Novel Remarks on the Cosmological Constant, the Bohm-Poisson Equation and Asymptotic Safety in Quantum Gravity *

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October, 2017

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Abstract

Recently, solutions to the *nonlinear* Bohm-Poisson equation were found [2] with cosmological applications. We were able to obtain a value for the vacuum energy density of the same order of magnitude as the extremely small observed vacuum energy density, and explained the origins of its *repulsive* gravitational nature. In this work we show how to obtain a value for the vacuum energy density which coincides *exactly* with the extremely small observed vacuum energy density. This construction allows also to borrow the results over the past two decades pertaining the study of the Renormalization Group (RG) within the context of Weinberg's Asymptotic Safety scenario. The RG flow behavior of G shows that G *increases* with distance, so that the magnitude of the repulsive force exemplified by $-G < 0$ becomes larger, and larger, as the universe expands. This is what is observed.

In a recent manuscript [2] we envisioned the present Universe's matter density distribution as being proportional to the probability density, in the same vain that one can view an electron orbiting the Hydrogen nucleus as an "electron probability cloud" surrounding the nucleus, and whose mass density distribution is $\rho = m_e \Psi^* \Psi$, where $\Psi(\vec{r})$ are the stationary wave-function solutions to the Schroedinger equation.

The mass density $\rho_m = m\rho$ was postulated to be a solution to the nonlinear quantum-like Bohm-Poisson equation

$$\nabla^2 V_Q \ = \ 4\pi G m \rho_m \ \Rightarrow \ - \ \frac{\hbar^2}{2m} \ \nabla^2 \ (\frac{\nabla^2 \sqrt{\rho}_m}{\sqrt{\rho}_m}) \ = \ 4\pi G m \rho_m \qquad (1)$$

and based on Bohm's quantum potential

*Dedicated to the memory of Ioannis Bakas, a nice friend and great scientist

$$V_Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{2}$$

where the fundamental quantity is *no* longer the wave-function Ψ (complex-valued in general) but the real-valued probability density $\rho = \Psi^*\Psi$.

If one wishes to introduce a temporal evolution to ρ via a Linblad-like equation, for instance, this would lead to an overdetermined system of differential equations for $\rho(\vec{r}, t)$. This problem might be another manifestation of the problem of time in Quantum Gravity. Naively replacing ∇^2 in eqs-(1,2) for the D'Alambertian operator $\nabla_\mu\nabla^\mu, \mu = 0, 1, 2, 3$ has the caveat that in QFT $\rho(x^\mu) = \rho(\vec{r}, t)$ *no* longer has the interpretation of a probability density (it is now related to the particle number current). For the time being we shall just focus on static solutions $\rho(\vec{r})$. Because the de Sitter space metric admits a *static* coordinate system $g_{tt}(r) = -(1 - \frac{\Lambda}{3}r^2); g_{rr}(r) = -(g_{tt})^{-1}, \cdots$ in terms of the cosmological constant $\Lambda = (3/R_H^2)$, where R_H is the Hubble radius, there is no inconsistency in focusing on static solutions $\rho_m(\vec{r})$ for the probability density.

It is straightforward to verify that a spherically symmetric solution to eq-(2) in $D = 3$ is

$$\rho_m(r) = \frac{A}{r^4}, \quad A = -\frac{\hbar^2}{2\pi G m^2} < 0 \tag{3}$$

At first glance, since $\rho_m(r) \leq 0$ one would be inclined to dismiss such solution as being unphysical. Nevertheless, we can bypass this problem by focusing instead on the *shifted* density $\tilde{\rho}_m(r) \equiv \rho_m(r) - \rho_0$ obeying the Bohm-Poisson equation

$$-\frac{\hbar^2}{2m}\nabla^2\left(\frac{\nabla^2\sqrt{\tilde{\rho}_m}}{\sqrt{\tilde{\rho}_m}}\right) = 4\pi G m \tilde{\rho}_m \tag{4}$$

and whose solution for the *shifted* mass density is given by

$$\tilde{\rho}_m = A/r^4 = \rho_m(r) - \rho_o \leq 0, \quad \Rightarrow \rho_m(r) = \frac{A}{r^4} + \rho_0, \quad A = -\frac{\hbar^2}{2\pi G m^2} \tag{5}$$

It is not problematic that the terms inside the square roots are less than zero, since a common factor of $i = \sqrt{-1}$ appears both in the numerator and denominator, and hence it cancels out. The idea now is to focus on the *domain* of values where $\rho_m(r) \geq 0$. And, in doing so, it will allows to show that the value of ρ_0 can be made to coincide exactly with the (extremely small) observed vacuum energy density, by simply introducing an ultraviolet length scale l that is *very close* to the Planck scale, and infrared length scale L equal to Hubble scale R_H.

