Potential energy deficit as an alternative for dark matter?

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1 Abstract

The problem of gravitational potential energy is analyzed within a simple model, in which the 3-dimensional ($3D$) space is a curved hypersurface of a 4-dimensional ($4D$) Euclidean space. The analysis shows that the effect of gravitational potential energy deficit is present in the model. For a particular profile of hypersurface representing $3D$ space, the effects of aforementioned deficit are similar to effects attributed to dark matter, while not being contrary to Newton's law of gravity.

keywords: gravitation – galaxies: kinematics and dynamics – cosmology: dark matter

2 Introduction

We are analyzing the gravitational potential energy of a particle in a simple, non-relativistic model similar to the well-known concept of $3D$ space being a deformed membrane of $4D$ Euclidean space. In this model, we take a similar assumption: that the $3D$ space is the P hypersurface, which, on account of the universe expansion, propagates itself with acceleration $\vec{A}'$ in a $4D$ Euclidean space. In the frame of reference linked to the P hypersurface, for every point of $4D$ space, the absolute acceleration $\vec{A} = -\vec{A}'$ is determined, generating the conservative force $\vec{F} = m\,\vec{A}$. This force shapes the P hypersurface and, depending on mass distribution, gives it a particular $y(r)$ profile in a $4D$ space, where y is the coordinate parallel to $\vec{F}$ force, and r is the particle's distance from the center in an undeformed hyperplane of $y = 0$ profile. For $y(r)$ profile we only assume, that: $y(0) = y_{min}$, $\mathrm{d}y/\mathrm{d}r \geq 0$ and $y(r \to \infty) \to 0$.

3 Potential energy

In a flat $4D$ space, the conservative force $\vec{F}$ is the field force. The elementary work of field force $\vec{F}$ performed over a particle (Fig.1) has the following form:

$$\mathrm{d}L_{4D} = \vec{F} \cdot \mathrm{d}\vec{y} = \vec{F_s} \cdot \mathrm{d}\vec{s}, \tag{1}$$

where $\vec{F_s} \cdot \mathrm{d}\vec{s}$ is the elementary work performed while moving a particle over the P hypersurface.
The elementary work of field force $\vec{F}$, contributes to potential energy:

$$E_{p4D}(y) = -\int_0^y \vec{F} \cdot \mathrm{d}\vec{y}, \tag{2}$$

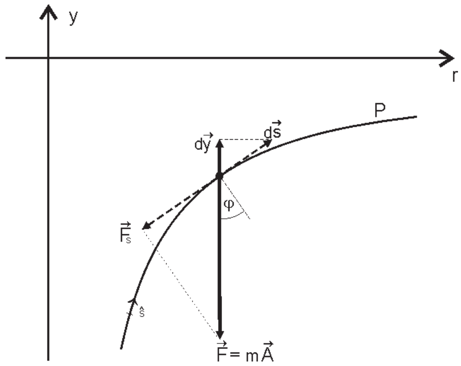

Figure 1: The work of $\vec{F} \cdot \mathrm{d}\vec{y}$ as an elementary work of field force $\vec{F}$, in a flat $4D$ space.

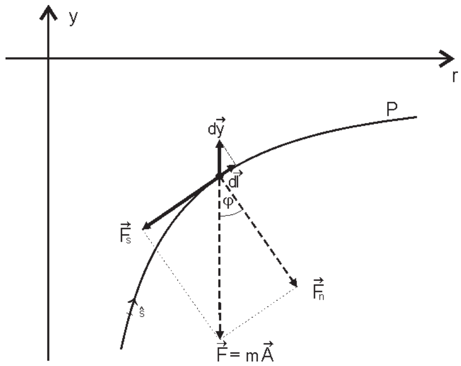

Figure 2: Work $\vec{F}_s \cdot \mathrm{d}\vec{y}$ as the part of elementary work of field force $\vec{F}$, that is available in curved $3D$ space.

where it is assumed that $E_{p4D}(y = 0) = 0$.

Therefore the potential energy of a particle on the P hypersurface takes the (2) form.

A particle located in a curved $3D$ space is present on the P hypersurface in $4D$ space. That is why its real potential energy takes the (2) form as well. However, unlike in $4D$ space, in curved $3D$ space only the part of elementary work $\mathrm{d}L_{4D}$ of field force $\vec{F}$, that depends on force $\vec{F}_s$, is available (Fig.2):

$$\mathrm{d}L_{3D} = \vec{F}_s \cdot \mathrm{d}\vec{y} = \vec{F}_s \cdot \mathrm{d}\vec{l}, \tag{3}$$

where: $\vec{F}_s \cdot \mathrm{d}\vec{l}$ is the equivalent of work $\vec{F}_s \cdot \mathrm{d}\vec{y}$ performed while moving a particle over the P hypersurface in a $4D$ space. The elementary work $\mathrm{d}L_{3D}$ allows an elementary increment of potential energy:

$$\mathrm{d}E_{ps} = -\,\mathrm{d}L_{3D}, \tag{4}$$

which results in potential energy:

$$E_{ps}(y) = -\int_0^y \vec{F}_s \cdot \mathrm{d}\vec{y}, \tag{5}$$

where it is assumed that $E_{ps}(y = 0) = 0$.

