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Solution of Maxwell's equations for a capacitor with alternating voltage

Contents

Annotation

A solution of the Maxwell equations for a variable voltage capacitor with a variable voltage is given, which is a development of inconsistency (corresponding to the energy conservation law) solution of the Maxwell equations for vacuum. It is shown that in an electromagnetic wave propagating through a capacitor, the flux of electromagnetic energy does not change with time. It is shown that there exists a longitudinal (along the radius) standing electromagnetic wave. For a simple verification of the findings, detailed proof is given.

1. Introduction

In [1, 2], a new solution of the Maxwell equations is proposed for a monochromatic wave in a nonconducting medium. The dielectric of the capacitor is also such a medium. If a monochromatic alternating voltage is present on the capacitor plates, then a monochromatic wave with electric and magnetic intensities should also be present in its dielectric. This wave propagates between the capacitor plates. This wave propagates between the capacitor plates. According to the existing concept, in the energy flow through the capacitor only the average (in time) value of the energy flux is conserved [3]. This contradicts the law of conservation of energy (this was already discussed in [1, 2] for a traveling wave). Therefore, a new solution of Maxwell's equations for a capacitor is proposed below.

The Maxwell equations for free electromagnetic oscillations in an unbounded medium have the form

$$\mathrm{rot}(E) + \mu \frac{\partial H}{\partial t} = 0, \tag{1}$$

$$\mathrm{rot}(H) - \varepsilon \frac{\partial E}{\partial t} = 0, \tag{2}$$

$$\mathrm{div}(E) = 0, \tag{3}$$

$$\mathrm{div}(H) = 0. \tag{4}$$

In [1, 2] the solution of these equations was obtained under the assumption that $E_z \equiv 0$. Below this restriction is removed.

2. Solution of the Maxwell's equations

As in [1, 2], we will use cylindrical coordinates r, φ, z and apply the following notation:

$$\mathrm{co} = \cos(\alpha\varphi + \chi z + \omega t), \tag{1}$$

$$\mathrm{si} = \sin(\alpha\varphi + \chi z + \omega t), \tag{2}$$

where α, χ, ω are some constants. We represent the unknown functions in the following form:

$$H_r. = h_r(r) \cdot \mathrm{co}, \tag{3}$$

$$H_\varphi. = h_\varphi(r) \cdot \mathrm{si}, \tag{4}$$

$$H_z. = h_z(r) \cdot \mathrm{si}, \tag{5}$$

$$E_r. = e_r(r) \cdot \mathrm{si}, \tag{6}$$

$$E_\varphi. = e_\varphi(r) \cdot \mathrm{co}, \tag{7}$$

$$E_z. = -e_z(r) \cdot \mathrm{co}. \tag{8}$$

Then the system of Maxwell's equations takes the form:

$$\frac{e_r(r)}{r} + e_r'(r) - \frac{e_\varphi(r)}{r}\alpha + \chi \cdot e_z(r) = 0, \tag{9}$$

$$\frac{e_z(r)}{r}\alpha + e_\varphi(r)\chi - \mu\omega h_r(r) = 0, \tag{10}$$

$$e_r(r)\chi + e_z'(r) + \mu\omega h_\varphi(r) = 0, \tag{11}$$

$$\frac{e_\varphi(r)}{r} + e'_\varphi(r) - \frac{e_r(r)}{r} \cdot \alpha + \mu\omega h_z(r) = 0, \tag{12}$$

$$\frac{h_r(r)}{r} + h'_r(r) + \frac{h_\varphi(r)}{r}\alpha - \chi \cdot h_z(r) = 0, \tag{13}$$

$$-\frac{h_z(r)}{r}\alpha - h_\varphi(r)\chi - \varepsilon\omega e_r(r) = 0, \tag{14}$$

$$-h_r(r)\chi - h'_z(r) + \varepsilon\omega e_\varphi(r) = 0, \tag{15}$$

$$\frac{h_\varphi(r)}{r} + h'_\varphi(r) + \frac{h_r(r)}{r} \cdot \alpha - \varepsilon\omega e_z(r) = 0, \tag{16}$$

where $h(r)$, $e(r)$ are some functions of coordinate r.

Here we can not use the solution obtained in [1, 2], since there in the search for a solution it was assumed that $e(r) \equiv 0$. Here such an assertion is not satisfied by the condition of the problem.

