Question 409: Integrals and Fractals

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abstract

This note presents some definite integrals.

1. Intoduction. Some definite integrals.

$$\pi=\frac{3}{2}\int_0^1 \cos^{-1}\left(\frac{\left(2+\sqrt{5}\right)^{2x}-\left(5+2\sqrt{5}\right)}{4+2\sqrt{5}}\right)dx \tag{1}$$

$$\pi=6\int_0^1 \sin^{-1}\left(\frac{\left(5+2\sqrt{5}\right)-\left(2+\sqrt{5}\right)^{2x}}{4+2\sqrt{5}}\right)dx \tag{2}$$

$$\pi=\frac{4}{3}\int_0^1 \cos^{-1}\left(\frac{2z^{2x}-\left(z^2+1\right)}{z^2-1}\right)dx \tag{3}$$

$$\pi=4\int_0^1 \sin^{-1}\left(\frac{\left(z^2+1\right)-2z^{2x}}{z^2-1}\right)dx \tag{4}$$

In (3) , (4) :

$$z=\frac{11}{3}+\frac{2}{3}\left(199+3\sqrt{33}\right)^{1/3}+\frac{68}{3\left(199+3\sqrt{33}\right)^{1/3}} \tag{5}$$

$$\pi=\frac{7}{5}\int_0^1 \cos^{-1}\left(\frac{2z^{2x}-\left(z^2+1\right)}{z^2-1}\right)dx \tag{6}$$

$$\pi=\frac{14}{3}\int_0^1 \sin^{-1}\left(\frac{\left(z^2+1\right)-2z^{2x}}{z^2-1}\right)dx \tag{7}$$

In (6) , (7) :

$$z=\frac{7}{3}+\frac{1}{3}\left(388+12\sqrt{69}\right)^{1/3}+\frac{52}{3\left(388+12\sqrt{69}\right)^{1/3}} \tag{8}$$

$$\pi=\frac{6}{5}\int_0^1 \cos^{-1}\left(\frac{2z^{2x}-\left(z^2+1\right)}{z^2-1}\right)dx \tag{9}$$

$$\pi=3\int_0^1 \sin^{-1}\left(\frac{\left(z^2+1\right)-2z^{2x}}{z^2-1}\right)dx \tag{10}$$

In (9) , (10) : $z=57.7341...$ is root of the equation:

$$z^5-57z^4-42z^3-22z^2-7z-1=0 \tag{11}$$

2. The equation $z^5-57z^4-42z^3-22z^2-7z-1=0$.

The equation

$$f\left(x\right)=x^5-57x^4-42x^3-22x^2-7x-1=0 \tag{12}$$

Is not solvable by radicals. Galois group $G\left(f\right)$ is not soluble.

$$f\left(x\right)=0\Rightarrow\begin{cases}x_1=57.7341095724734413...\\ x_2=-0.0925...+i\times 0.4268...\\ x_3=-0.0925...-i\times 0.4268...\\ x_4=-0.2745...+i\times 0.1242...\\ x_5=-0.2745...-i\times 0.1242...\end{cases} \tag{13}$$

3. Relations

$$f\left(x\right)=x^5-57x^4-42x^3-22x^2-7x-1 \tag{14}$$

$$g\left(x\right)=\left(1+x\right)^6-64x^5 \tag{15}$$

$$g\left(x\right)=\left(x-1\right)f\left(x\right) \tag{16}$$

4. Representations for root $x_1=z=57.7341...$

$$y=\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}+...\right)^{6}\right)^{6} \tag{17}$$

$$x_1=z=y^{-6} \tag{18}$$

5. Iterative methods

$$u_{n+1}=64\left(\frac{u_n}{1+u_n}\right)^{5}-1,\ u_1=57\Rightarrow u_n\to x_1=z=57.73410... \tag{19}$$

$$v_{n+1}=\left(\frac{1+v_n}{2}\right)^{6},\ v_1=0\Rightarrow v_n\to\frac{1}{x_1}=\frac{1}{z}=0.01732... \tag{20}$$

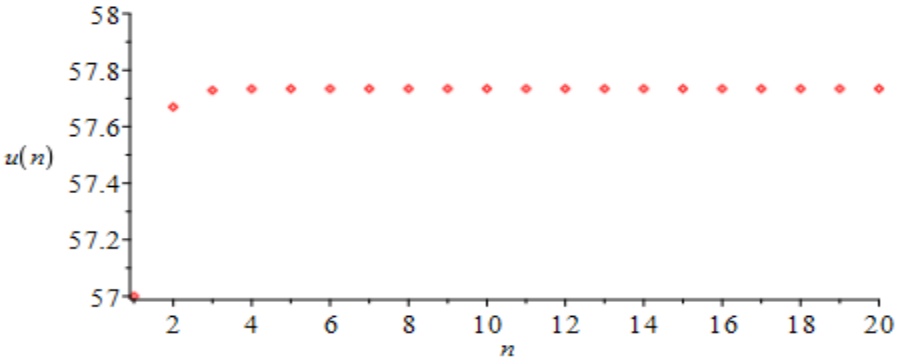


Figure 1.