The remaining part of elementary work $\mathrm{d}L_{4D}$, takes the following form:

$$\mathrm{d}L_{(4D-3D)} = \left(\vec{F} - \vec{F}_s\right) \cdot \mathrm{d}\vec{y} = \vec{F}_n \cdot \mathrm{d}\vec{y}. \tag{6}$$

In a curved $3D$ space it is the cause of potential energy deficit of the following range of values:

$$\big(E_{pn}(y_{min})\,;\, E_{pn}(0)\big). \tag{7}$$

The deficit indicates that even though the work against force field is being performed in curved $3D$ space, it is not possible for the potential energy increase (and, therefore, decrease) of (7) values range to appear. That is why in curved $3D$ space, the aforementioned energy remains on the lowest value level $E_{pn\,MIN} = E_{pn}(y_{min})$:

$$E_{pn\,MIN} = -\int_0^{y_{min}} \vec{F}_n \cdot \mathrm{d}\vec{y}, \tag{8}$$

where it is assumed that $E_{pn}(y = 0) = 0$.
Taking (5) and (8) into account, the potential energy E_{p3D} available in curved $3D$ space takes the following form:

$$E_{p3D}(y) = E_{ps}(y) + E_{pn\,MIN}. \tag{9}$$

The potential energy deficit (7) may be interpreted as the existence of a forbidden potential energy band $\Delta E'_p$ within the curved $3D$ space:

$$E_{p3D} \notin \Delta E'_p, \quad \text{where: } \Delta E'_p = \big(E_{pn\,MIN}\,;\,0\big). \tag{10}$$

Taking (9) into account, the real potential energy (2) manifests itself in a curved $3D$ space in the following form:

$$E_{p4D}(y) = E_{p3D}(y) + E_{p+}(y), \tag{11}$$

where E_{p+} is the positive gravitational potential energy:

$$E_{p+}(y) = m\,\Phi_+(y) \tag{12}$$

and Φ_+ is the gravitational potential positive:

$$\Phi_+(y) = -\frac{1}{m}\int_{y_{min}}^{y} \vec{F}_n \cdot \mathrm{d}\vec{y}. \tag{13}$$

The positive gravitational potential energy is the missing part of real potential energy, moved into the positive energy area:

$$E_{p+}(y) = E_{pn}(y) - E_{pn\,MIN}. \tag{14}$$

Basing on equation (11), the real potential energy E_{p4D} of a particle is represented in the curved $3D$ space by the sum of negative potential energy E_{p3D} and positive potential energy E_{p+}. This means, that the increase (decrease) of potential energy E_{p4D} manifests itself in this space with an increase (decrease) of negative potential energy E_{p3D} and an increase (decrease) of positive potential energy E_{p+}. However, in a curved $3D$ space, the surplus of elementary work that is performed against the field force, higher than the value $-\,\mathrm{d}L_{3D}$ (see eq. (4)) should contribute towards the increase of kinetic energy of a particle. This means, that the positive gravitational potential energy that appears in a curved $3D$ space, manifests itself in this space as a component of kinetic energy of a particle. Therefore, for the case where total mechanical energy E of a particle in $4D$ space is equal to zero:

$$E_k + E_{p4D} = 0, \tag{15}$$

after taking equation (11) into account, we obtain:

$$E_{k3D} = E_k + E_{p+} \qquad (\text{for } E = 0), \tag{16}$$

where: E_k is the kinetic energy of a particle, and E_{k3D} is the kinetic energy of a particle manifested in the curved $3D$ space.
Equation (16) shows that the positive potential energy overstates the kinetic energy of a particle exhibited in a curved $3D$ space. This means, that the consequence of gravitational potential energy deficit (PED) is the kinetic energy surplus (KES) in this space.

Then we assume, that the real potential energy (2) of a particle in a curved $3D$ space, is consistent with the potential energy $E_p(r) = m\Phi_N(r)$ of a particle in a flat $3D$ space of the Newtonian model. The consequence of this condition, is the P hypersurface profile:

$$y(r) = \Phi_N(r)/A, \tag{17}$$

where: Φ_N is the Newtonian potential.
In addition, we introduce the PED-KES model - a modification of the Newtonian model that takes into account the potential energy deficit and its effects. Unlike in the Newtonian model, in PED-KES model, the particle possesses potential energy $E_{p_{ped}}(r)$ compliant with its potential energy $E_{p3D}(y)$ available in the curved $3D$ space of the (17) profile. Taking (17) into account, the (11) equation for PED-KES model takes the following form:

$$E_p(r) = E_{p_{ped}}(r) + E_{p+}(r), \tag{18}$$

where:

$$E_{p_{ped}}(r) = m\left(\Phi_N(r) - \Phi_+(r)\right), \tag{19}$$

$$E_{p+}(r) = m\,\Phi_+(r) \tag{20}$$

and:

$$\Phi_+(r) = \int_0^r \frac{\mathrm{d}\Phi_N}{\mathrm{d}r}\left[1+\left(\frac{1}{A}\frac{\mathrm{d}\Phi_N}{\mathrm{d}r}\right)^2\right]^{-1}\mathrm{d}r. \tag{21}$$