In particular, the ultraviolet scale l is chosen at the node of $\rho_m(r)$, so

$$\rho_m(r = l) = -\frac{\hbar^2}{2\pi G m^2}\frac{1}{l^4} + \rho_o = 0 \Rightarrow \rho_o = \frac{\hbar^2}{2\pi G m^2}\frac{1}{l^4} \tag{6}$$

The domain of physical values of r must be $r \geq l$ in order to ensure a positive-definite density $\rho_m(r) \geq 0$.

In natural units of $\hbar = c = 1$, introducing the infrared scale $L = R_H$ in the normalization condition (otherwise the mass would diverge) it yields

$$m = \int_l^{R_H} \rho(r)\, 4\pi r^2\, dr = \int_l^{R_H} (\frac{A}{r^4} + \rho_0)\, 4\pi r^2\, dr = \int_l^{R_H} \left(- \frac{1}{2\pi G m^2} \frac{1}{r^4} + \rho_0 \right) 4\pi r^2\, dr \tag{7}$$

Upon performing the integral in eq-(7), after plugging in the value of ρ_0 derived from eq-(6), with the provision that $R_H >> l$, the dominant contribution to the integral stems solely from ρ_o, and one ends up with the following relationship

$$\frac{4\pi R_H^3}{3} \rho_o = \frac{4\pi R_H^3}{3} \frac{1}{2\pi G m^2 l^4} = m \Rightarrow m^3 = \frac{2}{3} \frac{R_H^3}{G l^4} \tag{8}$$

solving for m one gets

$$m = (\frac{2}{3 G l^4})^{1/3} R_H \tag{9}$$

One can verify that when the ultraviolet scale l is chosen to be *very* close to the Planck scale, given by

$$l^4 = \frac{4}{3} L_P^4 \Rightarrow l = (\frac{4}{3})^{1/4} L_p = 1.0745\ L_p \tag{10}$$

and then upon inserting the values for m and l obtained in eqs-(9,10) into the expression for ρ_o *derived* in eq-(6), after setting $L_p^2 = 2G$, [1] it gives in natural units of $\hbar = c = 1$

$$\rho_o = \frac{1}{2\pi G m^2} \frac{1}{l^4} = \frac{1}{2\pi G} (\frac{3\ G\ l^4}{2})^{2/3} \frac{1}{R_H^2\ l^4} = \frac{3}{8\pi G} \frac{L_p^4}{R_H^2\ L_p^4} = \frac{3}{8\pi G R_H^2} \tag{11}$$

which is precisely *equal* to the observed vacuum energy density $\rho = (2\Lambda/16\pi G)$ associated with a cosmological constant $\Lambda = (3/R_H^2)$ and corresponding to a de Sitter expanding universe whose throat size is the Hubble radius R_H.

The physical reason behind the choice of the ultraviolet scale l in eq-(10) is based on re-interpreting ρ_o as the uniform energy (mass) density inside a black hole region of Schwarzschild radius $R = 2Gm$

$$\rho_{bh} = \frac{m}{(4\pi/3) R^3} = \frac{3}{8\pi G R^2}, \quad L_P^2 = 2G, \quad \hbar = c = 1 \tag{12}$$

In the regime $R = 2Gm >> l$, when the dominant contribution to the integral (7) stems from the ρ_o term, we may equate the expression for ρ_o in eq-(6) to ρ_{bh} in eq-(12) giving

$$\frac{1}{2\pi G m^2 l^4} = \frac{1}{2\pi l^4} \frac{(2G)^2}{G R^2} = \frac{1}{2\pi l^4} \frac{L_p^4}{G R^2} = \frac{3}{8\pi G R^2} \Rightarrow l = (\frac{4}{3})^{1/4} L_p, \quad \hbar = c = 1 \tag{13}$$

[1]Some authors absorb the factor of 2 inside the definition of L_p, we define the Planck scale such that the Compton wavelength coincides with the Schwarzschild radius

and leading once again to the value of $l = 1.0745 L_p$ in eq-(10). Therefore, when $R = 2Gm >> l$, the value of l in eqs-(10,13) is always *very* close to the Planck scale, and *independent* of $R = 2Gm$, because the scale R has *decoupled* in eq-(13).