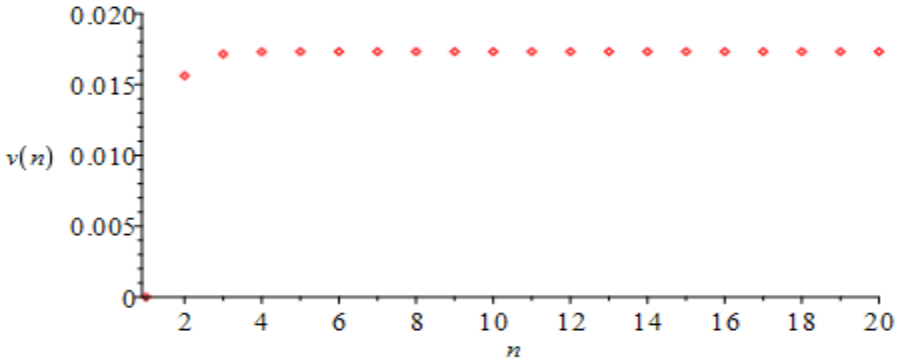
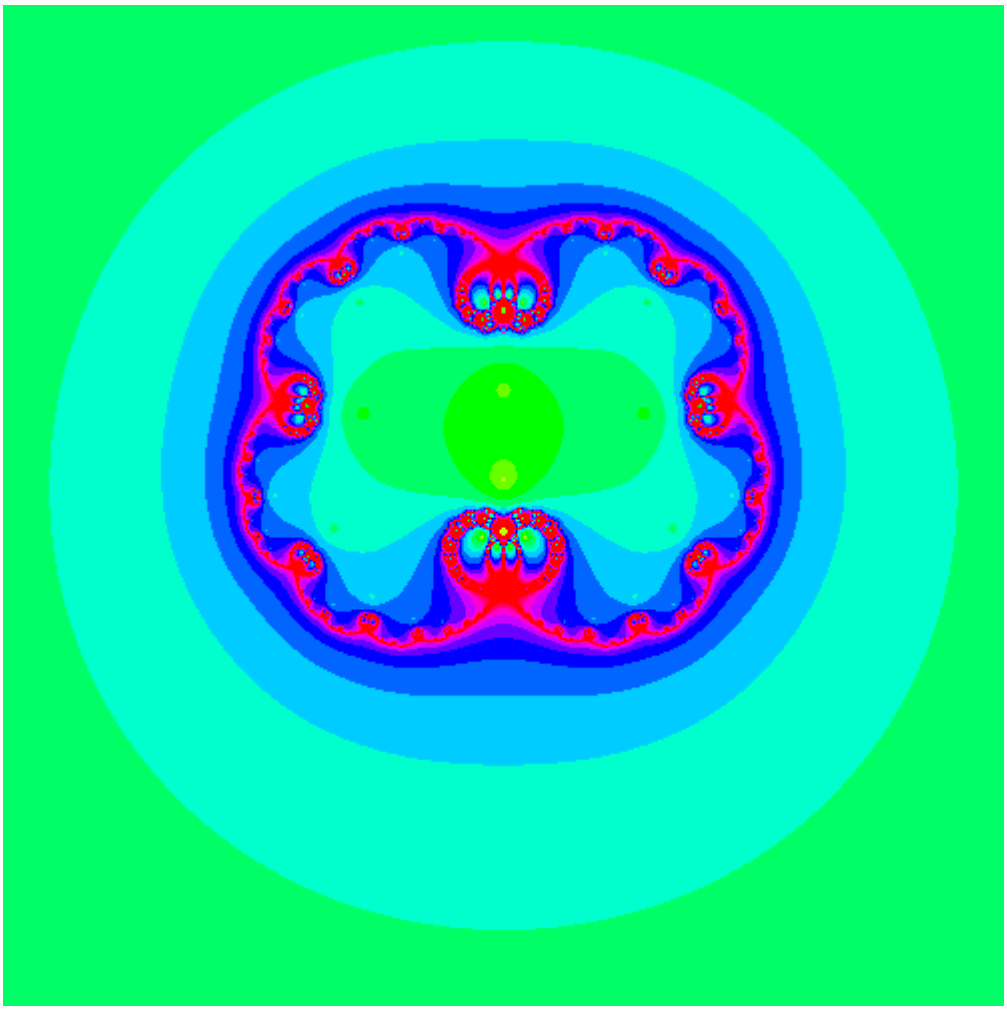


Figure 2.

6. Fractals

❖ Fractals for $F(x) = x - 64\left(\frac{x}{1+x}\right)^5 + 1$



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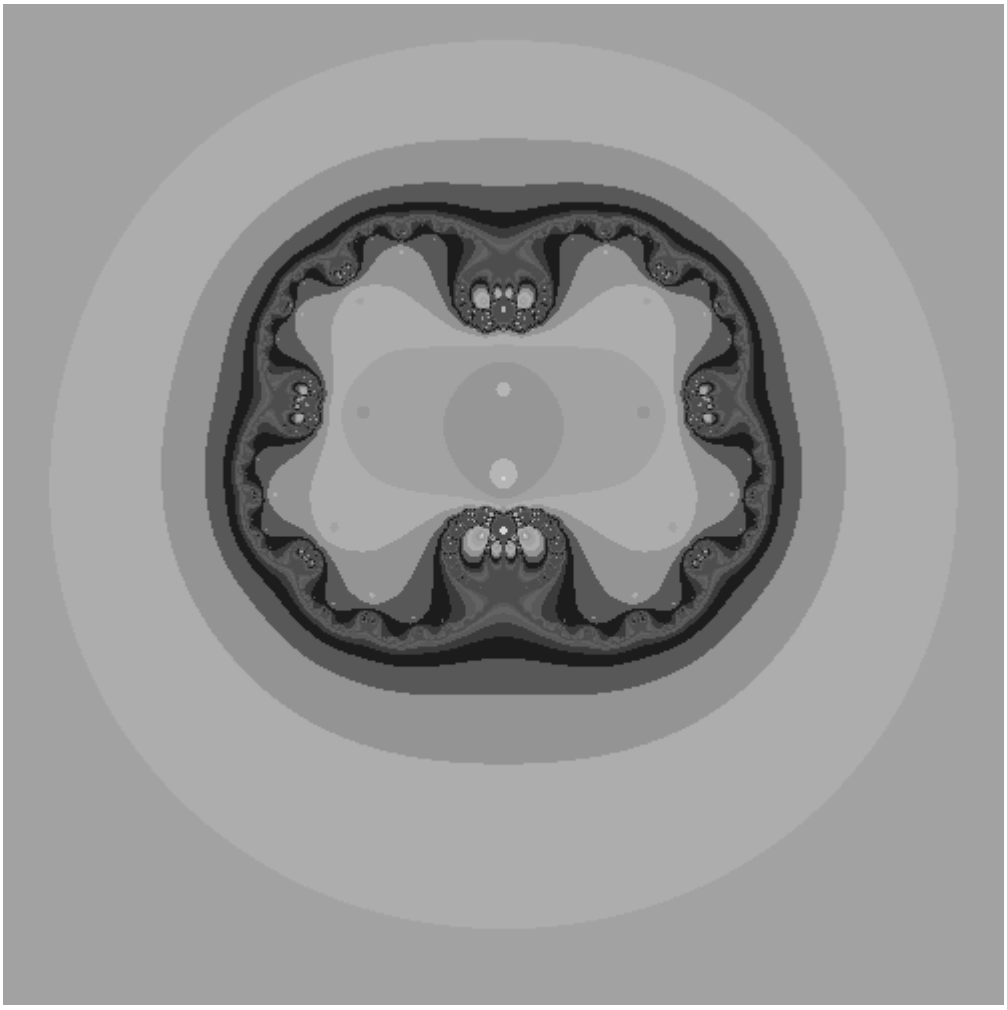