The forbidden gravitational potential energies $\Delta E'_p$ band takes the following form:

$$E_{p_{ped}} \notin \left(E_{pn}(0)\ ;\ 0\right), \quad \text{where: } E_{pn}(0) = -\,E_{p+}(\infty). \tag{22}$$

In the Newtonian model, the deficit of potential energy does not exist. The potential energy E_p can be therefore presented as the sum of potential energies E_{ps} and E_{pn}:

$$E_p(r) = E_{ps}(r) + E_{pn}(r), \tag{23}$$

where:

$$E_{ps}(r) = m\,\Phi_N(r) - E_{pn}(r), \tag{24}$$

$$E_{pn}(r) = m\int_\infty^r \frac{\mathrm{d}\Phi_N}{\mathrm{d}r}\left[1+\left(\frac{1}{A}\frac{\mathrm{d}\Phi_N}{\mathrm{d}r}\right)^2\right]^{-1}\mathrm{d}r. \tag{25}$$

Figure 3 presents the potential energy $E_p(r)$ coming from point mass and its components, for both the potential energy deficit presence (equation (18)) and the lack of it (equation (23)).

4 Centripetal force

Taking (18) into account, the real centripetal force affecting a particle in the PED-KES model equals:

$$F(r) = -\frac{\mathrm{d}E_p(r)}{\mathrm{d}r} = -\frac{\mathrm{d}}{\mathrm{d}r}\left[E_{p_{ped}}(r) + E_{p+}(r)\right] = -m\frac{\mathrm{d}\Phi_N}{\mathrm{d}r} \tag{26}$$

and is compliant with the Newton's law of gravitation.

5 Circular speed

The real circular speed v_c around mass center presented in the PED-KES model, is a consequence of known dependencies. Taking equation (18) into account, we obtain:

$$v_c^2(r) = \frac{r}{m}\frac{\mathrm{d}E_p}{\mathrm{d}r} = \frac{r}{m}\frac{\mathrm{d}}{\mathrm{d}r}\left[E_{p_{ped}}(r) + E_{p+}(r)\right]. \tag{27}$$

Taking equation (26) into consideration, from equation (27) we obtain:

$$v_c^2(r) = \frac{r}{m}\left|F(r)\right|. \tag{28}$$

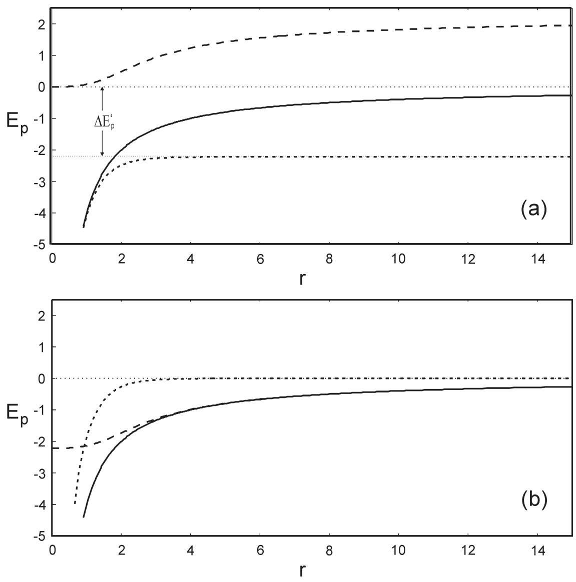

Figure 3: Potential energy E_p in the function of distance from point mass (solid line), as a sum of two components: (a) for the presence of potential energy deficit (the PED-KES model): the available potential energy $E_{p_{ped}}(r)$ (dotted line) and positive potential energy $E_{p+}(r)$ (dashed line), $\Delta E_p'$ - the forbidden gravitational potential energy band; (b) for the lack of potential energy deficit (equivalent of the Newtonian model): the potential energy $E_{ps}(r)$ (dotted line) and the potential energy $E_{pn}(r)$ (dashed line). Presented in arbitrary units.

