prod_{j=1}^n (1 - T(a_j))^{\omega_j}\right), \prod_{j=1}^n I^{\omega_j}(a_j), \prod_{j=1}^n F^{\omega_j}(a_j)\right\rangle. \end{aligned}\tag{3}$$

Proof The Equation (2) can be derived from Equation (1) using Definition 5 in Section 2.3. In the following, we first prove

$$\begin{aligned}&\omega_1 a_1^\lambda + \omega_2 a_2^\lambda + \cdots + \omega_n a_n^\lambda\\ =&\left\langle s_{\sum_{j=1}^n \omega_j \theta^\lambda(a_j)}, \left(1 - \prod_{j=1}^n (1 - T^\lambda(a_j))^{\omega_j},\right.\right.\\ &\left.\left.\prod_{j=1}^n (1 - (1 - I(a_j))^\lambda)^{\omega_j}, \prod_{j=1}^n (1 - (1 - F(a_j))^\lambda)^{\omega_j}\right)\right\rangle, \end{aligned}\tag{4}$$

by using mathematical induction on n.

1) For $n = 2$: Since

$$a_1^\lambda = \left\langle s_{\theta^\lambda(a_1)}, (T^\lambda(a_1), 1 - (1 - I(a_1))^\lambda, 1 - (1 - I(a_1))^\lambda)\right\rangle,$$

$$a_2^\lambda = \left\langle s_{\theta^\lambda(a_2)}, (T^\lambda(a_2), 1 - (1 - I(a_2))^\lambda, 1 - (1 - I(a_2))^\lambda)\right\rangle,$$

then

$$\begin{aligned}
&\omega_1 a_1^\lambda + \omega_2 a_2^\lambda \\
=&\left\langle s_{\sum_{j=1}^{2}\omega_j\theta^\lambda(a_j)}, 1-\prod_{j=1}^{2}(1-T^\lambda(a_j))^{\omega_j},\right. \\
&\left.\prod_{j=1}^{2}(1-(1-I(a_j))^\lambda)^{\omega_j}, \prod_{j=1}^{2}(1-(1-F(a_j))^\lambda)^{\omega_j}\right\rangle.
\end{aligned}$$

2) If Equation (4) holds for $n=k$, that is,

$$\begin{aligned}
&\omega_1 a_1^\lambda + \omega_2 a_2^\lambda + \cdots + \omega_k a_k^\lambda \\
=&\left\langle s_{\sum_{j=1}^{k}\omega_j\theta^\lambda(a_j)}, \left(1-\prod_{j=1}^{k}(1-T^\lambda(a_j))^{\omega_j},\right.\right. \\
&\left.\left.\prod_{j=1}^{k}(1-(1-I(a_j))^\lambda)^{\omega_j}, \prod_{j=1}^{k}(1-(1-F(a_j))^\lambda)^{\omega_j}\right)\right\rangle,
\end{aligned}$$

then, when $n=k+1$, by the operational laws in Definition 5,

$$\begin{aligned}
&\omega_1 a_1^\lambda + \omega_2 a_2^\lambda + \cdots + \omega_{k+1} a_{k+1}^\lambda \\
=&\left\langle s_{\sum_{j=1}^{k}\omega_j\theta^\lambda(a_j)}, \left(1-\prod_{j=1}^{k}(1-T^\lambda(a_j))^{\omega_j}, \prod_{j=1}^{k}(1-(1-I(a_j))^\lambda)^{\omega_j},\right.\right. \\
&\left.\left.\prod_{j=1}^{k}(1-(1-F(a_j))^\lambda)^{\omega_j}\right)\right\rangle \\
&+\langle s_{\omega_{k+1}\theta^\lambda(a_{k+1})}, (1-(1-T^\lambda(a_{k+1}))^{\omega_{k+1}}, (1-(1-I(a_{k+1}))^\lambda)^{\omega_{k+1}}, \\
&(1-(1-F(a_{k+1}))^\lambda)^{\omega_{k+1}})\rangle \\
=&\left\langle s_{\sum_{j=1}^{k+1}\omega_j\theta^\lambda(a_j)}, \left(1-\prod_{j=1}^{k+1}(1-T^\lambda(a_j))^{\omega_j}, \prod_{j=1}^{k+1}(1-(1-I(a_j))^\lambda)^{\omega_j},\right.\right. \\
&\left.\left.\prod_{j=1}^{k+1}(1-(1-F(a_j))^\lambda)^{\omega_j}\right)\right\rangle,
\end{aligned}$$

i.e., Equation (4) holds for $n=k+1$. Thus, Equation (4) holds for all n.