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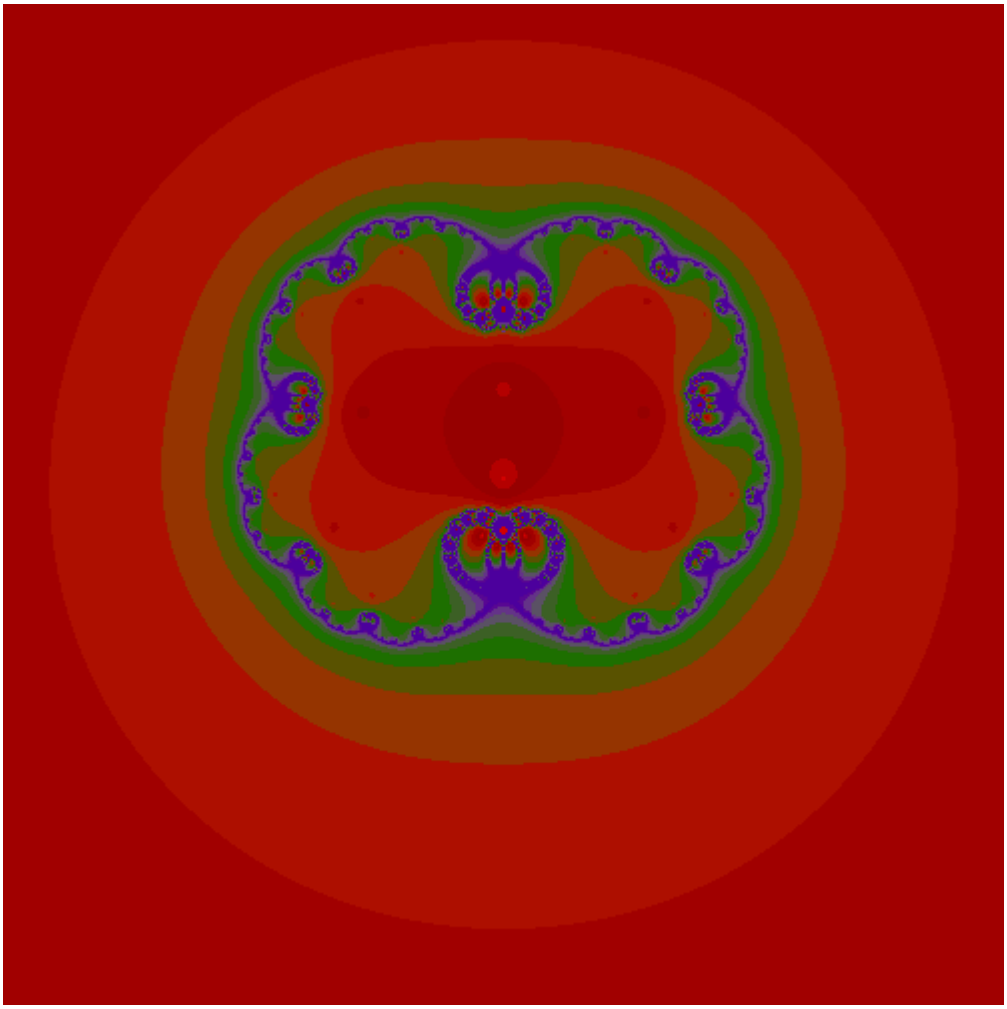

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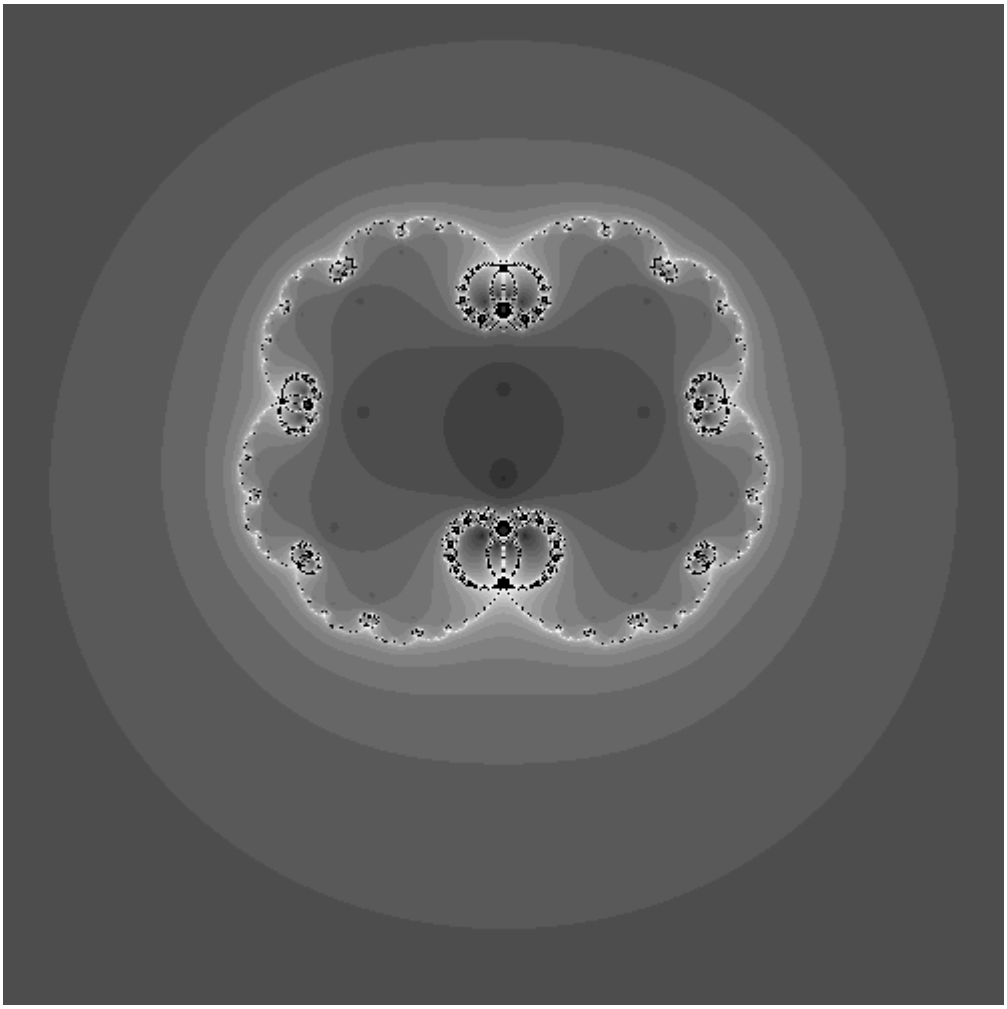

