Question 404: Three formulas and Some Fractals

Edgar Valdebenito

abstract

This note presents three formulas involving pi and some fractals.

Introduction: three formulas involving π .

For $\alpha>0,\beta>0,\gamma>0,\sqrt{3}\sin\alpha=\sinh\alpha,\left(\sqrt{2}+1\right)\sin\beta=\sinh\beta,\left(2+\sqrt{3}\right)\sin\gamma=\sinh\gamma$ we have:

$$\pi+6\sum_{n=2}^{\infty}\tan^{-1}\left(\frac{\sin(n\alpha)}{\sinh(n\alpha)}\right)=12\sum_{n=0}^{\infty}\frac{e^{-(2n+1)\alpha}\sin((2n+1)\alpha)}{(2n+1)\left(1-2e^{-(2n+1)\alpha}\cos((2n+1)\alpha)+e^{-(4n+2)\alpha}\right)}\qquad(1)$$

$$\pi+8\sum_{n=2}^{\infty}\tan^{-1}\left(\frac{\sin(n\beta)}{\sinh(n\beta)}\right)=16\sum_{n=0}^{\infty}\frac{e^{-(2n+1)\beta}\sin((2n+1)\beta)}{(2n+1)\left(1-2e^{-(2n+1)\beta}\cos((2n+1)\beta)+e^{-(4n+2)\beta}\right)}\qquad(2)$$

$$\pi+12\sum_{n=2}^{\infty}\tan^{-1}\left(\frac{\sin(n\gamma)}{\sinh(n\gamma)}\right)=24\sum_{n=0}^{\infty}\frac{e^{-(2n+1)\gamma}\sin((2n+1)\gamma)}{(2n+1)\left(1-2e^{-(2n+1)\gamma}\cos((2n+1)\gamma)+e^{-(4n+2)\gamma}\right)}\qquad(3)$$

Remark: $\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=3.141592...$

The numbers α,β,γ .

$$\alpha>0,\sqrt{3}\sin\alpha=\sinh\alpha\Rightarrow\alpha=1.2800214...\qquad(4)$$

$$\beta > 0, \left(\sqrt{2}+1\right)\sin\beta = \sinh\beta \Rightarrow \beta = 1.6140273... \tag{5}$$

$$\gamma > 0, \left(2+\sqrt{3}\right)\sin\gamma = \sinh\gamma \Rightarrow \gamma = 1.9547593... \tag{6}$$

Newton fractals for:

$$f(z) = \left(\sinh z - \sqrt{3}\sin z\right)\left(\sinh z - \left(\sqrt{2}+1\right)\sin z\right)\left(\sinh z - \left(2+\sqrt{3}\right)\sin z\right)$$

