ON THE MOTION OF QUANTUM PARTICLES AND EUCLIDEAN RELATIVITY

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Abstract: In this work we discuss the motion of quantum particles when they are viewed as three-dimensional Riemannian manifolds by extending the isometric transformations in classical physics to the isometric embedding between smooth manifolds. According to the Whitney embedding theorem, in order to smoothly embed three-dimensional Riemannian manifolds we would need an ambient six-dimensional Euclidean space. As has been shown in our previous works, a six-dimensional Minkowski pseudo-Euclidean spacetime can be obtained by extending one-dimensional temporal continuum to three-dimensional temporal manifold. While the question of whether it is possible to smoothly embed three-dimensional Riemannian manifolds in six-dimensional pseudo-Euclidean spacetime remains, we will show that it is possible to apply the principle of relativity and the postulate of a universal speed to formulate a special theory of relativity in which the geometry of spacetime has a positive definite metric by modifying the Lorentz transformation. The modified Lorentz transformation gives rise to new interesting features, such as there is no upper limit for the relative speed between inertial reference frames, the assumed universal speed is not the speed of any physical object or physical field but rather the common speed of expansion of the spatial space of all inertial frames. Furthermore, we also show that when the relative speed approaches infinite values, there will be a conversion between space and time.

In mathematics and physics, the motion of physical objects in an ambient space can be described by geometric transformations under which the properties of the configuration of the objects remain unchanged, such as isometric transformations that preserve the distance from a configuration space onto itself. In classical dynamics, the motion of solid objects can be described by the Poincaré group, which is the non-abelian Lie group of Minkowski spacetime isometries [1,2]. In our previous works on the quantum structures of elementary particles [3,4], we suggested that instead of viewing elementary particles as point-particles we consider elementary particles as three-dimensional differentiable manifolds, therefore we will need to extend the description of the dynamics of elementary particles in classical physics as point-particles to the dynamics of elementary particles as three-dimensional differentiable manifolds in an ambient space. Being viewed as three-dimensional differentiable manifolds, elementary particles are assumed to possess internal geometrical and topological structures that in turns possess internal symmetries that give rise to intrinsic dynamics. Furthermore, if elementary particles are assumed to remain as stable structures then their intrinsic dynamics should be described by continuous isometric transformations. However, since in our previous works we could only describe the evolution of elementary particles as a change of their geometric structures through evolution processes, such as the Ricci flow, rather than their motion in an ambient space, in this work we would like to discuss further how the dynamics of elementary particles, which are assumed to be three-dimensional differentiable manifolds, can be described in an ambient space. We will assume that the motion of elementary particles is an isometric transformation, which is a continuous isometric embedding into spacetime. The continuous isometric embedding of three-dimensional Riemannian manifolds can also be viewed as geometric solitons which are formed by a continuous process of materialising spacetime structures rather than the motion of a solid physical object through space with respect to time as described in classical physics. Solitons have interesting features such as they can travel only either forward or backward in time. For example, the dynamics of an elementary particle described as a soliton can be formulated by a covariant Ricci flow using the Lie differentiation given by the equation

$$L_X g_{\alpha\beta} = \kappa R_{\alpha\beta} \tag{1}$$

where κ is a dimensional constant, $g_{\alpha\beta}$ is a covariant metric tensor and $R_{\alpha\beta}$ is the Ricci curvature tensor [5]. It should be remarked here that it is possible to suggest that the process of materialisation from spacetime structures under a continuous isometric embedding will disturb the geometric structure of spacetime which in turns will give rise to physical effects which manifest as physical fields, such as the gravitational field or the electromagnetic field. Furthermore, it may also be suggested that the mass of an elementary particle is related to the conformal factor of conformal isometric transformation. In the following we will address the fundamental problem that emerges from the above considerations how a three-dimensional Riemannian manifold can be isometrically embedded in an ambient Euclidean space and we will also show how a six-dimensional Euclidean spacetime with positive definite metric can be constructed from a modified Lorentz transformation.