Then

$$\begin{aligned}
&\mathrm{GSVNLWA}(a_1, a_2, \cdots, a_n) \\
=&(\omega_1 a_1^\lambda + \omega_2 a_2^\lambda + \cdots + \omega_n a_n^\lambda)^{1/\lambda} \\
=&\left(\left\langle s_{\sum_{j=1}^{k+1}\omega_j\theta^\lambda(a_j)}, \left(1-\prod_{j=1}^{k+1}(1-T^\lambda(a_j))^{\omega_j}, \prod_{j=1}^{k+1}(1-(1-I(a_j))^\lambda)^{\omega_j},\right.\right.\right. \\
&\left.\left.\left.\prod_{j=1}^{k+1}(1-(1-F(a_j))^\lambda)^{\omega_j}\right)\right\rangle\right)^{1/\lambda} \\
=&\left\langle s_{(\sum_{j=1}^{n}\omega_j\theta^\lambda(a_j))^{1/\lambda}}, \left(\left(1-\prod_{j=1}^{n}(1-T^\lambda(a_j))^{\omega_j}\right)^{1/\lambda},\right.\right.
\end{aligned}$$

$$1-\left(1-\prod_{j=1}^{n}\left(1-(1-I(a_j))^{\lambda}\right)^{\omega_j}\right)^{1/\lambda},$$
$$1-\left(1-\prod_{j=1}^{n}\left(1-(1-F(a_j))^{\lambda}\right)^{\omega_j}\right)^{1/\lambda}\Bigg)\Bigg\rangle.$$

Theorem 2 (Idempotency) *Let* $a_j =< s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ *be a collection of SVNLNs, where* $\lambda > 0$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ *is a weight vector associated with the GSVNLWA operator, with* $\omega_j \in [0, 1]$, *and* $\sum_{j=1}^{n}\omega_j=1$, *if* $a_j (j = 1, 2, \cdots, n)$ *is equal, i.e.,* $a_j = a$ *for* $j = 1, 2, \cdots, n$, *then* $GSVNLWA(a_1, a_2, \cdots, a_n) = a$.

Proof By Definition 5, we have

$$\begin{aligned}GSVNLWA(a_1, a_2, \cdots, a_n) &= (\omega_1 a_1^{\lambda} + \omega_2 a_2^{\lambda} + \cdots + \omega_n a_n^{\lambda})^{1/\lambda}\\ &= (\omega_1 a^{\lambda} + \omega_2 a^{\lambda} + \cdots + \omega_n a^{\lambda})^{1/\lambda}\\ &= ((\omega_1 + \omega_2 + \cdots + \omega_n) a^{\lambda})^{1/\lambda}\\ &= \left(\left(\sum_{j=1}^{n}\omega_j\right)a^{\lambda}\right)^{1/\lambda} = a.\end{aligned}$$

Theorem 3 (Boundedness) *Let* $a_j =< s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ *be a collection of SVNLNs, where* $\lambda > 0$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ *is a weight vector associated with the GSVNLWA operator, with* $\omega_j \in [0, 1]$, *and* $\sum_{j=1}^{n}\omega_j=1$, *and let* $a_{\min} = \min(a_1, a_2, \cdots, a_n)$, $a_{\max} = \max(a_1, a_2, \cdots, a_n)$, *then*

$$a_{\min} \le \mathrm{GSVNLWA}(a_1, a_2, \cdots, a_n) \le a_{\max}. \tag{5}$$

Proof Since $a_{\min} = \min(a_1, a_2, \cdots, a_n)$ and $a_{\max} = \max(a_1, a_2, \cdots, a_n)$, then

$$\begin{aligned}&a_{\min} \le a_j \le a_{\max},\\ &a_{\min}^{\lambda} \le a_j^{\lambda} \le a_{\max}^{\lambda},\\ &\sum_{j=1}^{n}\omega_j a_{\min}^{\lambda} \le \sum_{j=1}^{n}\omega_j a_j^{\lambda} \le \sum_{j=1}^{n}\omega_j a_{\max}^{\lambda},\\ &a_{\min}^{\lambda} \le \sum_{j=1}^{n}\omega_j a_j^{\lambda} \le a_{\max}^{\lambda},\\ &(a_{\min}^{\lambda})^{1/\lambda} \le \left(\sum_{j=1}^{n}\omega_j a_j^{\lambda}\right)^{1/\lambda} \le (a_{\max}^{\lambda})^{1/\lambda},\\ &a_{\min} \le \mathrm{GSVNLWA}(a_1, a_2, \cdots, a_n) \le a_{\max}.\end{aligned}$$

Thus, Equation (5) always holds.