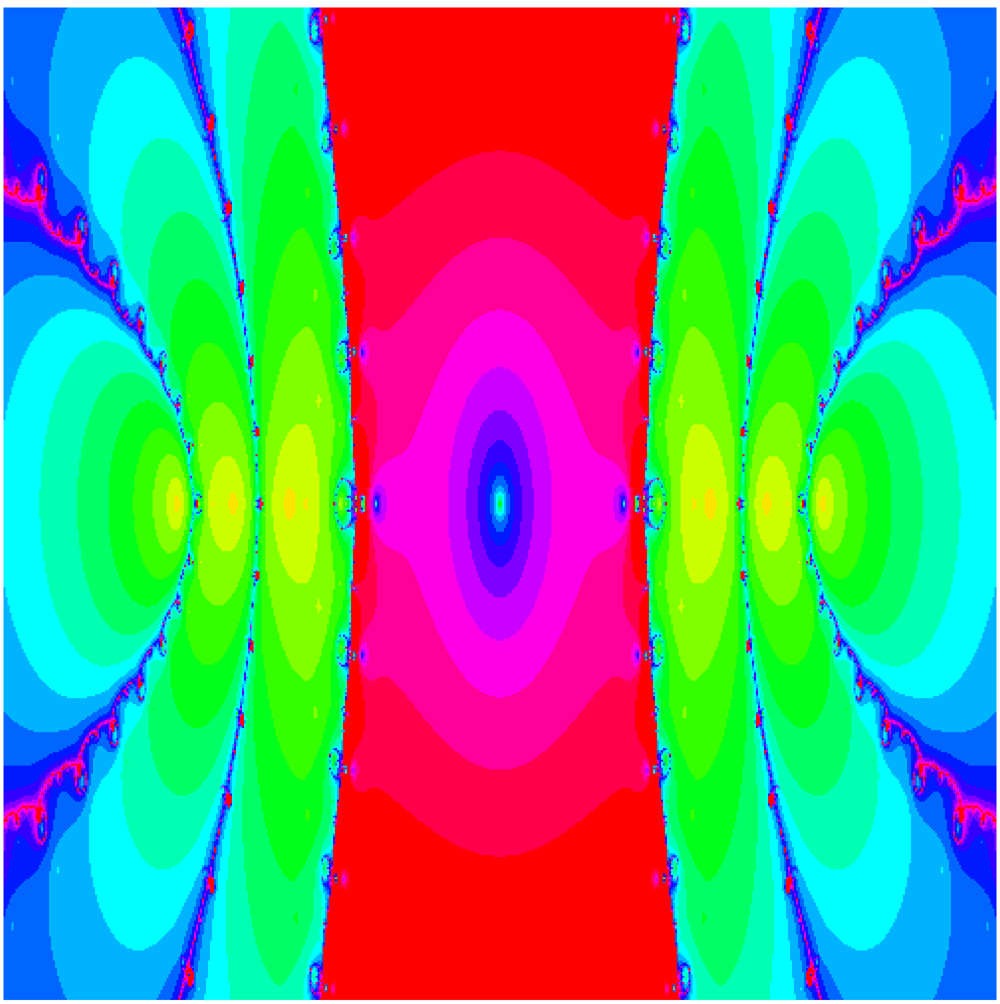

Figure 1. $(bl, ur) = (-3-i, 3+i)$

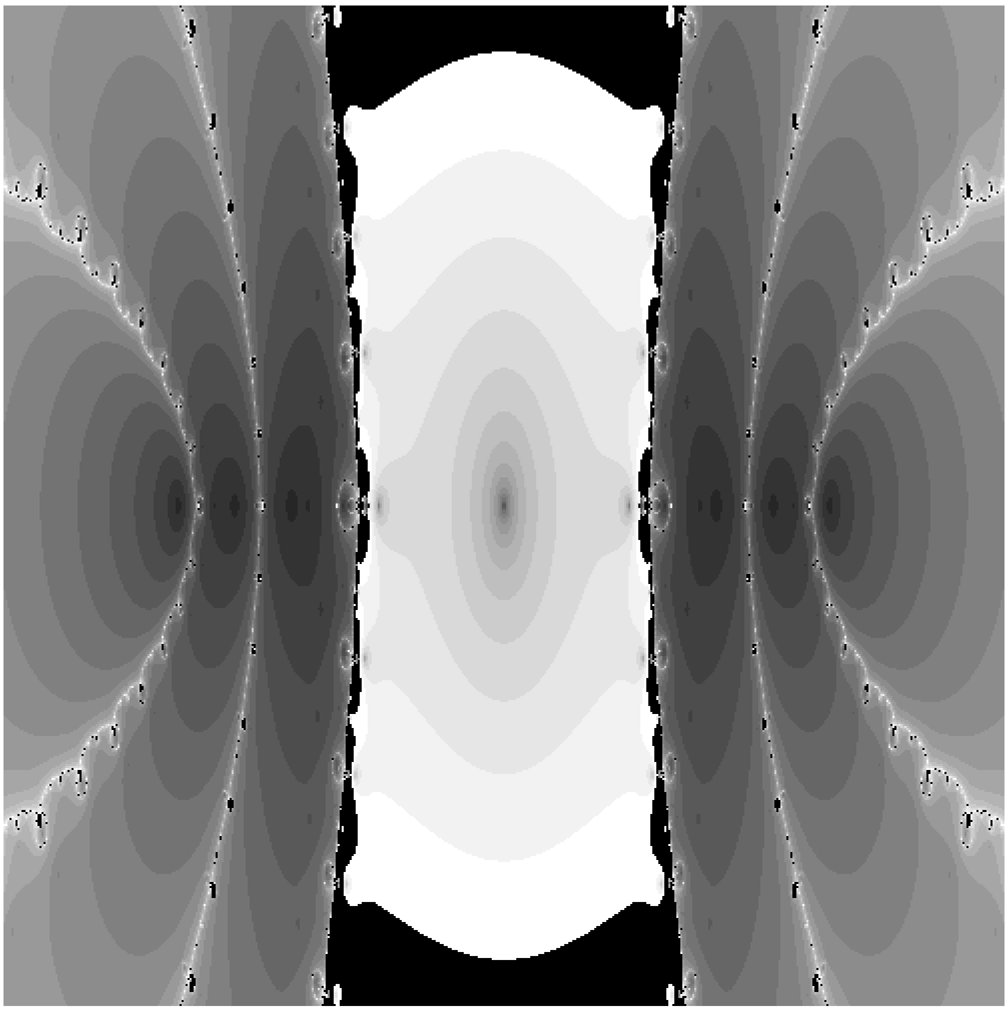

Figure 2. $(bl, ur) = (-3-i, 3+i)$