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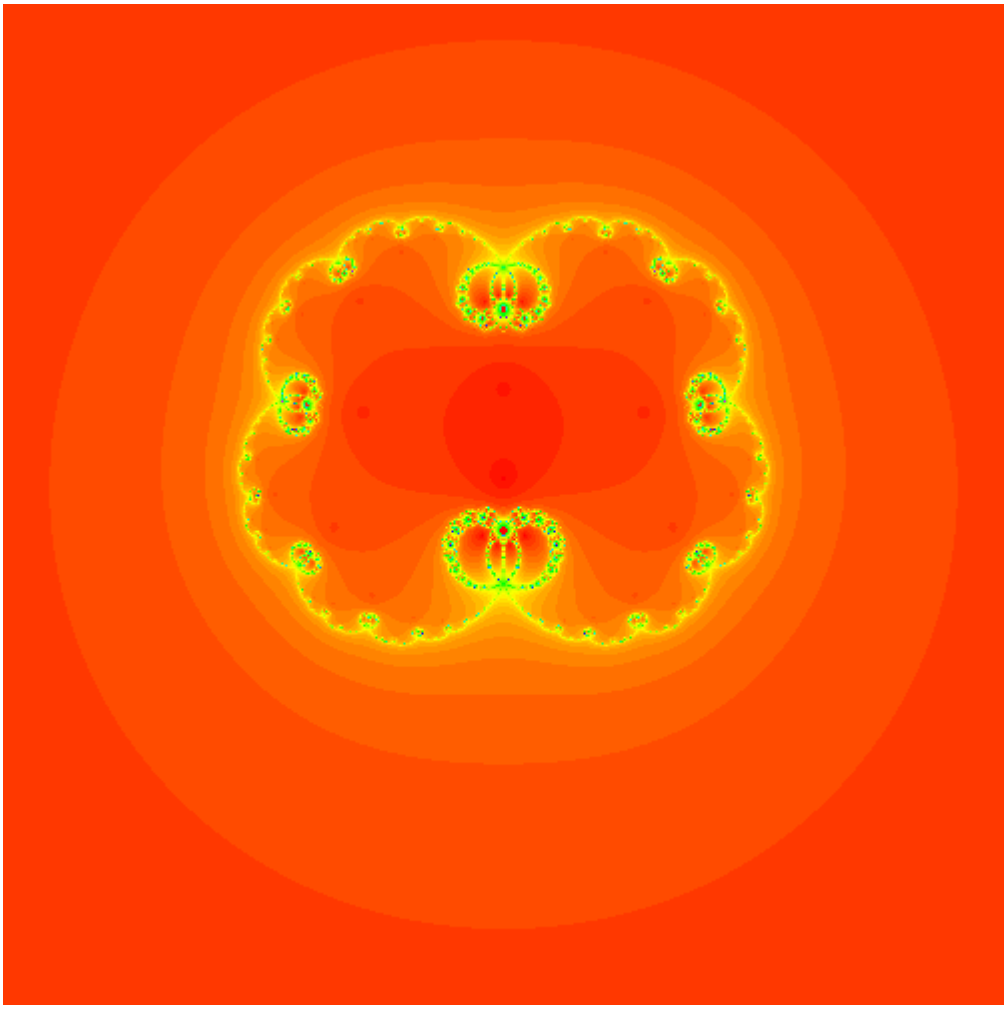

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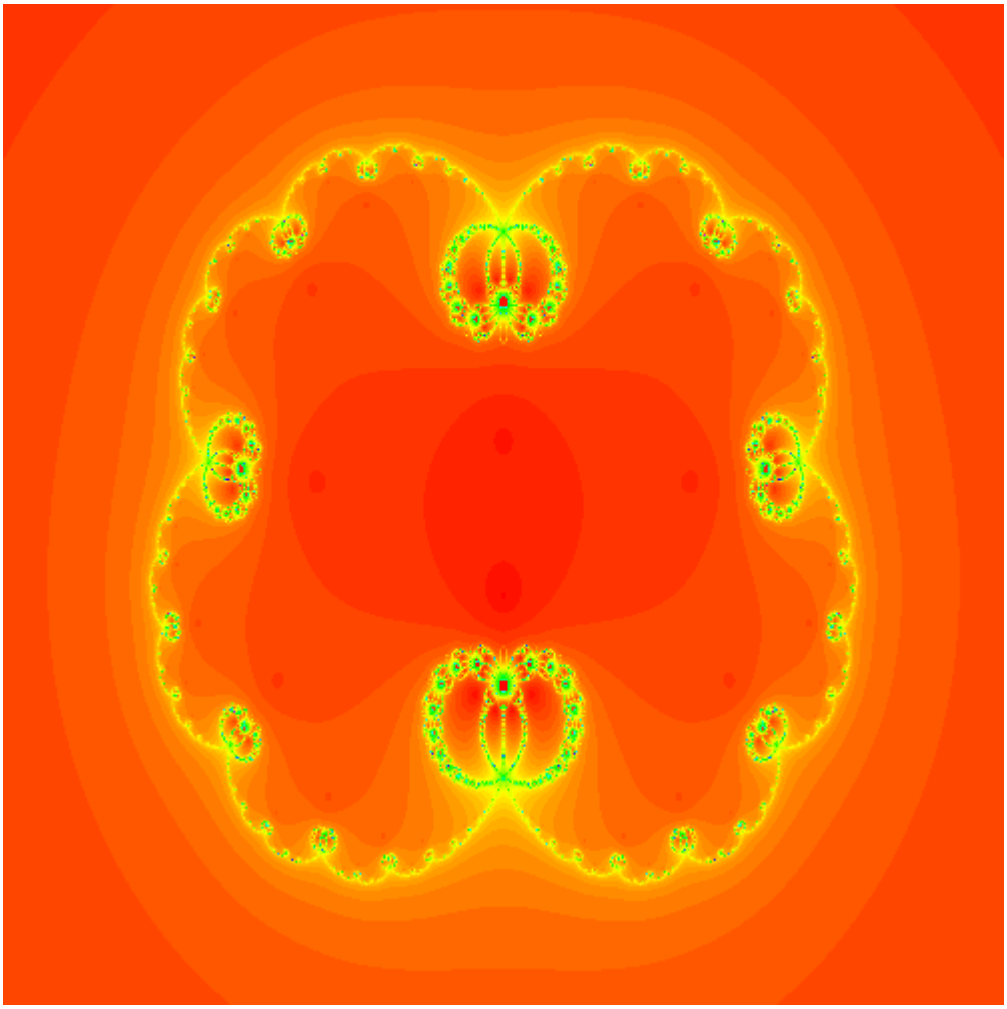