Theorem 4 (Monotonity) *Let* $a_j =< s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ *be a collection of SVNLNs, where* $\lambda > 0$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ *is a weight vector associated with the GSVNLWA operator, with* $\omega_j \in [0, 1]$, *and* $\sum_{j=1}^{n}\omega_j=1$, *and if* $a_j \le a_j^*$, *then*

$$\mathrm{GSVNLWA}(a_1, a_2, \cdots, a_n) \le \mathrm{GSVNLWA}(a_1^*, a_2^*, \cdots, a_n^*). \tag{6}$$

Proof Since $a_j \le a_j^*$, then

$$\begin{aligned}
&a_j^\lambda \le (a_j^*)^\lambda,\\
&\sum_{j=1}^n \omega_j a_j^\lambda \le \sum_{j=1}^n \omega_j (a_j^*)^\lambda,\\
&\left(\sum_{j=1}^n \omega_j a_j^\lambda\right)^{1/\lambda} \le \left(\sum_{j=1}^n \omega_j (a_j^*)^\lambda\right)^{1/\lambda},\\
&\mathrm{GSVNLWA}(a_1, a_2, \cdots, a_n) \le \mathrm{GSVNLWA}(a_1^*, a_2^*, \cdots, a_n^*).
\end{aligned}$$

Thus, Equation (6) always holds.

Definition 10 Generalized single valued neutrosophic linguistic ordered weighted averaging (GSVNLOWA) operator

Let $a_j =< s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ be a collection of SVNLNs, and GSVNLOWA: $\Omega^n \to \Omega$, if

$$\mathrm{GSVNLOWA}(a_1, a_2, \cdots, a_n) = \left(\sum_{j=1}^n w_j a_{\sigma(j)}^\lambda\right)^{1/\lambda}, \tag{7}$$

where $s_\theta \in S$ is a linguistic variable, $\lambda > 0$, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is an associated weight vector such that $w_j \in [0, 1]$, $j = 1, 2, \cdots, n$, and $\sum_{j=1}^n w_j$=1, $a_{\sigma(j)}$ is the jth largest of a_j, $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, with the condition $a_{\sigma(j-1)} \ge a_{\sigma(j)}$, then the function GSVNLOWA is called a GSVNLOWA operator.

Theorem 5 *Let $a_j =< s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ be a collection of SVNLNs, then their aggregated value by using the $GSVNLOWA$ also an SVNLN, and*

$$\begin{aligned}
&\mathrm{GSVNLOWA}(a_1, a_2, \cdots, a_n)\\
&= \left\langle s_{(\sum_{j=1}^n \omega_j \theta^\lambda(a_{\sigma(j)}))^{1/\lambda}}, \left(\left(1 - \prod_{j=1}^n (1 - T^\lambda(a_{\sigma(j)}))^{w_j}\right)^{1/\lambda},\right.\right.\\
&\quad 1 - \left(1 - \prod_{j=1}^n (1 - (1 - I(a_{\sigma(j)}))^\lambda)^{w_j}\right)^{1/\lambda},\\
&\quad \left.\left. 1 - \left(1 - \prod_{j=1}^n (1 - (1 - F(a_{\sigma(j)}))^\lambda)^{w_j}\right)^{1/\lambda}\right)\right\rangle,
\end{aligned} \tag{8}$$

where $\lambda > 0$, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is an associated weight vector such that $w_j \in [0, 1]$, $j = 1, 2, \cdots, n$, and $\sum_{j=1}^n w_j$=1, $a_{\sigma(j)}$ is the jth largest of a_j, $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, with the condition $a_{\sigma(j-1)} \ge a_{\sigma(j)}$.

The parameter λ plays a regulatory role during the information aggregation process. When the parameter λ was set to a special number, the GSVNLOWA operator can be reduced.