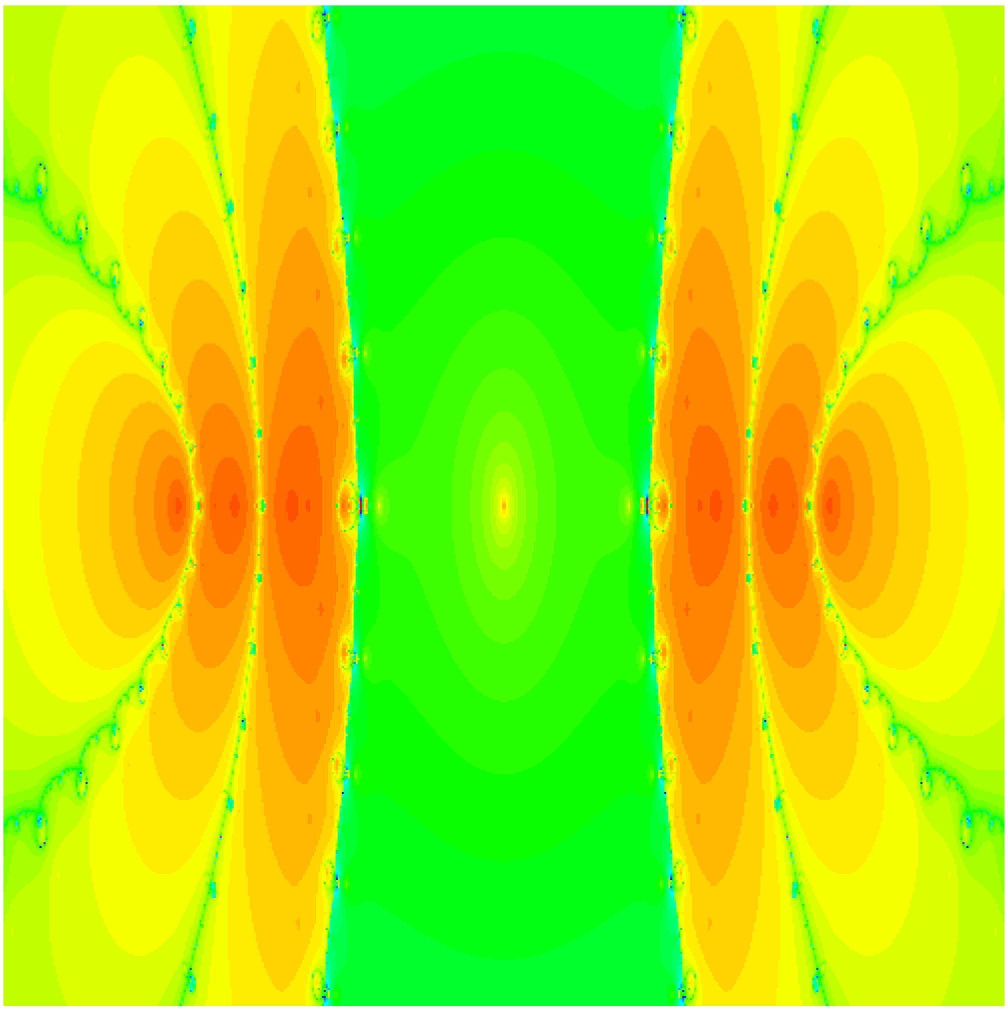

Figure 3. $(bl, ur) = (-3 - i, 3 + i)$

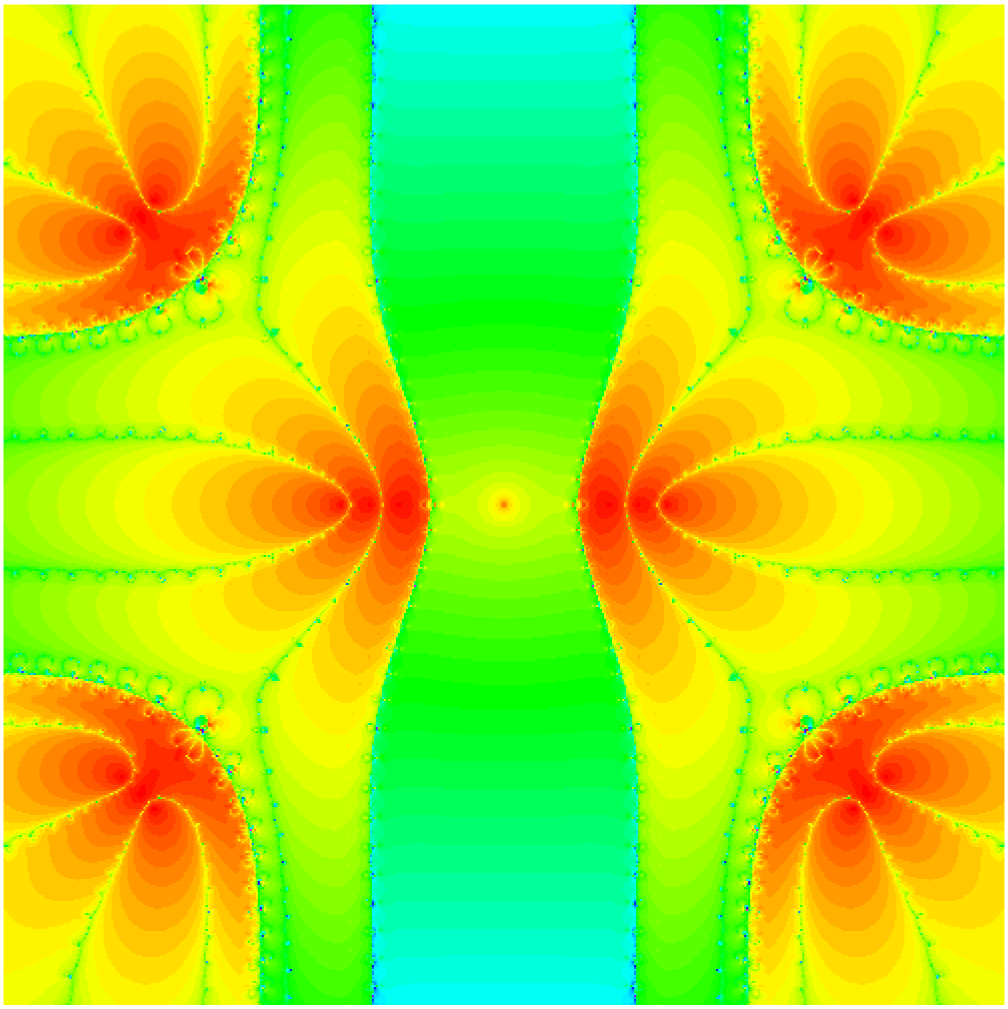