The real circular speed around mass center in the PED-KES model is determined by the value of centripetal force, and is identical to the one present in the Newtonian model. Out of equation (28) emerges the dependency of circular motion kinetic energy E_{k_c} from centripetal force value:

$$E_{k_c}(r) = \frac{1}{2} r \left| F(r) \right| . \tag{29}$$

In order to determine the influence of potential energy deficit on circular speed around mass center, we use the Newtonian potential equation for spherical mass distribution (see [1], equation 2-22):

$$\Phi_N(r) = -4\pi G \left[\frac{1}{r} \int_0^r \rho(r')\, r'^2 \mathrm{d}r' + \int_r^\infty \rho(r')\, r' \mathrm{d}r' \right] . \tag{30}$$

Taking into account:

$$M(r) = 4\pi \int_0^r \rho(r')\, r'^2 \mathrm{d}r', \tag{31}$$

as well as the following relation:

$$\frac{v_c^2(r)}{r} = \frac{G\, M(r)}{r^2}, \tag{32}$$

out of equation (30), we are able to obtain the connection between potential energy and kinetic rotation energy E_{k_c} for the Newtonian model:

$$E_p(r) = -2E_{k_c}(r) - 4m\pi G \int_r^\infty \rho(r')\, r' \mathrm{d}r'. \tag{33}$$

Taking into consideration, that positive potential energy manifests itself as a component of exhibited kinetic energy, out of equations (33) and (18), for the PED-KES model, we obtain:

$$E_{p_{ped}}(r) = -2 \left(E_{k_c}(r) + \frac{1}{2} E_{p+}(r) \right) - 4m\pi G \int_r^\infty \rho(r')\, r' \mathrm{d}r', \tag{34}$$

therefore the circular motion kinetic energy around mass center $E_{k_{kes}}$, exhibited in the PED-KES model, takes the following form:

$$E_{k_{kes}}(r) = E_{k_c}(r) + \frac{1}{2} E_{p+}(r). \tag{35}$$

Equation (35) shows, that positive potential energy overstates the exhibited kinetic energy of particle's rotation around mass center when compared to the real kinetic energy E_{k_c} that results from centripetal force (see eq. (29)). The consequence of equation (35) is the equation for circular speed around mass center $v_{c_{kes}}$ exhibited in the PED-KES:

$$v_{c_{kes}}(r) = \sqrt{\frac{2}{m}\left[E_{k_c}(r) + \frac{1}{2} E_{p+}(r)\right]} = \sqrt{v_c^2(r) + \Phi_+(r)}. \tag{36}$$

Equation (36) shows that positive potential energy manifests itself in the area of circular speed in a similar way, as the additional circular speed $v_{c_{ex}}$ that has a value of:

$$v_{c_{ex}}(r) = \sqrt{\Phi_+(r)}. \tag{37}$$

This is not, however, additional circular speed, since it is not accompanied by centripetal force (see eq. (28)). This is a pseudo-circular speed, that results from manifestation of positive potential energy in the area of kinetic energy.

The limit value of circular speed exhibited in PED-KES is determined by pseudo-circular speed and equals:

$$v_{c_{kes}}(\infty) = \lim_{r\to\infty} \sqrt{\Phi_+(r)}. \tag{38}$$

For mass distribution in which the maximum centripetal acceleration $g_{N_{max}}$ of matter's rotation around center is considerably lower than A, from equations (21) and (38) we obtain:

$$v_{c_{kes}}(\infty) = \sqrt{|\,\Phi_N(0)\,|} \qquad (\text{for} \quad g_{N_{max}} \ll A). \tag{39}$$

Taking equation (35) into account, the real kinetic energy of a particle E_{k_c} takes the following form:

$$E_{k_c}(r) = E_{k_{kes}}(r) - \frac{1}{2} E_{p+}(r), \tag{40}$$

therefore the real circular speed equals:

$$v_c(r) = \sqrt{v_{c_{kes}}^2(r) - \Phi_+(r)}. \tag{41}$$

6 Escape speed

The consequence of equation (16) is the connection between escape speed $v_{e_{kes}}$ exhibited in the PED-KES model and the real escape speed v_e:

$$v_{e_{kes}}(r) = \sqrt{\frac{2}{m}\Big[E_k(r) + E_{p+}(r)\Big]} = \sqrt{v_e^2(r) + 2\,\Phi_+(r)}, \tag{42}$$

where the influence of positive potential energy can be described as pseudo-escape speed $v_{e_{ex}}$ that has a value of:

$$v_{e_{ex}}(r) = \sqrt{2\,\Phi_+(r)}. \tag{43}$$

A particle moving with the real escape speed v_e, that equals:

$$v_e(r) = \sqrt{\frac{2\,|E_p\,|}{m}} = \sqrt{\frac{2}{m}\left|E_{p_{ped}}(r) + E_{p+}(r)\right|}, \tag{44}$$

exhibits kinetic energy (16) that suggests it is moving with speed $v_{e_{kes}}$, which is the result of equation (42). The limit value of escape speed exhibited in PED-KES is determined by pseudo-escape speed, and equals:

$$v_{e_{kes}}(\infty) = \lim_{r\to\infty} \sqrt{2\,\Phi_+(r)}. \tag{45}$$

For mass distribution in which the maximum centripetal acceleration $g_{N_{max}}$ of matter's rotation around center is considerably lower than A, from equations (21) and (45) we obtain:

$$v_{e_{kes}}(\infty) = \sqrt{2\,|\,\Phi_N(0)\,|} \qquad (\text{for} \quad g_{N_{max}} \ll A). \tag{46}$$