We will seek a solution in which the tensions are related by the relation

$$h_z(r) \equiv 0, \tag{17}$$

which follows from physical considerations. Then the system of equations (9-16) takes the form:

$$\frac{e_r(r)}{r} + e'_r(r) - \frac{e_\varphi(r)}{r}\alpha + \chi \cdot e_z(r) = 0, \tag{18}$$

$$\frac{e_z(r)}{r}\alpha + e_\varphi(r)\chi - \mu\omega h_r(r) = 0, \tag{19}$$

$$e_r(r)\chi + e'_z(r) + \mu\omega h_\varphi(r) = 0, \tag{20}$$

$$\frac{e_\varphi(r)}{r} + e'_\varphi(r) - \frac{e_r(r)}{r} \cdot \alpha = 0, \tag{21}$$

$$\frac{h_r(r)}{r} + h'_r(r) + \frac{h_\varphi(r)}{r}\alpha = 0, \tag{22}$$

$$-h_\varphi(r)\chi - \varepsilon\omega e_r(r) = 0, \tag{23}$$

$$-h_r(r)\chi + \varepsilon\omega e_\varphi(r) = 0, \tag{24}$$

$$\frac{h_\varphi(r)}{r} + h'_\varphi(r) + \frac{h_r(r)}{r} \cdot \alpha - \varepsilon\omega e_z(r) = 0. \tag{25}$$

In Appendix 1 it is shown that there exists a definite Bessel function, denoted as $F_\alpha(r)$, on which the functions of intensities depend, namely

$$e_z(r) = F_\alpha(r),$$

$$e_\varphi(r) \equiv \frac{1}{r}F_\alpha(r),\ h_r(r) \equiv \frac{1}{r}F_\alpha(r),$$

$$e_r(r) \equiv \frac{d}{dt}F_\alpha(r),\ h_\varphi(r) \equiv \frac{d}{dt}F_\alpha(r).$$

More precisely,

$$e_z(r) = F_\alpha(r), \tag{26}$$

$$e_z'(r) = \frac{d}{dt}F_\alpha(r), \tag{27}$$

$$e_r(r) = \frac{\chi}{q}e_z'(r), \tag{28}$$

$$e_\varphi(r) = -\frac{\chi\alpha}{q}\frac{e_z(r)}{r}, \tag{29}$$

$$h_r(r) = \frac{\varepsilon\omega}{\chi}e_\varphi(r), \tag{30}$$

$$h_\varphi(r) = -\frac{\varepsilon\omega}{\chi}e_r(r), \tag{31}$$

where

$$q = \chi^2 - \mu\varepsilon\omega^2 > 0\,. \tag{32}$$

The function $F_\alpha(r)$ is a solution of the equation

$$e_z''(r) + \frac{e_z'(r)}{r} + e_z(r)\cdot\left(q - \frac{\alpha^2}{r^2}\right) = 0\,. \tag{33}$$

For the existence of this solution, the **quantity** q **must be positive.**

3. Velocity of electromagnetic wave propagation

It was shown in [1, 2] that in such a solution for a free wave propagating at the velocity of light,

$$\chi = \pm\omega\sqrt{\mu\varepsilon} = \pm\frac{\omega}{c}\,. \tag{1}$$

In the case under consideration, the quantity (2.32) must be positive, i.e.

$$\chi^2 - \mu\varepsilon\omega^2 \geq 0 \tag{2}$$

or

$$\chi \geq \left|\omega\sqrt{\mu\varepsilon}\right| = \frac{\omega}{c}\text{, причем } \chi_{\min} = \frac{\omega}{c}\,. \tag{3}$$

Obviously, this velocity is equal to the derivative $\frac{dz}{dt}$ of the function given implicitly in the form (2.3-2.8). Having determined the derivative of these functions $z(t)$, we find the propagation velocity of a monochromatic electromagnetic wave

$$v_m = \frac{dz}{dt} = -\frac{\omega}{\chi}. \tag{4}$$

Combining (3, 4), we obtain:

$$v_m = \left| \frac{\omega}{\chi \geq \left| \omega\sqrt{\mu\varepsilon} \right|} \right| \leq \frac{1}{\left| \geq \sqrt{\mu\varepsilon} \right|} \leq \frac{1}{\left| \geq \frac{1}{c} \right|}. \tag{5}$$

So,

$$v_m \leq c. \tag{6}$$

Consequently, the propagation velocity of the electromagnetic wave in the capacitor **is less** than the velocity of light.