For example, when $\lambda = 1$, then,

$$\begin{aligned}
&\mathrm{GSVNLOWA}(a_1, a_2, \cdots, a_n)\\
&= \left\langle s_{\sum_{j=1}^n \omega_j \theta(a_{\sigma(j)})}, \left(1 - \prod_{j=1}^n (1 - T(a_{\sigma(j)}))^{w_j}, \prod_{j=1}^n (I(a_{\sigma(j)}))^{w_j}, \prod_{j=1}^n (F(a_{\sigma(j)}))^{w_j}\right)\right\rangle.
\end{aligned} \tag{9}$$

The GSVNLOWA operator has some properties similar to those of the GSVNLWA operator.

Definition 11 Generalized single valued neutrosophic linguistic hybrid averaging (GSVNLHA) operators

Let $a_j = < s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ be a collection of SVNLNs, and GSVNLHA: $\Omega^n \to \Omega$, if

$$\mathrm{GSVNLHA}(a_1, a_2, \cdots, a_n) = \left(\sum_{j=1}^{n} w_j \dot{a}_{\sigma(j)}^{\lambda} \right)^{1/\lambda}, \tag{10}$$

where $s_\theta \in S$ is a linguistic variable, $\lambda > 0$, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is an associated weight vector such that $w_j \in [0, 1]$, $j = 1, 2, \cdots, n$, and $\sum_{j=1}^{n} w_j$=1, $\dot{a}_{\sigma(j)}$ is the jth largest of $\dot{a}_j (\dot{a}_j = n\omega_j a_j \ j = 1, 2, \cdots, n)$, $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, with the condition $\dot{a}_{\sigma(j-1)} \geq \dot{a}_{\sigma(j)}$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is the weight vector of $a_j (j = 1, 2, \cdots, n)$ with $\omega_j \in [0, 1]$, and $\sum_{j=1}^{n} \omega_j$=1 and n is the balancing coefficient, which plays a role of balance, then the function GSVNLHA is called a GSVNLHA operator.

Theorem 6 *Let $a_j = < s_{\theta(a_j)}, (T(a_j), I(a_j), F(a_j)) >$ $(j = 1, 2, \cdots, n)$ be a collection of SVNLNs, then their aggregated value by using the GSVNLHA also an SVNLN, and*

$$\begin{aligned} \mathrm{GSVNLHA}(a_1, a_2, \cdots, a_n) = \Bigg\langle & s_{(\sum_{j=1}^{n} \omega_j \theta^{\lambda}(\dot{a}_{\sigma(j)}))^{1/\lambda}}, \Bigg(\left(1 - \prod_{j=1}^{n} (1 - T^{\lambda}(\dot{a}_{\sigma(j)}))^{w_j} \right)^{1/\lambda}, \\ & 1 - \left(1 - \prod_{j=1}^{n} (1 - (1 - I(\dot{a}_{\sigma(j)}))^{\lambda})^{w_j} \right)^{1/\lambda}, \\ & 1 - \left(1 - \prod_{j=1}^{n} (1 - (1 - F(\dot{a}_{\sigma(j)}))^{\lambda})^{w_j} \right)^{1/\lambda} \Bigg) \Bigg\rangle, \end{aligned} \tag{11}$$

where $\lambda > 0$, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is an associated weight vector of GSVNLHA operator, with $w_j \in [0, 1]$, $j = 1, 2, \cdots, n$, and $\sum_{j=1}^{n} w_j$=1. $\dot{a}_{\sigma(j)}$ is the jth largest of the weighted SVNLNs $\dot{a}_j (\dot{a}_j = n\omega_j a_j,\ j = 1, 2, \cdots, n)$, the condition $\dot{a}_{\sigma(j-1)} \geq \dot{a}_{\sigma(j)}$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is the weight vector of a_j $(j = 1, 2, \cdots, n)$, with $\omega_j \in [0, 1]$, and $\sum_{j=1}^{n} \omega_j = 1$, and n is the balancing coefficient, which plays a role of balance.

The parameter λ plays a regulatory role during the information aggregation process. When the parameter λ was set to a special number, the GSVNLHA operator can be reduced.