7 Absolute acceleration A

For the case of point mass, from equation (21) we obtain:

$$\begin{aligned}\Phi_+(r) = \frac{A\,r_k}{2\sqrt{2}} &\left[\frac{1}{2}\ln\left(\frac{r^2}{r_k^2} - \sqrt{2}\,\frac{r}{r_k} + 1\right)\right.\\ &- \frac{1}{2}\ln\left(\frac{r^2}{r_k^2} + \sqrt{2}\,\frac{r}{r_k} + 1\right) + \arctan\left(\sqrt{2}\,\frac{r}{r_k} + 1\right)\\ &\left.+ \arctan\left(\sqrt{2}\,\frac{r}{r_k} - 1\right)\right],\end{aligned} \tag{47}$$

where $r_k = \sqrt{\mathrm{G}M/A}$ is the distance from point mass, for which the centripetal acceleration equals A. The r_k form is similar to MOND radius $r_M = \sqrt{\mathrm{G}M/a_0}$ [2, 3], present in the MOND concept, where a_0 is the acceleration introduced in this concept as a new physical constant. It turns out, that assuming the value of absolute acceleration as equal to $A = 1.2 \times 10^{-10}\ \mathrm{m/s^2} = a_0$, gives good compliance of the PED-KES concept with observation. This may suggest that acceleration a_0 is the absolute acceleration in the PED-KES concept. However, unlike a_0, the A acceleration has an unambiguously determined physical sense. Therefore, in the PED-KES concept, we use A to denote absolute acceleration.

8 Results

The following assumes spherical mass distribution and $A = 1.2 \times 10^{-10}\ \mathrm{m/s^2}$.
Figure 4 presents a graph of relations (36) and (42) for an exemplary point mass equal to the baryonic mass of the Galaxy [4]. As seen on the graph, the PED-KES rotation can be conventionally divided into two areas: Keplerian, for $r < r_k$, and non-Keplerian for $r > r_k$. r_k can thus be regarded as radius of the Keplerian area. Formula (38) indicates that the terminal value of PED-KES circular speed is defined by pseudo-circular speed $v_{c_{ex}}$ and, for the assumed value of mass, the v_∞ matches the Galaxy's terminal circular speed value [4].
Taking equations (47) and (38) into account, for point mass we obtain:

$$v_{c_\infty} = \sqrt{\frac{\pi A\,r_k}{2\sqrt{2}}} = \left(\frac{\pi^2}{8} A\mathrm{G}M\right)^{1/4}. \tag{48}$$

From the equation (48) we obtain the relation between point mass M and terminal circular speed around mass M exhibited in PED-KES:

$$M \;=\; q\,v_\infty^4, \qquad \text{where:} \quad q = \frac{8}{\pi^2\,A\,\mathrm{G}} \approx \frac{0.81}{A\,\mathrm{G}}. \tag{49}$$

Figure 5 presents BTFR relations graph [5] with the graph resulting from equation (49) drawn over it.
Taking equations (36) and (47) into account, for point mass, we can determine the $v_{c_{kes}}^2/v_c^2$ ratio as a

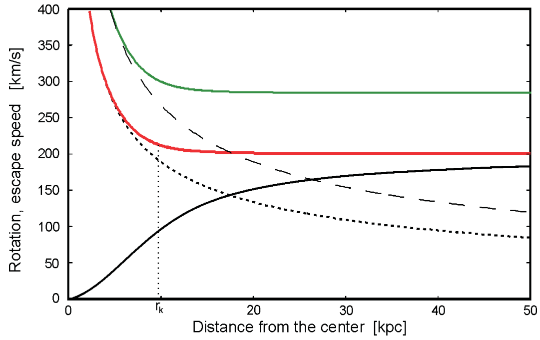

Figure 4: Rotation curve and escape curve, which result from kinetic energy exhibited in the PED-KES model for the case of gravitational field coming from point mass of exemplary mass $M = 8,3 \times 10^{10} M_{\odot}$ (baryonic mass of the Galaxy): red line - PED-KES circular speed; dotted line - Keplerian circular speed; black solid line - pseudo-circular speed $\sqrt{\Phi_{+}(r)}$, establishing the terminal circular speed for this mass at 200 $\frac{\text{km}}{\text{s}}$; green line - escape speed exhibited in the PED-KES model; dashed line - actual excape speed.

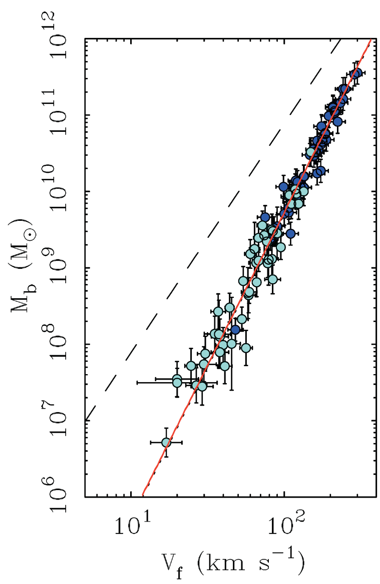

Figure 5: Graph of relations between baryonic mass and PED-KES terminal rotation for point mass - red line, drawn over BTFR relations graph [5]. Based on [5].