Let (M^n, g) and (N^m, h) be two Riemannian manifolds. It is shown in differential geometry that the manifolds M^n and N^m are isometric if there exists a diffeomorphism f which preserves the distances. Let x^α, $\alpha = 1,2,\dots,n$ be the local coordinates on the manifold M^n with the Riemannian metric $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ and $y^\beta = f(x^\alpha)$, $\beta = 1,2,\dots,m$ be the local coordinates in the manifold N^m with the Riemannian metric $ds^2 = h_{\alpha\beta}dy^\alpha dy^\beta$, then the two manifolds M^n and N^m are said to be locally isometric if the following condition holds

$$g_{\alpha\beta} = \frac{\partial y^\mu}{\partial x^\alpha}\frac{\partial y^\nu}{\partial x^\beta}h_{\mu\nu} \tag{2}$$

In topology, a topological embedding between topological spaces is a homeomorphism, which is an injective continuous transformation. If the topological spaces are smooth manifolds then the topological embedding is a diffeomorphism and the image is a submanifold of the codomain manifold. According to the Whitney embedding theorem, a manifold of dimension n can be smoothly embedded in the Euclidean space of dimension $2n$ [6,7]. Consider a smooth embedding f of a Riemannian manifold (M^n, g) in the Euclidean space R^m whose Euclidean metric is given by

$$ds^2 = \sum_{\beta=1}^{m} \left(dy^{\beta}\right)^2 \tag{3}$$

With the Euclidean metric given in Equation (3), from Equation (1), we obtain

$$g_{\alpha\beta} = \sum_{\mu=1}^{n} \frac{\partial y^{\mu}}{\partial x^{\alpha}} \frac{\partial y^{\mu}}{\partial x^{\beta}} \tag{4}$$

Equation (4) is a system of $n(n+1)/2$ non-linear partial differential equations in m unknown functions. It is also conjectured in differential geometry that any Riemannian manifold of dimension n can be isometrically embedded in a Euclidean space of dimension $n(n+1)/2$ [6]. In our situation, we consider elementary particles as Riemannian manifolds of dimension three therefore the required ambient Euclidean space must have a dimension six. As discussed in our previous works [8,9], a six-dimensional Euclidean space can be formulated as the union of a three-dimensional spatial manifold and a three-dimensional temporal manifold. However, there are two possibilities that can be considered when we attempt to formulate such a unified spacetime, because a spacetime may be endowed either with a Euclidean metric or a pseudo-Euclidean metric. If the ambient Euclidean space is a pseudo-Euclidean spacetime of signature $(3,3)$ then there remains the question of whether it is possible to embed three-dimensional quantum particles with positive definite metric in such ambient space. It is interesting to note that if a three-dimensional Riemannian spatial manifold can be embedded in a six-dimensional pseudo-Euclidean spacetime of signature $(3,3)$ then there would also exist three-dimensional temporal manifolds that could also be embedded and these temporal manifolds could also exist as physical objects. As an illustration of isometric embedding in pseudo-Euclidean space, consider the case of differentiable manifolds with dimension $n = 2$. According to the Whitney embedding theorem, two-dimensional Riemannian manifolds can be isometrically embedded in the ambient Euclidean space R^3. For example, it is possible to induce a Riemannian metric on a two-dimensional manifold of a two-sheeted hyperboloid from the ambient pseudo-Euclidean space of signature $(2,1)$. A two-sheeted hyperboloid with equation $z^2 - x^2 - y^2 = a^2$ can be parameterised in polar coordinates as $\mathbf{r}(\theta, \phi) = (a\sinh\theta\cos\phi, a\sinh\theta\sin\phi, a\cosh\theta)$. From the pseudo-Riemannian metric of the ambient space $ds^2 = dx^2 + dy^2 - dz^2$ we obtain the Riemannian metric for the two-sheeted hyperboloid as $ds^2 = a^2 d\theta^2 + a^2 \mathrm{shin}^2\theta d\phi^2$[10].

We will now discuss the possibility to formulate a Euclidean relativity so that spacetime has a positive definite Euclidean metric instead of a pseudo-Euclidean metric so that the Whitney embedding theorem can be applied. In physics, the concept of a pseudo-Euclidean spacetime was introduced by Minkowski in order to accommodate Einstein theory of special relativity in which the coordinate transformation between the inertial frame S with spacetime coordinates (ct, x, y, z) and the inertial frame S' with coordinates (ct', x', y', z') are derived from the principle of relativity and the postulate of a universal speed c. The coordinate transformation is the Lorentz transformation given by