For example, when $\lambda = 1$, then,

$$\begin{aligned} \mathrm{GSVNLHA}(a_1, a_2, \cdots, a_n) = \Bigg\langle & s_{\sum_{j=1}^{n} w_j \theta(\dot{a}_{\sigma(j)})}, \Bigg(1 - \prod_{j=1}^{n} (1 - T(\dot{a}_{\sigma(j)}))^{w_j}, \\ & \prod_{j=1}^{n} (I(\dot{a}_{\sigma(j)})^{w_j}, \prod_{j=1}^{n} F(\dot{a}_{\sigma(j)})^{w_j} \Bigg) \Bigg\rangle. \end{aligned} \tag{12}$$

The GSVNLHA operator has some properties similar to those of the GSVNLWA operator.

4 Group Decision Making Method Based on the Entropy of Neutrosophic Linguistic Sets and the GSVNLOWA and GSVNLHA Operators

This section proposes a method for multiple attribute group decision making problems by means of the entropy of neutrosophic linguistic sets and the GSVNLOWA and GSVNLHA operators under single valued neutrosophic linguistic environment.

Considering the multiple attribute group decision making problems based on SVNLNs, let $A = \{a_1, a_2, \cdots, a_m\}$ be the set of alternatives, and $C = \{c_1, c_2, \cdots, c_n\}$ be the set of attributes. ω_j is the weight of the attribute c_j $(j = 1, 2, \cdots, n)$, where $\omega_j \in [0, 1]$ $(j = 1, 2, \cdots, n)$, and $\sum_{j=1}^{n} \omega_j = 1$. Suppose that $D = \{d_1, d_2, \cdots, d_l\}$ is the set of decision makers, and $e_k(k = 1, 2, \cdots, l)$ is a weight of decision maker d_k with $e_k \in [0, 1]$ $(k = 1, 2, \cdots, l)$, $\sum_{k=1}^{l} e_k=1$. Suppose that $R^k = (a_{ij}^k)_{m\times n}$ is the decision matrix, where $a_{ij}^k =< s_{\theta(a_{ij})}^k, (T^k(a_{ij}), I^k(a_{ij}), F^k(a_{ij})) >$ takes the form of SVNLN and $s_{\theta(a_{ij})}^k \in S$, $T^k(a_{ij})$, $I^k(a_{ij})$, $F^k(a_{ij}) \in [0,1]$, $0 \le T^k(a_{ij}) + I^k(a_{ij}) + F^k(a_{ij}) \le 3$, which describes the evaluation information of the attribute c_j with respect to the alternative a_i given by the decision maker d_k. Then, we can rank the order of the alternatives and obtain the best alternatives based on the given information.

The decision procedure for the proposed method is as follows:

Step 1 Transform the decision matrix.

Generally, attributes can be categorized into two types: benefit attributes and cost attributes. In order to eliminate the influence of the attribute types, we need to convert the cost type to the benefit type. The SVNLN decision matrix $R^k = (a_{ij}^k)_{m\times n}$ can be transformed into a normalized SVNLN decision matrix $\tilde{R}^k = (\tilde{a}_{ij}^k)_{m\times n}$, where $\tilde{a}_{ij}^k = (a_{ij}^k)^c = \langle s_{\tau-\theta(a_{ij})}^k, (T^k(a_{ij}), I^k(a_{ij}), F^k(a_{ij}))\rangle$.

Step 2 Determine the weights of attributes.

Based on the entropy of neutrosophic linguistic set, we can obtain the attribute weight vector $\omega^k = (\omega_1^k, \omega_2^k, \cdots, \omega_n^k)$ of each normalized decision matrix. Then the final attribute weight ω_j can be determined by weighted average, where $\omega_j = \sum_{k=1}^{l} e_k\omega_j^k$.

Step 3 Calculate the comprehensive evaluation value of each alternative.

According to the GSVNLWA operator, we can calculate the comprehensive evaluation value a_i^k of each alternative for each decision maker, where $a_i^k = \text{GSVNLWA}(a_{i1}^k, a_{i2}^k, \cdots, a_{in}^k) = (\sum_{j=1}^{n} \omega_j a_j^\lambda)^{1/\lambda}$.

Step 4 Aggregate the decision information of each decision maker to the collective information by GSVNLHA operator, and get the group comprehensive evaluation value a_i of each alternative, where $a_i = \text{GSVNLHA}(a_i^1, a_i^2, \cdots a_i^l) = \left(\sum_{j=1}^{n} w_j \dot{a}_{\sigma(j)}^\lambda\right)^{1/\lambda}$, and $\dot{a}_{\sigma(j)}$ is the jth largest of $\dot{a}_j(\dot{a}_j = le_k(a_i^k))$.