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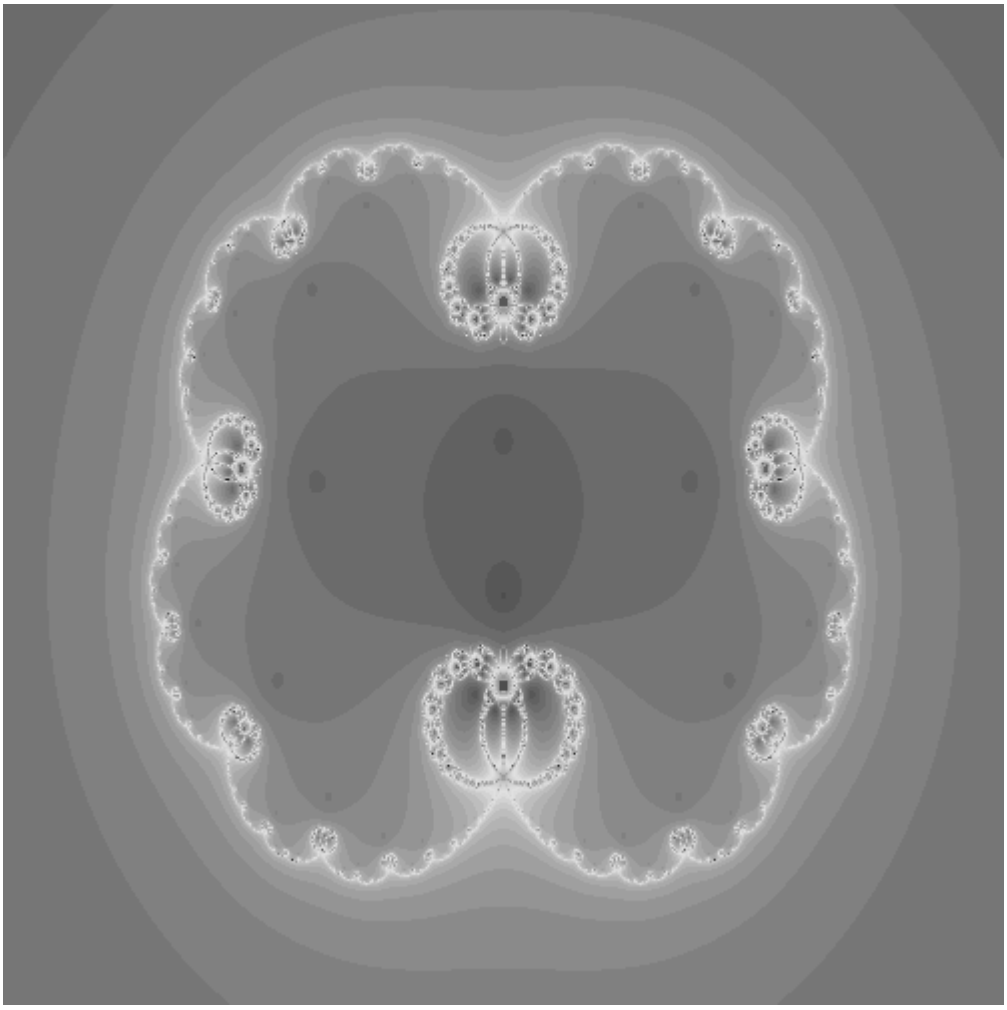

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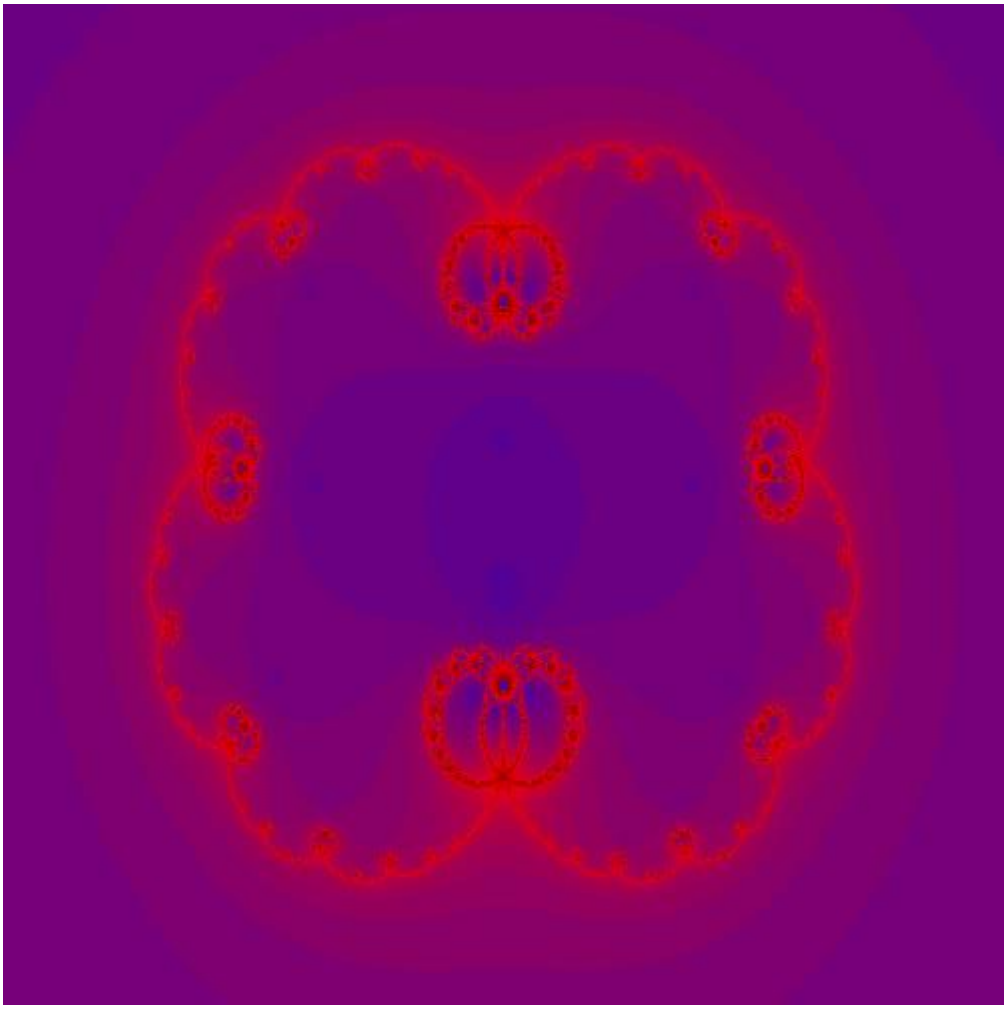