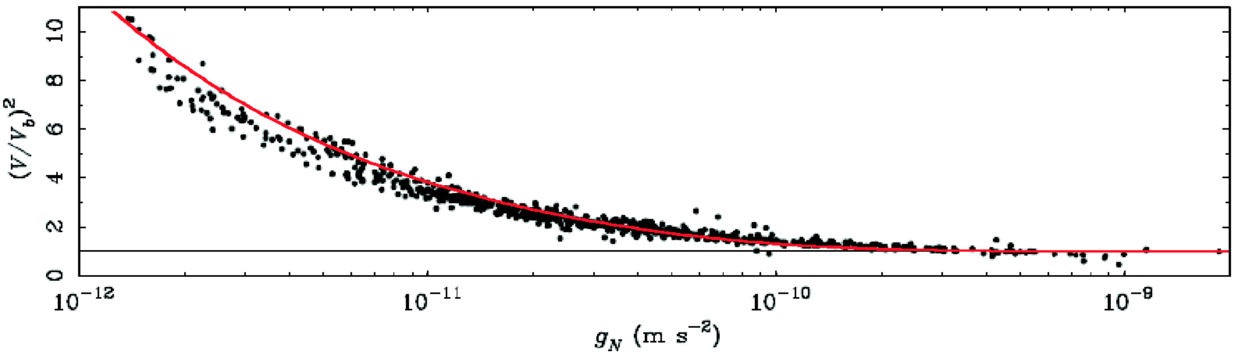

Figure 6: Graph of $(v^2_{c_{kes}}/v^2_c)\,(g_N)$ relations for point mass (red line), drawn over a graph of $(V^2/V^2_b)\,(g_N)$ relations, resulting from hundreds of measurements taken for nearly a hundred spiral galaxies. Based on [5].

function of Newton's acceleration $g_N = v^2_c/r = Ar^2_k/r^2$:

$$\begin{aligned} \frac{v^2_{c_{kes}}}{v^2_c}(g_N) \;=\; & 1 + \frac{1}{2\sqrt{2}}\sqrt{\frac{A}{g_N}}\;\left[\frac{1}{2}\ln\left(\frac{A}{g_N} - \sqrt{\frac{2A}{g_N}} + 1\right)\right. \\ & - \frac{1}{2}\ln\left(\frac{A}{g_N} + \sqrt{\frac{2A}{g_N}} + 1\right) + \arctan\left(\sqrt{\frac{2A}{g_N}} + 1\right) \\ & \left. + \arctan\left(\sqrt{\frac{2A}{g_N}} - 1\right)\right]. \end{aligned} \tag{50}$$

Figure 6 shows relation (50) graph drawn over V^2/V^2_b graph, where V is the observed velocity and V_b is the velocity attributable to visible baryonic matter in Newton's acceleration function $g_N = V^2_b/r$, resulting from hundreds of measurements taken for nearly a hundred spiral galaxies [5].

Figures 7 and 8 show a number of examples of determined PED-KES rotations drawn over galaxies rotation graph from [6, 5]. Basing on baryonic rotation curves shown on the sourcing graphs, from equation (21) we numerically designated a gravitational potential positive $\Phi_+(r)$. Then, from equation (36), PED-KES rotations have been determined. Figure 8 also includes escape speed curves obtained from equation (42).

Figure 9 shows three examples of galaxies from [5, 7], for which the maximum centripetal acceleration of matter's rotation around center is significantly lower than A. For those galaxies, the compliance of PED-KES rotation with the observed has been achieved for masses larger than usually assumed. Basing on the original baryonic rotations, baryonic rotations providing similar compliance between PED-KES and MOND rotations have been determined.

9 Conclusions

The consequence of negative gravitational potential energy deficit is the positive gravitational potential energy. Because this energy manifests itself in the area of particle's kinetic energy, it naturally overstates its exhibited value. That's why PED-KES concept can also be called NES (Natural Energy Surplus)[1]. PED-KES concept is a simple concept based on elementary physics. It does not require any additional non-baryonic matter to exist and, at the same time, is not contradictory to Newton's gravity model. Therefore further verification of this concept seems justified.