Step 5 Rank $S(a_i, a^+)$ in descending order according to the Definition 6 and Definition 7 described in 2.3, where the ideal solution $a^+ = \langle s_6, (1, 0, 0)\rangle$. Generally speaking, the greater the similarity value, the better the alternative.

Step 6 End.

5 Illustrative Example

An illustrative example is used to demonstrate the application of the proposed decision making method under single valued neutrosophic linguistic environment. It is about investment alternatives for a multiple attribute group decision making problem adapted from [24]. An investment company wants to invest a sum of money in the best option. There is panel with four possible alternatives to invest the money: 1) a_1 is a car company; 2) a_2 is a food company; 3) a_3 is a computer company; 4) a_4 is an arms company. The investment company must take a decision according to the following three attributes: 1) c_1 is the risk; 2) c_2 is the growth; 3) c_3 is the environmental impact. The four possible alternatives of $a_i (i = 1, 2, 3, 4)$ are to be evaluated using the single valued neutrosophic linguistic information by some decision makers or experts under the three attributes of $c_j (j = 1, 2, 3)$. Assume that the set of three experts is $D = \{d_1, d_2, d_3\}$, and the weight vector of the three experts is $e = (0.3700, 0.3300, 0.3000)^{\mathrm{T}}$. The experts evaluate these alternatives by SVNLNs under the linguistic term set $S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor, $s_3 =$ medium, $s_4 =$ good, $s_5 =$ very good, $s_6 =$ extremely good.

The evaluation of an alternative a_i $(i = 1, 2, 3, 4)$ with respect to an attribute c_j $(j = 1, 2, 3)$ is obtained from the experts. The evaluation values can be represented by SVNLNs. For example, the SVNLNs of an alternative a_1 with respect to an attribute c_1 is given as $< s_4^1, (0.4, 0.2, 0.3) >$ by the expert d_1, which indicates that the mark of the alternative a_1 with respect to an attribute c_1 is about the linguistic term s_4 with the satisfaction degree 0.4, dissatisfaction degree 0.3, and indeterminacy degree 0.2. Thus, evaluation matrix of three experts as follows:

$$R^1 = \begin{bmatrix} \langle s_4^1, (0.4, 0.2, 0.3)\rangle & \langle s_4^1, (0.4, 0.2, 0.3)\rangle & \langle s_2^1, (0.3, 0.2, 0.5)\rangle \\ \langle s_5^1, (0.6, 0.1, 0.2)\rangle & \langle s_4^1, (0.6, 0.1, 0.2)\rangle & \langle s_2^1, (0.5, 0.2, 0.2)\rangle \\ \langle s_3^1, (0.3, 0.2, 0.3)\rangle & \langle s_3^1, (0.5, 0.2, 0.3)\rangle & \langle s_3^1, (0.5, 0.3, 0.1)\rangle \\ \langle s_4^1, (0.7, 0.0, 0.1)\rangle & \langle s_3^1, (0.6, 0.1, 0.2)\rangle & \langle s_3^1, (0.3, 0.1, 0.2)\rangle \end{bmatrix},$$

$$R^2 = \begin{bmatrix} \langle s_4^2, (0.4, 0.3, 0.4)\rangle & \langle s_5^2, (0.5, 0.1, 0.2)\rangle & \langle s_1^2, (0.3, 0.1, 0.6)\rangle \\ \langle s_5^2, (0.7, 0.2, 0.3)\rangle & \langle s_4^2, (0.7, 0.2, 0.3)\rangle & \langle s_1^2, (0.6, 0.2, 0.2)\rangle \\ \langle s_5^2, (0.4, 0.2, 0.4)\rangle & \langle s_5^2, (0.6, 0.3, 0.4)\rangle & \langle s_2^2, (0.6, 0.1, 0.3)\rangle \\ \langle s_3^2, (0.8, 0.1, 0.2)\rangle & \langle s_4^2, (0.7, 0.2, 0.3)\rangle & \langle s_2^2, (0.4, 0.2, 0.2)\rangle \end{bmatrix},$$

$$R^3 = \begin{bmatrix} \langle s_5^3, (0.5, 0.2, 0.3)\rangle & \langle s_5^3, (0.6, 0.2, 0.4)\rangle & \langle s_1^3, (0.2, 0.1, 0.6)\rangle \\ \langle s_4^3, (0.5, 0.2, 0.3)\rangle & \langle s_3^3, (0.7, 0.2, 0.2)\rangle & \langle s_1^3, (0.7, 0.2, 0.1)\rangle \\ \langle s_5^3, (0.5, 0.1, 0.3)\rangle & \langle s_4^3, (0.6, 0.1, 0.3)\rangle & \langle s_2^3, (0.6, 0.2, 0.1)\rangle \\ \langle s_3^3, (0.6, 0.1, 0.2)\rangle & \langle s_3^3, (0.5, 0.2, 0.2)\rangle & \langle s_3^3, (0.4, 0.1, 0.1)\rangle \end{bmatrix}.$$