In this way, one can effectively view the observable universe as a "black-hole" whose Hubble radius R_H encloses a mass M_U given by $2GM_U = R_H$. From eq-(14) it follows that when $R = R_H$, the black hole density $\rho_{bh} = \rho_o = \rho_{obs}$ coincides with the observed vacuum energy density. It is well known that inside the black hole horizon region the roles of t and r are exchanged due to the switch in the signature of the g_{tt}, g_{rr} metric components. Cosmological solutions based on this $t \leftrightarrow r$ exchange were provided by the Kantowski-Sachs metric.

To sum up the results in [2] : After applying the Bohm-Poisson equation to the observable Universe as a *whole*, and by introducing an ultraviolet (very close to the Planck scale) and an infrared (Hubble) scale, one can naturally obtain a value for the vacuum energy density which coincides *exactly* with the extremely small observed vacuum energy density. It is remarkable that the Bohm-Poisson equation chooses for us a lower scale to be basically equal to the Planck scale. It was not put it in by hand, but is a direct result of the solutions to the Bohm-Poisson equation. The only assumption made was to choose the Hubble scale R_H for the infrared cutoff, and which makes physical sense since R_H is the cosmological horizon. Is it a numerical coincidence or design ? Because Bohm's formulation of QM is by construction non-local, it is this non-locality which casts light into the crucial ultraviolet/infrared entanglement of the Planck/Hubble scales which was required in order to obtain the observed values of the vacuum energy density.

Furthermore, one can also explain the origins of its *repulsive* gravitational nature. The Bohm-Poisson's (BP) equation is invariant under $\tilde{\rho}_m \rightarrow -\tilde{\rho}_m$, and $G \rightarrow -G$. Consequently $-\tilde{\rho}_m \geq 0$ is a solution to a BP equation associated to a *negative* gravitational coupling $-G < 0$ which is tantamount to *repulsive* gravity. This is perhaps the most salient feature of the results in [2].

Because some readers might object to this construction since $\rho_m(r) = (A/r^4) + \rho_o$, and ρ_o are not actual solutions of the BP equation, but only their $difference$ $\tilde{\rho}_m(r)$ *is* a solution, because the Bohm-Poisson equation is *nonlinear*, we shall show below that there are two scenarios yielding the observed vacuum energy density.

Asymptotic Safety

The Renormalization Group (RG) improvement of Einstein's equations is based on the possibility that Quantum Einstein Gravity might be non-perturbatively renormalizable and asymptotically safe due to the presence of interacting ultraviolet fixed points [8]. In this program one has k (energy) dependent modifications to the Newtonian coupling $G(k)$, the cosmological constant $\Lambda(k)$ and energy-dependent spacetime metrics $g_{ij,(k)}(x)$.

In $D = 4$ there is a nontrivial interacting (non-Gaussian) ultraviolet fixed point $G_* = G(k)k^2 \neq 0$. The fixed point G_* by definition is *dimensionless* and the running gravitational coupling has the form [9], [8]

$$G(k) \ = \ G_N \ \frac{1}{1 + [G_N k^2 / G_*]} \tag{14}$$

The scale dependence of $\Lambda(k)$ in the de Sitter case was found to be [9]

$$\Lambda(k) \ = \ \Lambda_0 \ + \ \frac{b\ G(k)}{4}\ k^4, \quad \Lambda_0 > 0 \tag{15}$$

where b is positive numerical constant.

In $D = 4$, the dimensionless gravitational coupling has a nontrivial fixed point $G = G(k)k^2 \to G_*$ in the $k \to \infty$ limit, and the dimensionless variable $\Lambda = \Lambda(k)k^{-2}$ has also a nontrivial ultraviolet fixed point $\Lambda_* \neq 0$ [9]. The infrared limits are $\Lambda(k \to 0) = \Lambda_0 > 0,\ G(k \to 0) = G_N$. Whereas the ultraviolet limit is $\Lambda(k = \infty) = \infty; G(k = \infty) = 0$.