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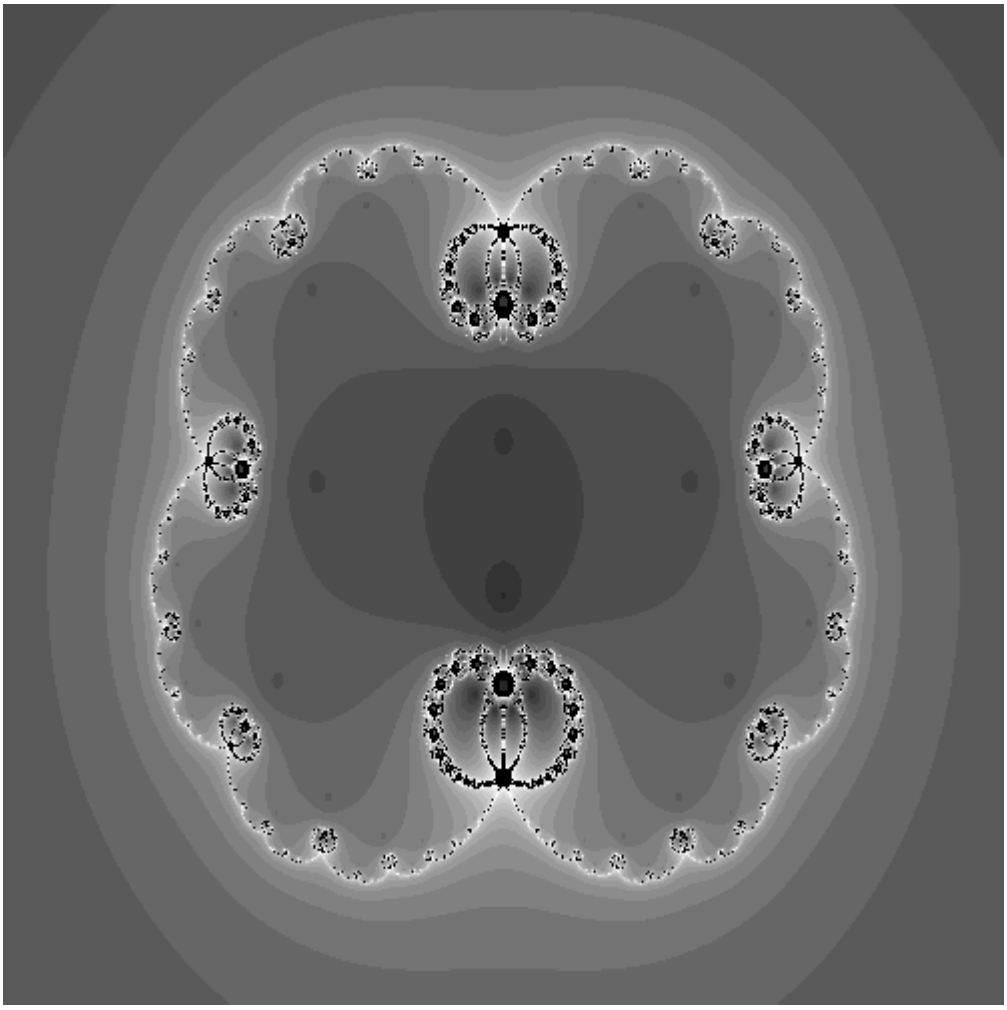