Step 1 Normalize decision matrices. Here c_1 and c_2 are benefit attributes, and c_3 is a cost attribute, so c_3 need to be standardized, then the normalized matrices can be obtained as follows:

$$\tilde{R}^1=\begin{bmatrix}\left\langle s_4^1,(0.4,0.2,0.3)\right\rangle & \left\langle s_4^1,(0.4,0.2,0.3)\right\rangle & \left\langle s_4^1,(0.3,0.2,0.5)\right\rangle\\ \left\langle s_5^1,(0.6,0.1,0.2)\right\rangle & \left\langle s_4^1,(0.6,0.1,0.2)\right\rangle & \left\langle s_4^1,(0.5,0.2,0.2)\right\rangle\\ \left\langle s_3^1,(0.3,0.2,0.3)\right\rangle & \left\langle s_3^1,(0.5,0.2,0.3)\right\rangle & \left\langle s_3^1,(0.5,0.3,0.1)\right\rangle\\ \left\langle s_4^1,(0.7,0.0,0.1)\right\rangle & \left\langle s_3^1,(0.6,0.1,0.2)\right\rangle & \left\langle s_3^1,(0.3,0.1,0.2)\right\rangle\end{bmatrix},$$

$$\tilde{R}^2=\begin{bmatrix}\left\langle s_4^2,(0.4,0.3,0.4)\right\rangle & \left\langle s_5^2,(0.5,0.1,0.2)\right\rangle & \left\langle s_5^2,(0.3,0.1,0.6)\right\rangle\\ \left\langle s_5^2,(0.7,0.2,0.3)\right\rangle & \left\langle s_4^2,(0.7,0.2,0.3)\right\rangle & \left\langle s_5^2,(0.6,0.2,0.2)\right\rangle\\ \left\langle s_5^2,(0.4,0.2,0.4)\right\rangle & \left\langle s_5^2,(0.6,0.3,0.4)\right\rangle & \left\langle s_4^2,(0.6,0.1,0.3)\right\rangle\\ \left\langle s_3^2,(0.8,0.1,0.2)\right\rangle & \left\langle s_4^2,(0.7,0.2,0.3)\right\rangle & \left\langle s_4^2,(0.4,0.2,0.2)\right\rangle\end{bmatrix},$$

$$\tilde{R}^3=\begin{bmatrix}\left\langle s_5^3,(0.5,0.2,0.3)\right\rangle & \left\langle s_5^3,(0.6,0.2,0.4)\right\rangle & \left\langle s_5^3,(0.2,0.1,0.6)\right\rangle\\ \left\langle s_4^3,(0.5,0.2,0.3)\right\rangle & \left\langle s_3^3,(0.7,0.2,0.2)\right\rangle & \left\langle s_5^3,(0.7,0.2,0.1)\right\rangle\\ \left\langle s_5^3,(0.5,0.1,0.3)\right\rangle & \left\langle s_4^3,(0.6,0.1,0.3)\right\rangle & \left\langle s_4^3,(0.6,0.2,0.1)\right\rangle\\ \left\langle s_3^3,(0.6,0.1,0.2)\right\rangle & \left\langle s_3^3,(0.5,0.2,0.2)\right\rangle & \left\langle s_3^3,(0.4,0.1,0.1)\right\rangle\end{bmatrix}.$$

Step 2 By Definition 8 we can calculate the weight vector of attribute for each decision matrix:

$$\omega^1=(0.4111,0.3411,0.2478),\quad \omega^2=(0.3131,0.3310,0.3559),\quad \omega^3=(0.3538,0.3283,0.3178).$$