Let us choose now an actual positive-definite solution $\hat\rho_m \equiv -\tilde\rho_m = |A|/r^4 \geq 0$; $|A| = \hbar^2/2\pi G m^2$, of the BP equation associated to *repulsive* gravity $-G < 0$, as explained earlier

$$-\ \frac{\hbar^2}{2m}\ \nabla^2\ (\frac{\nabla^2\sqrt{-\tilde\rho_m}}{\sqrt{-\tilde\rho_m}}) \ = \ 4\pi\ (-G)\ m\ (-\tilde\rho_m) \tag{16}$$

The mass density solution of (16) to focus on (in $\hbar = c = 1$ units) is

$$\hat\rho_m(r) \ = \ -\ \tilde\rho_m(r) \ = \ \frac{1}{2\pi G m^2 r^4}\ \geq 0 \tag{17}$$

If one selects $m = (1/R_H)$ to coincide with the Compton mass of a particle corresponding to the Hubble scale R_H, then at the Hubble scale $r = R_H$ one has

$$\hat\rho_m(r = R_H) \ = \ \frac{1}{2\pi G m^2 R_H^4} \ = \ \frac{1}{2\pi G\ (R_H)^{-2}\ R_H^4} \ \sim \ (L_P R_H)^{-2} \sim 10^{-122} M^4_{Planck} \tag{18a}$$

and which agrees with the observed vacuum energy density. It is well known (to the experts) that such extremely small value is of the same order of magnitude as $(m_{neutrino})^4$.

The problem arises when one evaluates $\hat\rho_m(r)$ at L_p, given $m = 1/R_H$. One gets a huge value

$$\hat\rho_m(r = L_p) \ \sim \ \frac{1}{G m^2 L_p^4} \ = \ (\frac{R_H}{L_p})^2\ L_p^{-4} \ \sim \ 10^{122}\ M_p^4 \tag{18b}$$

We will see how the Asymptotic Safety scenario comes to our rescue by realizing that a Renormalization Group flow of G and m^2 solves the problem. The key idea, based on dimensional grounds, is simply to postulate that the flow of $m^2(k)$ has the *same* functional form as the flow of $\Lambda(k)$ in eq-(15)

$$m^2(k) \ = \ m_o^2 \ + \ \frac{b\ G(k)}{4}\ k^4, \quad m_o^2 > 0 \tag{19}$$

The only thing remaining is to related the scale r in eq-(17) with the energy (momentum) scale k. The authors [9] expressed k as the inverse of $d(r)$ where $d(r)$ was a proper distance derived from the Schwarzschild metric. If one opts for the simplest choice $k = 1/r$, eq-(17) can be rewritten as

$$\hat{\rho}_m(r) \;=\; \frac{1}{2\pi}\;\frac{1}{G(k)\;m^2(k)\;r^4} \;=\; \frac{1}{2\pi}\;\frac{k^4}{G(k)m^2(k)} \qquad (20)$$

note that strictly speaking eq-(20) is *not* a solution to the BP equation, because if it were one must have that $G(k)m^2(k) = constant$, for all values of k, which is *not* the case. Similarly, the renormalization-group-improved black hole solutions [9] are *not* solutions to the Einstein vacuum field equations [6]. Nevertheless, from eqs-(14,15) one learns that

$$lim_{k\rightarrow 0}\;(G(k)\;m^2(k)) \;=\; G_N\;m_o^2 \qquad (21a)$$

$$lim_{k\rightarrow\infty}\;(G(k)\;m^2(k)) \;=\; \frac{b}{4}\;(G_*)^2 \qquad (21b)$$

whereas at the Planck scale $k = M_p$

$$lim_{k\rightarrow M_p}\;(G(k)\;m^2(k)) \;\sim\; \frac{b}{4}\;(G_*)^2 \qquad (22)$$

Consequently, eqs-(20,22) lead to

$$\hat{\rho}_m(r = L_p) \;\sim\; \frac{1}{2\pi\;G(k = M_p)\;m^2(k = M_p)\;L_p^4} \;\sim\; \frac{2}{b\pi}\;(G_*)^{-2}\;L_p^{-4} \;\sim\; M_p^4 \qquad (23)$$

which is the expected result for the vacuum energy density at the Planck scale.

To sum up, the renormalization group machinery (Asymptotic Safety) can be implemented such that eq-(18a) furnishes the observed vacuum energy density at the Hubble scale, while eq-(23) is the expected vacuum energy density at Planck scale. Naturally, one needs to generalize the BP equation to the fully relativistic regime. As said earlier, we could replace the Laplacian for the D'Alambertian. The key question is what is the "particle" represented by the mass m in the BP equation (16) ? i.e. a mass that experiences a renormalization group flow (19) similar to the flow experienced by Λ (15).