Figure 4. $(bl, ur) = (-6-6i, 6+6i)$

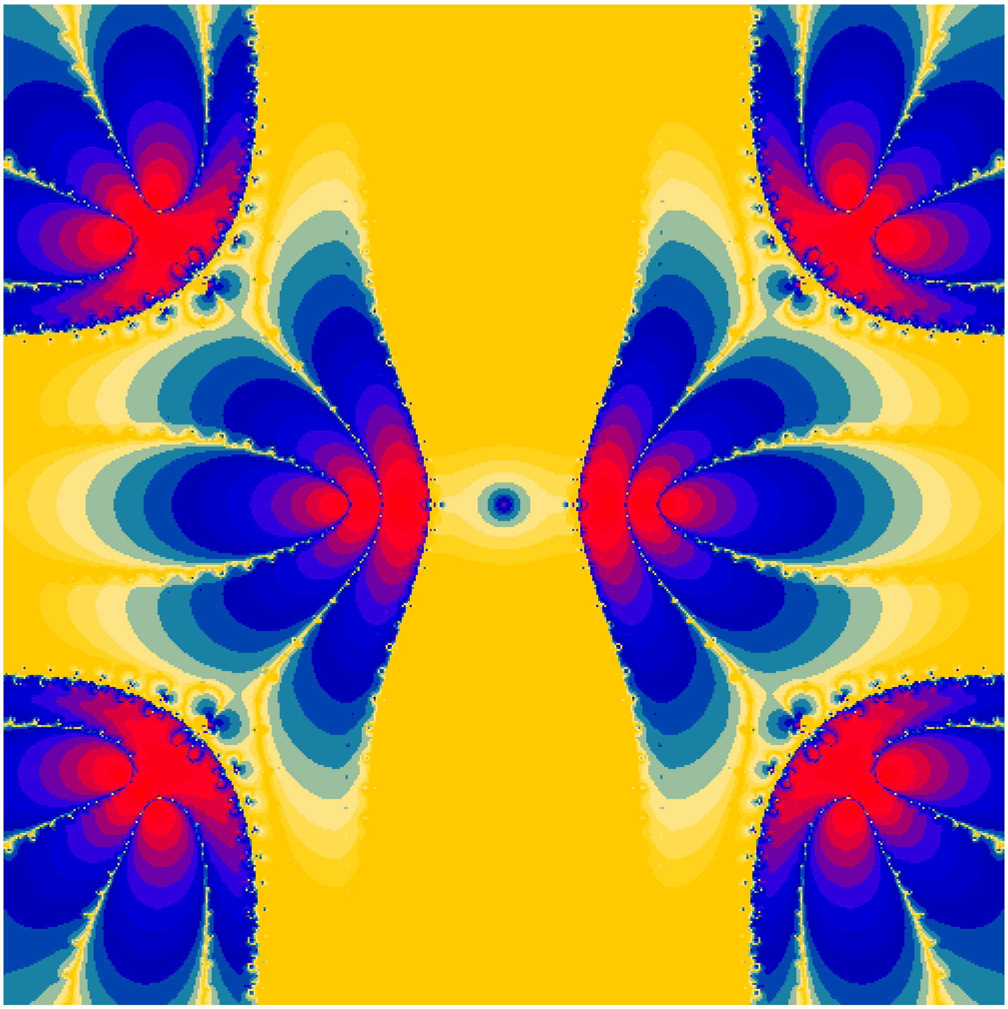

Figure 5. $(bl, ur) = (-6-6i, 6+6i)$

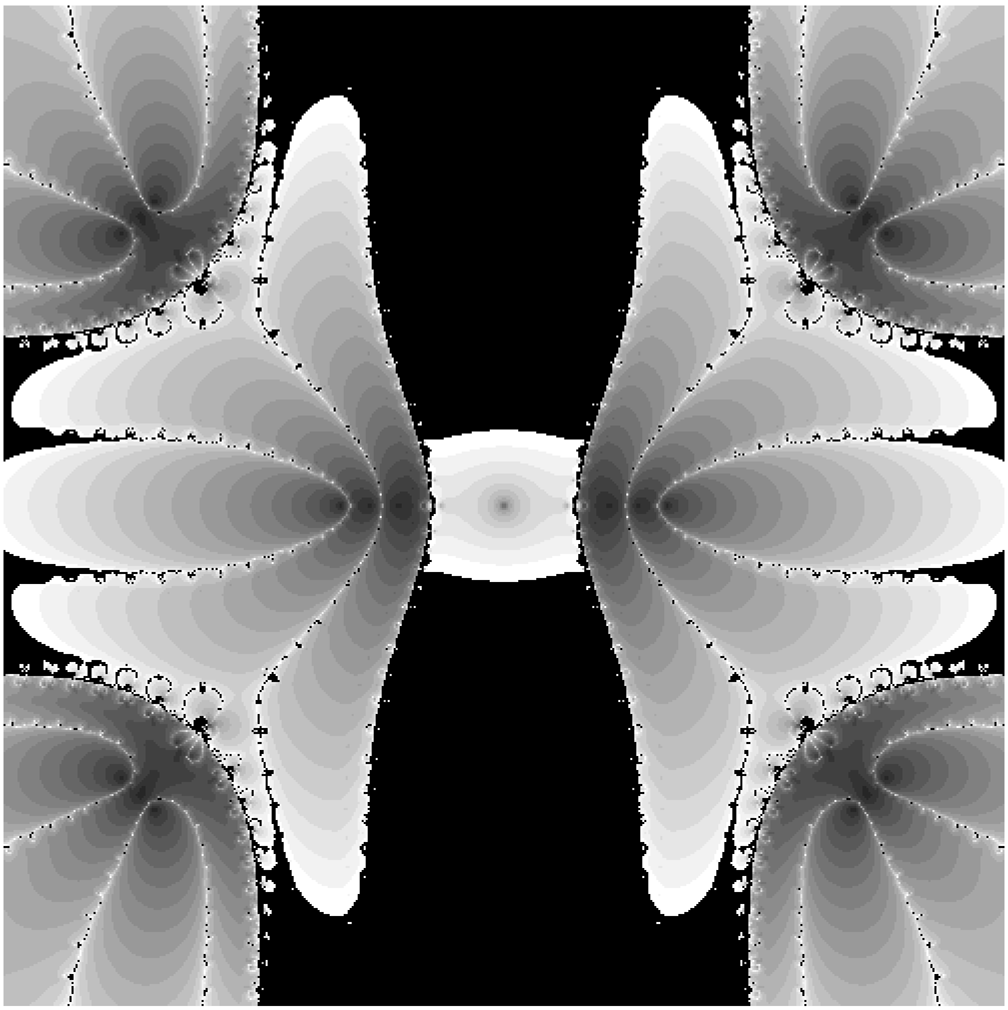