[1]see also viXra:1506.0040

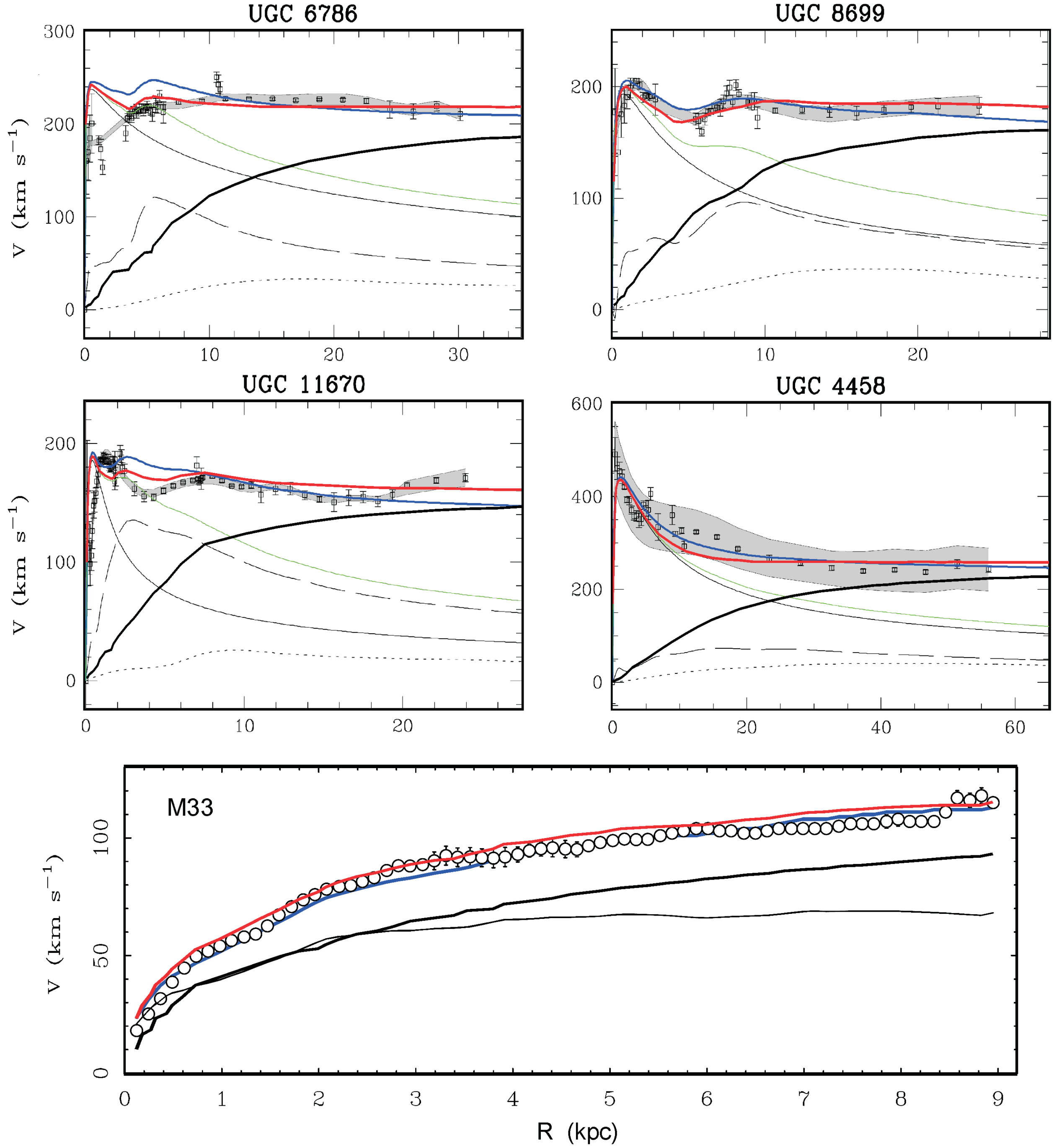

Figure 7: PED-KES rotation curves determined for exemplary spiral galaxies: observed rotation - gray strip (M33 - circles); PED-KES rotation curves - red line; pseudo-rotation speed $\sqrt{\Phi_+(r)}$ as a consequence of potential energy deficit - thick black line; resultant baryonic rotation - thin green line (M33 - thin black line). All other markings as described in [6, 5]. Based on [6, 5].