$$x' = \gamma(x - \beta ct) \tag{5}$$

$$y' = y \tag{6}$$

$$z' = z \tag{7}$$

$$ct' = \gamma(-\beta x + ct) \tag{8}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ [11]. It is shown that the Lorentz transformation given in Equations (5-8) leaves the Minkowski spacetime interval $-c^2t^2 + x^2 + y^2 + z^2$ invariant. Spacetime with this metric is a pseudo-Euclidean space. We now show that it is possible to construct a special relativistic transformation that will make spacetime a Euclidean space rather than a pseudo-Euclidean space as in the case of the Lorentz transformation. Consider the following modified Lorentz transformation

$$x' = \gamma_E(x - \beta ct) \tag{9}$$

$$y' = y \tag{10}$$

$$z' = z \tag{11}$$

$$ct' = \gamma_E(\beta x + ct) \tag{12}$$

where $\beta = v/c$ and γ_E will be determined from the principle of relativity and the postulate of a universal speed of a physical field. Instead of the invariance of the Minkowski spacetime interval, if we now assume the invariance of the Euclidean interval $c^2t^2 + x^2 + y^2 + z^2 = c^2t'^2 + x'^2 + y'^2 + z'^2$ then from the modified Lorentz transformation given in Equations (9-12), we obtain the following expression for γ_E

$$\gamma_E = \frac{1}{\sqrt{1+\beta^2}} \tag{13}$$

It is seen from the expression of γ_E given in Equation (13) that there is no upper limit in the relative speed v between inertial frames. The value of γ_E at the universal speed $v = c$ is $\gamma_E = 1/\sqrt{2}$. For the values of $v \ll c$, the modified Lorentz transformation given in Equations (9-12) also reduces to the Galilean transformation. However, it is interesting to observe that when $v \to \infty$ we have $\gamma_E \to 0$ and $\beta\gamma_E \to 1$, and in this case we have $x' \to -ct$ and $ct' \to x$. This result shows that there is a conversion between space and time when $v \to \infty$. As in the case of the Lorentz transformation given in Equations (5-8), we can also derive the relativistic kinematics from the modified Lorentz transformation given in Equations (9-12), such as the transformation of a length, the transformation of a time interval and the transformation of velocities. Let L_0 be the proper length then the length transformation can be found as

$$L = \sqrt{1+\beta^2}L_0 \tag{14}$$

It is observed from the length transformation given in Equation (14) that the length of a moving object is expanding rather than contracting as in Einstein theory of special relativity.

Now if Δt_0 is the proper time interval then the time interval transformation can also be found to be given by the relation

$$\Delta t = \frac{1}{\sqrt{1+\beta^2}}\Delta t_0 \tag{15}$$

It is also observed from the time interval transformation given in Equation (15) that the proper time interval is longer than the same time interval measured by a moving observer. With the modified Lorentz transformation given in Equations (9-12), the transformation of velocities can be found as follows

$$v'_x = \frac{dx'}{dt'} = \frac{v_x - \beta c}{1 + \frac{\beta v_x}{c}} \tag{16}$$

$$v'_y = \frac{dy'}{dt'} = \frac{v_y}{\gamma_E\left(1 + \frac{\beta v_x}{c}\right)} \tag{17}$$

$$v'_z = \frac{dz'}{dt'} = \frac{v_z}{\gamma_E\left(1 + \frac{\beta v_x}{c}\right)} \tag{18}$$

Form Equation (16), if we let $v_x = c$ then we obtain $v'_x = \left(\frac{c-v}{c+v}\right)c$. Therefore in this case $v'_x = c$ only when the relative speed v between two inertial frames vanishes, $v = 0$. In other words, the universal speed c is not the common speed of any moving physical object or physical field in inertial reference frames. In order to specify the nature of the assumed universal speed we observe that in Einstein theory of special relativity it is assumed that spatial space of an inertial frame remains steady and this assumption is contradicted to Einstein theory of general relativity that shows that spatial space is actually expanding. Therefore it seems reasonable to suggest that the universal speed c in the modified Lorentz transformation given in Equations (9-12) is the universal speed of expansion of the spatial space of all inertial frames.

With the modified Lorentz transformation, it is possible to formulate a six-dimensional Euclidean spacetime that would comply with the Whitney embedding theorem of differentiable manifolds. Even though it is straightforward to develop and show that the effects on physical measurements due to the modified Lorentz transformation are opposite to those due to the Lorentz transformation in Einstein theory of special relativity, as shown above for the length transformation and the time interval transformation, we will leave possible developments of the Euclidean relativity that follows from the modified Lorentz transformation for further, especially experimental, investigations.

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