Figure 6. $(bl, ur) = (-6-6i, 6+6i)$

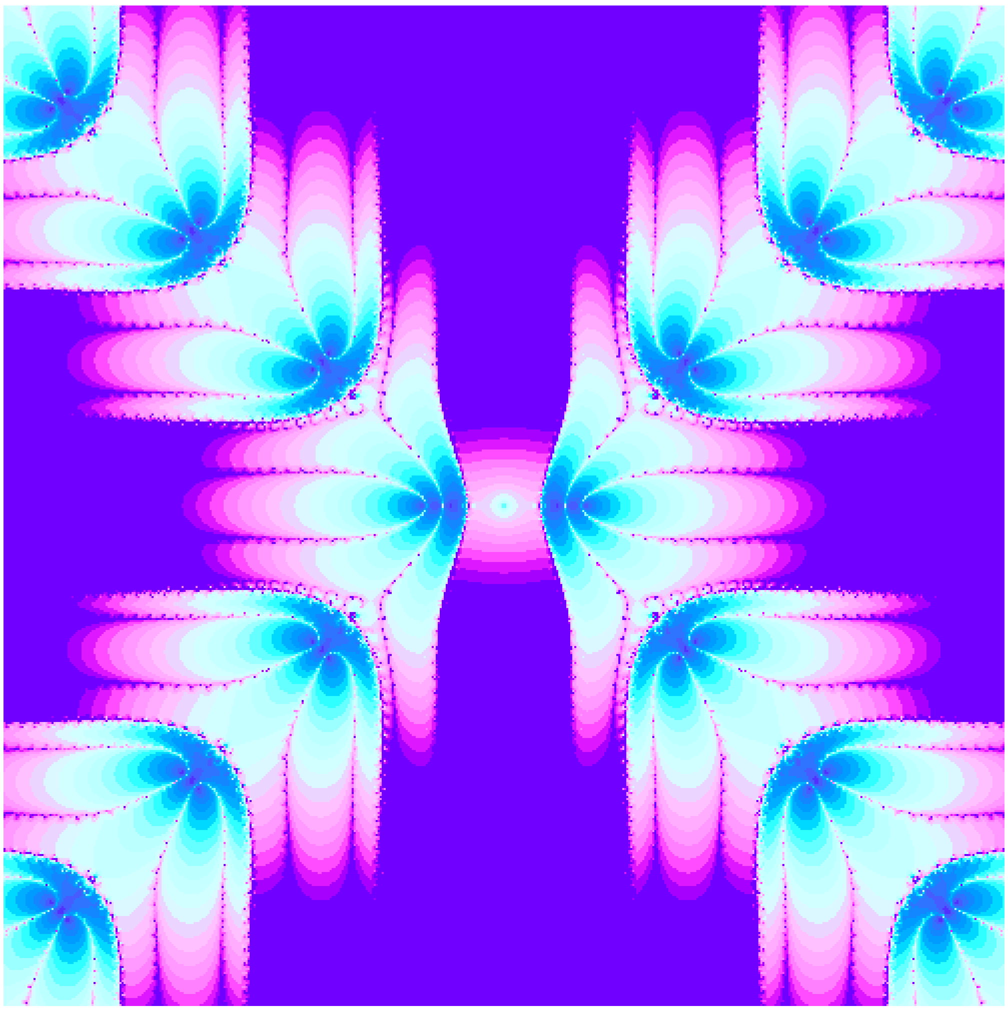

Figure 7. $(bl, ur) = (-12-12i, 12+12i)$

References

1. Berndt, B.: Ramanujan's Notebooks. Part I. Springer-Verlag, 1985.
2. Berndt, B.: Ramanujan's Notebooks. Part IV. Springer-Verlag, 1994.