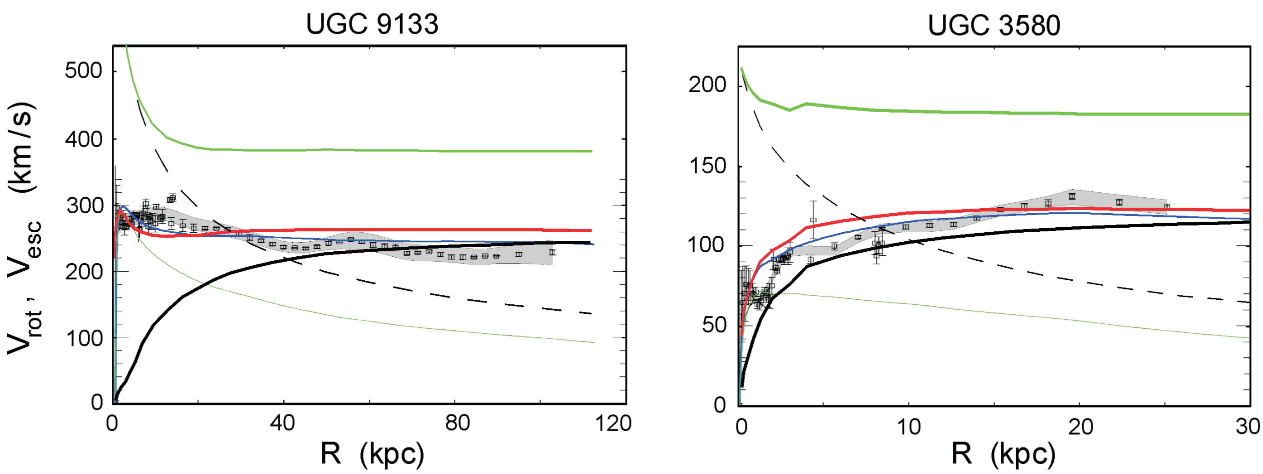

Figure 8: PED-KES rotation curves and escape curve determined for exemplary spiral galaxies: observed rotation - gray strip; PED-KES rotation curves - red line; pseudo-rotation speed $\sqrt{\Phi_+(r)}$ as a consequence of potential energy deficit - thick black line; resultant baryonic rotation - thin green line; Newtonian escape velocity curve - dashed line; PED-KES escape speed curve - thick green line. All other markings as described in [6]. Based on [6].

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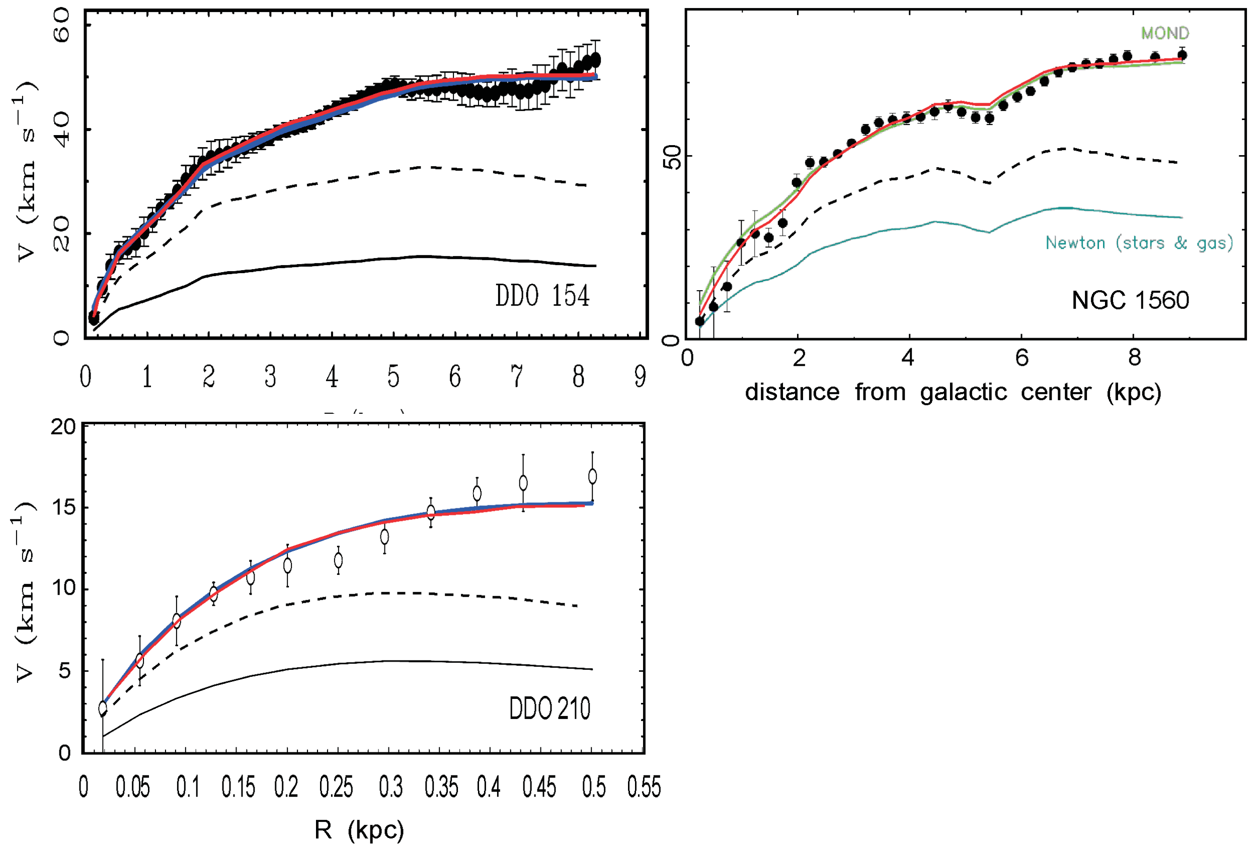

Figure 9: Baryonic rotation curve suggested by PED-KES (dashed line) that provides similar compliance of PED-KES and MOND rotation curves for three examples of galaxies, for which $g_{N_{max}} \ll A$. PED-KES rotation curves - red line. All other markings as described in [5, 7].