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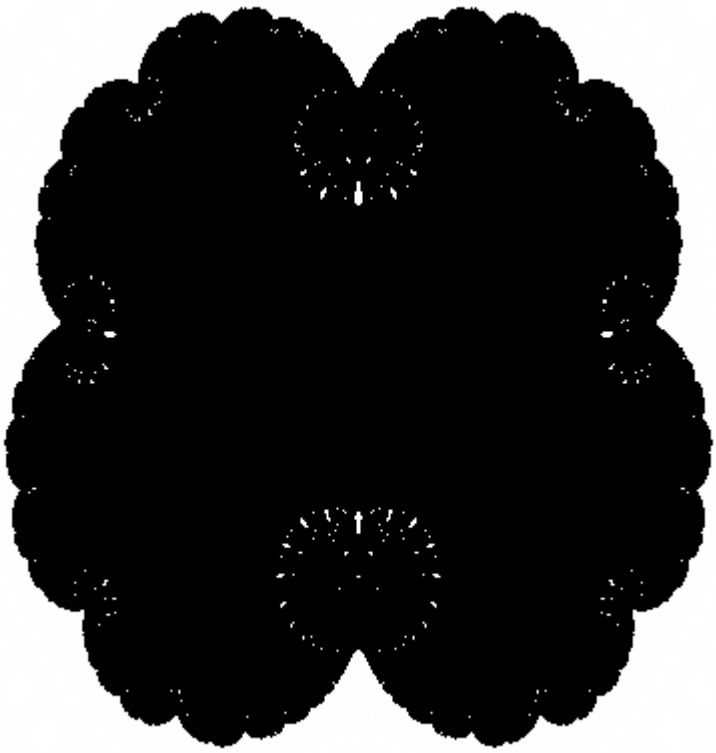
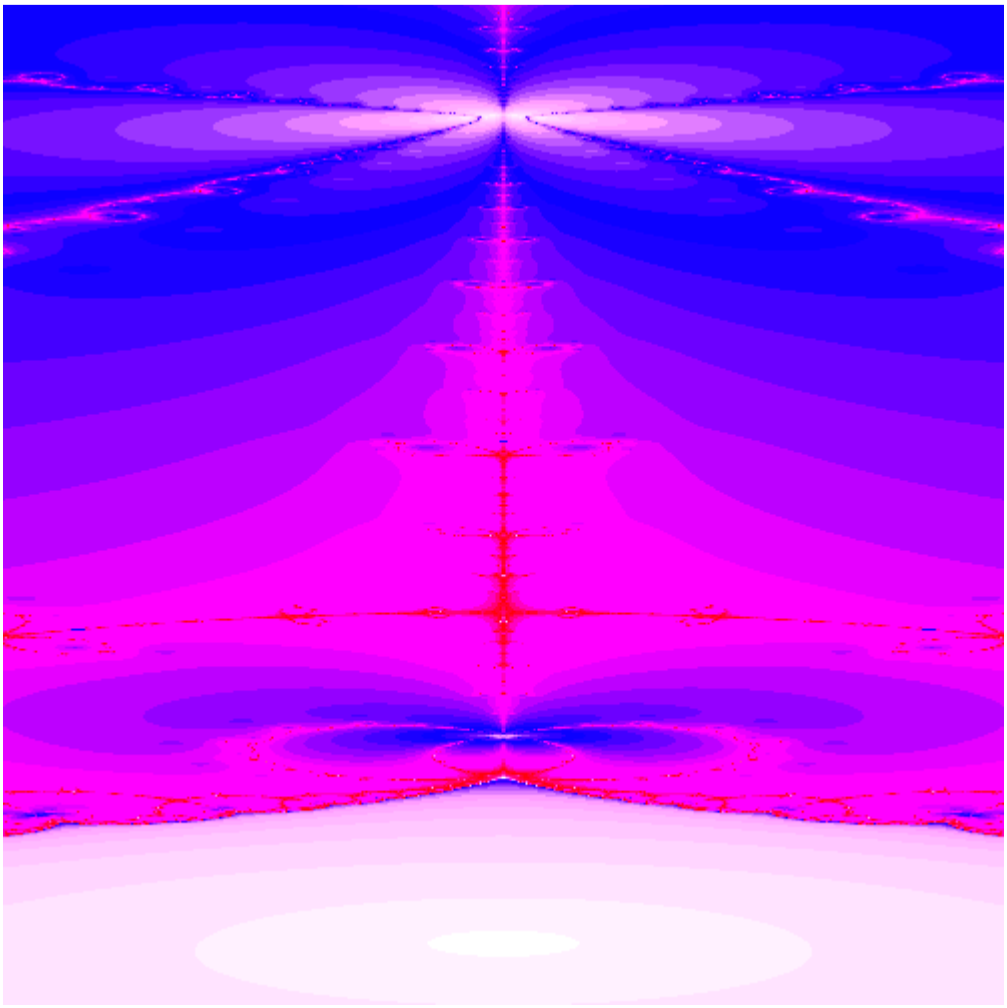


Figure 12.

❖ Fractals for $F(x) = x^5 - 57x^4 - 42x^3 - 22x^2 - 7x - 1$.



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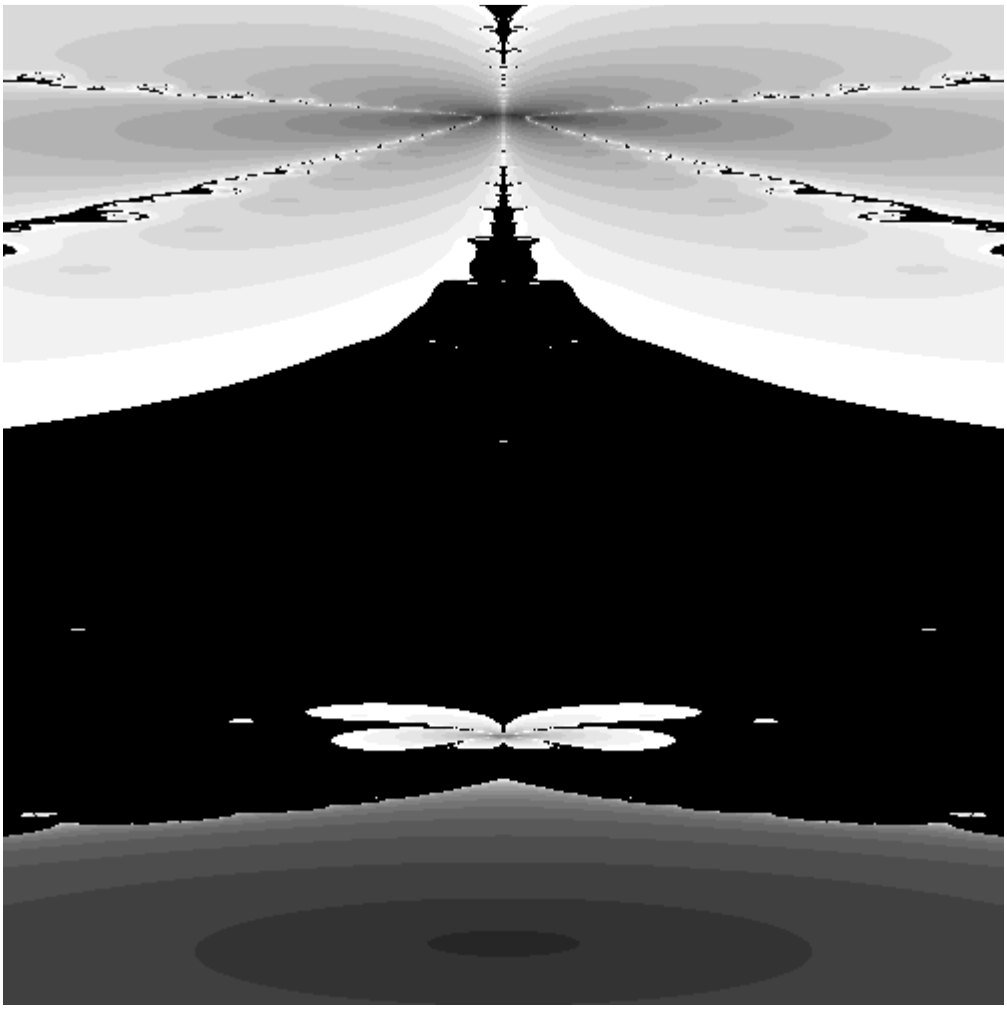