We emphasized earlier the key role that $-G < 0$ plays in all of this and which *stems* directly from the invariance of the BP equation under $\rho \rightarrow -\rho; G \rightarrow -G$. Our solutions for $\hat{\rho}_m(r) \geq 0$ correspond to $-G < 0$, thus the "particle" in question exerts a *repulsive* gravitational force which mimics "dark energy". We can coin such "particle" the "Bohmion". The RG flow behavior of G displayed in eq-(14) shows that G grows as k decreases. Meaning that G *increases* with distance, so that the magnitude of the repulsive force exemplified by $-G < 0$ becomes larger, and larger, as the universe expands. This is what is observed. Next we shall provide a different view of our findings so far.

Matter Creation from the Vacuum

The second interpretation of the solution (17) to the BP equation (16) is that involving matter creation from the vacuum, as advocated by Hoyle long ago. Imagine one pumps matter out of the vacuum in lumps/units of Planck masses. Let us assume that Universe expands in such a way that matter is being replenish from the vacuum so that the mass at any moment is linearly proportional to the size of the Universe. As the mass of the universe grows the vacuum energy density decreases since the vacuum is being depleted. In this scenario, at the Hubble scale R_H, one has $M_U \sim R_H$. This result is also compatible with Mach's principle. By equating GmM_U/R_H to the rest mass m of a particle one arrives at $GM_U = R_H$, which once again is very close to the Schwarzschild radius $2GM_U$. Hence, one arrives at the scaling relation

$$\frac{M_p}{L_P} = \frac{M_U}{R_H} \tag{24}$$

which we interpreted long ago [7] as equating the proper forces (after re-introducing c) $M_U c^2/R_H = M_p c^2/L_p$ and leading to some sort of maximal/minimal acceleration duality.

Inserting the values of M_P, M_U, and $r = L_P$, into the solution (17) of the BP equation gives

$$\hat{\rho}(r = L_p; m = M_p) = \frac{1}{2\pi G M_P^2\ L_p^4} \sim L_p^{-4} = M_p^4, \quad L_p^2 = 2G \tag{24a}$$

which is compatible with the large density at the Planck scale, and

$$\hat{\rho}(r = L_p; m = M_U) = \frac{1}{2\pi G M_U^2\ L_p^4} \sim \frac{G}{2\pi R_H^2\ L_p^4} \sim (L_p\ R_H)^{-2} \tag{24b}$$

which agrees with the observed vacuum energy density at the Hubble scale and obtained above in eq-(11).

To conclude we add some remarks pertaining the Dirac-Eddington large numbers coincidences. Nottale [5] found long ago a direct relationship between the fine structure constant α and the cosmological constant Λ. In $\hbar = c = 1$ units, $\alpha = e^2 = 1/137$, the expression is

$$\Lambda \simeq \frac{(L_p)^4}{(r_e)^6} = \frac{(m_e)^6\ (L_p)^4}{(\alpha)^6} = 10^{-56}\ cm^{-2} \tag{25}$$

the classical electron radius r_e is defined in terms of the charge e, and electron mass m_e, as

$$\frac{e^2}{r_e} = \frac{\alpha}{r_e} = m_e \tag{26}$$

This important relation between Λ and α [5] warrants further investigation within the context of the Bohm-Poisson equation and the Dirac-Eddington large number coincidences.

Finally, we should add that of the many articles surveyed in the literature pertaining the role of Bohm's quantum potential and cosmology, [3], [4] we did not find any related to the Bohm-Poisson equation proposed in this work. [2] The authors [4], for instance, have shown that replacing classical geodesics with quantal (Bohmian) trajectories gives rise to a quantum corrected Raychaudhuri equation (QRE). They derived the second

[2] A Google Scholar search provided the response "Bohm-Poisson equation and cosmological constant did not match any articles"

order Friedmann equations from the QRE, and showed that this also contains a couple of quantum correction terms, the first of which can be interpreted as cosmological constant (and gives a correct estimate of its observed value), while the second as a radiation term in the early universe, which gets rid of the big-bang singularity and predicts an infinite age of our universe.

Acknowledgments

We are indebted to M. Bowers for assistance and Tony Smith for discussions pertaining Armand Wyler's derivation of the fine structure constant and its plausible connection to the Bohm-Poisson equation.

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