Then the final attribute weight is the followings:

$$\omega=(0.3616,0.3340,0.3045).$$

Step 3 By Equation (3) we can calculate the comprehensive evaluation value of each alternative for each decision matrix as follows:

$$a_1^1=\left\langle s^1_{4.0004},(0.3712,0.2000,0.3504)\right\rangle,\quad a_2^1=\left\langle s^1_{4.3620},(0.5719,0.1235,0.2000)\right\rangle,$$

$$a_3^1=\left\langle s^1_{3.0003},(0.4353,0.2262,0.2147)\right\rangle,\quad a_4^1=\left\langle s^1_{3.3619},(0.5726,0.0000,0.1556)\right\rangle,$$

$$a_1^2=\left\langle s^2_{4.6389},(0.4083,0.1487,0.3590)\right\rangle,\quad a_2^2=\left\langle s^2_{4.6665},(0.6726,0.2000,0.2651\right\rangle,$$

$$a_3^2=\left\langle s^2_{4.6960},(0.5369,0.1854,0.3664\right\rangle,\quad a_4^2=\left\langle s^2_{3.6388},(0.6801,0.1556,0.2290\right\rangle,$$

$$a_1^3=\left\langle s^3_{5.0005},(0.4645,0.1619,0.4078)\right\rangle,\quad a_2^3=\left\langle s^3_{3.9709},(0.6392,0.2000,0.1875\right\rangle,$$

$$a_3^3=\left\langle s^3_{4.3620},(0.5664,0.1235,0.2147)\right\rangle,\quad a_4^3=\left\langle s^3_{3.0003},(0.5125,0.1260,0.1619)\right\rangle.$$

Step 4 By Equation (12) we can calculate the group comprehensive evaluation value of each alternative as follows:

$$a_1=\langle s_{4.5332},(0.4107,0.1650,0.3678\rangle,$$

$$a_2=\langle s_{4.4197},(0.6394,0.1764,0.2286)\rangle,$$

$$a_3=\langle s_{3.7953},(0.5004,0.1807,0.2332\rangle,$$

$$a_4 = \langle s_{3.4147}, (0.6195, 0.0000, 0.1920)\rangle,$$

where the associated weight vector is $w = (0.2429, 0.5142, 0.2429)^{\mathrm{T}}$, which determined by the literature [29]. And the sorting method is based on the distance between an alternative and the ideal solution $a^+ = \langle s_6, (1, 0, 0)\rangle$.

Step 5 By using Definition 6 and Definition 7, we can calculate the similarity value between the alternative a_i and ideal solution a^+ as follows:

$$S(a_1, a^+) = 0.7331, \quad S(a_2, a^+) = 0.7838, \quad S(a_3, a^+) = 0.7604, \quad S(a_4, a^+) = 0.7986.$$

According to the similarity value, the ranking order of the four alternatives is $a_4 \succ a_2 \succ a_3 \succ a_1$, and the best alternative is a_4, that is to say, the investment company should invest in arms companies to get the maximum benefits.

Obviously, the ranking orders and the best alternative in this paper are in accordance with ones of [24].

In addition, the method proposed in this paper differs from the existing single valued neutrosophic linguistic multiple attribute group decision making method[24], and has some special characteristics. First, the method of weight calculation is different, and the method based on entropy is more objective than the subjective assumption of the paper [24]. Second, the method of aggregation for decision information is different. The method proposed in this paper using GSVNLWA operator and GSVNLHA operator not only considers the importance of attributes, but also takes the importance of position into account, so it is more general than the method in [24] which only using SVNLWA operator. Last, the subscript of linguistic variable starts from zero in this paper. This representation is more accurate than the one in [24]. For example, in our paper, the $neg(s_2 = \text{poor}) = s_{6-2} = s_4 = \text{good}$ but in paper [24], the $neg(s_3 = \text{poor}) = s_{7-3} = s_4 = \text{medium}$, so it is not in conformity with the language habits. Therefore, our method is more reasonable and effective.

6 Conclusion

This paper proposed a group decision making method based on entropy of neutrosophic linguistic sets and generalized single valued neutrosophic linguistic operators. The entropy of neutrosophic linguistic sets was defined to determine the weight of attribute. It introduced three generalized single valued neutrosophic linguistic operators and analyzed their properties, and introduced the similarity measure method of single valued neutrosophic linguistic numbers. Finally, an illustrative example was given to demonstrate the application of the proposed method. Therefore, the proposed method enriched and developed the theory and method of group decision making problems and provided a new way to solving group decision making problems.

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