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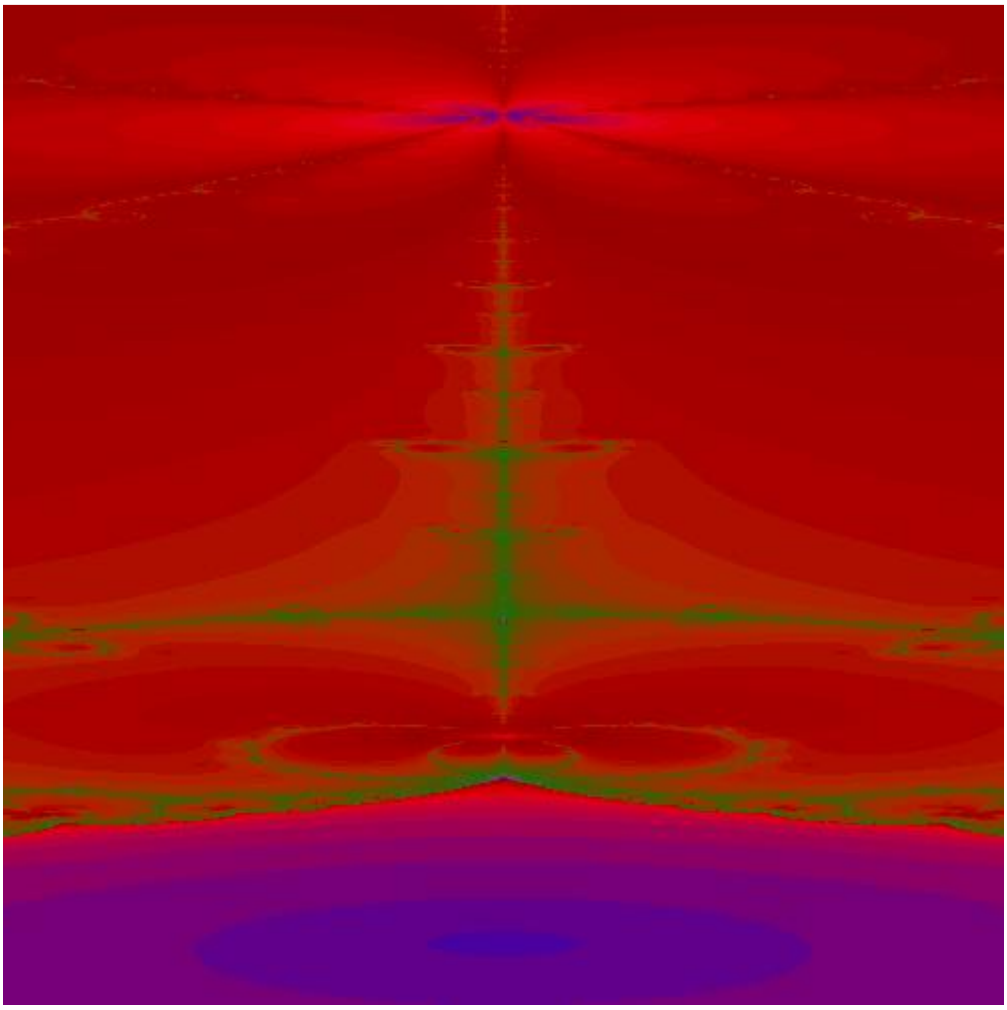

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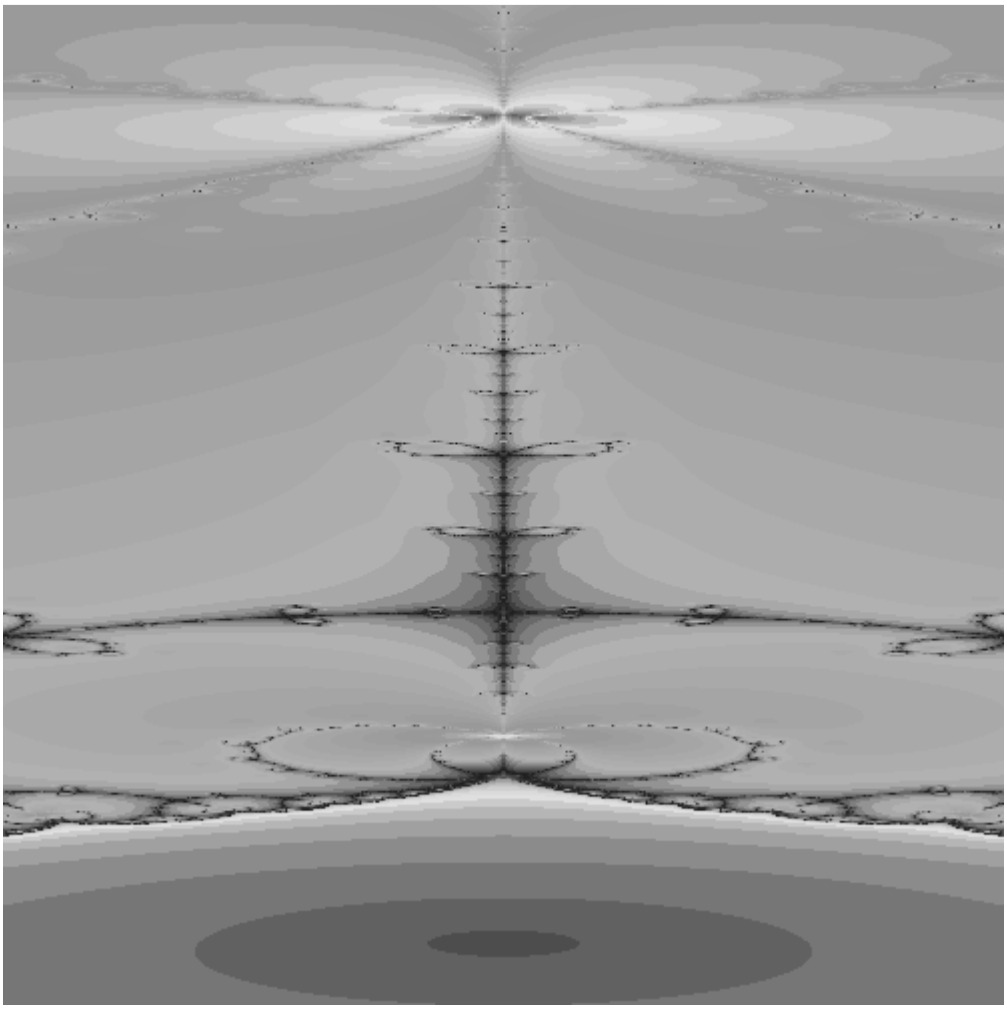

Figure 16.

References

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