4. Energy density

The energy density is

$$W = \left(\frac{\varepsilon}{2} E^2 + \frac{\mu}{2} H^2 \right) \tag{1}$$

or, taking into account previous formulas,

$$W = \left\{ \begin{array}{l} \frac{\varepsilon}{2} \left((e_r(r)\mathrm{si})^2 + (e_\varphi(r)\mathrm{co})^2 + (e_z(r)\mathrm{co})^2 \right) + \\ + \frac{\mu}{2} \left((h_r(r)\mathrm{co})^2 + (h_\varphi(r)\mathrm{si})^2 \right) \end{array} \right\} \tag{2}$$

Taking into account (2.29, 2.30), we obtain:

$$W = \left\{ \begin{array}{l} \frac{\varepsilon}{2} \left((e_r(r)\mathrm{si})^2 + (e_\varphi(r)\mathrm{co})^2 + (e_z(r)\mathrm{co})^2 \right) + \\ + \left(\frac{\varepsilon\omega}{\chi} \right)^2 \frac{\mu}{2} \left((e_\varphi(r)\mathrm{co})^2 + (e_r(r)\mathrm{si})^2 \right) \end{array} \right\}$$

or

$$W = \left\{ \frac{\varepsilon}{2} (e_z(r)\mathrm{co})^2 + \left(\left(\frac{\varepsilon\omega}{\chi} \right)^2 \frac{\mu}{2} + \frac{\varepsilon}{2} \right) \left((e_\varphi(r)\mathrm{co})^2 + (e_r(r)\mathrm{si})^2 \right) \right\} \tag{3}$$

Thus, **<u>electromagnetic wave energy density in condenser is constant in time and equal in all points of the cylinder of given radius</u>**.

5. Energy Flows

The density of electromagnetic flow is Umov-Pointing vector

$$S = \eta E \times H \,, \tag{1}$$

where

$$\eta = c/4\pi \,. \tag{2}$$

In cylindrical coordinates $r,\ \varphi,\ z$ the flux density of electromagnetic energy has three components $S_r,\ S_\varphi,\ S_z$ directed along the radius, along the circumference, along the axis, respectively. They are determined by the formula (as shown in [1, 2])

$$S = \begin{bmatrix} S_r \\ S_\varphi \\ S_z \end{bmatrix} = \eta\left(E \times H\right) = \eta \begin{bmatrix} E_\varphi H_z - E_z H_\varphi \\ E_z H_r - E_r H_z \\ E_r H_\varphi - E_\varphi H_r \end{bmatrix}. \tag{3}$$

or, taking into account previous formulas,

$$S = \begin{bmatrix} S_r \\ S_\varphi \\ S_z \end{bmatrix} = \eta \begin{bmatrix} s_r \cdot si^2 \\ s_\varphi \cdot si \cdot co \\ s_z \cdot si \cdot co \end{bmatrix}. \tag{4}$$

where

$$\begin{aligned} s_r &= \left(e_\varphi h_z - e_z h_\varphi\right) \\ s_\varphi &= \left(e_z h_r - e_r h_z\right). \\ s_z &= \left(e_r h_\varphi - e_\varphi h_r\right) \end{aligned} \tag{5}$$

Taking into account (5, 2.27-2.31), we obtain:

$$s_r = -e_z h_\varphi = e_z \frac{\varepsilon\omega}{\chi} e_r = -e_z \left(\frac{\varepsilon\omega}{q}\right) e_z' \,, \tag{7}$$

$$s_\varphi = \left(e_z h_r\right) = e_z \frac{\varepsilon\omega}{\chi} e_\varphi = -\frac{\varepsilon\omega\alpha}{q} \frac{e_z^2}{r} \,, \tag{8}$$

$$s_z = \left(e_r h_\varphi - e_\varphi h_r\right) = -\frac{\varepsilon\omega}{\chi}\left(e_r^2 + \alpha e_\varphi^2\right). \tag{9}$$

Поток энергии, который распространяется по радиусу <u>из всей окружности</u> данного радиуса, как следует из (4), равен

$$\overline{S_r} = \eta \int_0^{2\pi} - e_z \left(\frac{\varepsilon\omega}{q} \right) e_z' \cdot \mathrm{si}^2 \cdot r \cdot d\varphi = \eta \frac{\varepsilon\omega}{q} \cdot e_z e_z' \cdot r \int_0^{2\pi} \mathrm{si}^2 \cdot d\varphi . \qquad (10)$$

We call this flow a radial flow of energy. The integral in (10) is a constant. In Appendix 3 shows that the quantity $\Phi = \left(e_z e_z' \cdot r \right)$ is a periodic function of r. This means that the **radial energy flux varies along the radius, and its total value is zero**.

The energy flow that propagates along a circle of a given radius, as follows from (4), is equal to

$$\overline{S_r} = -\eta \int_0^{2\pi} \frac{\varepsilon\omega\alpha}{q} \frac{e_z^2}{r} \cdot \mathrm{co} \cdot \mathrm{si} \cdot r \cdot d\varphi = -\eta \frac{\varepsilon\omega\alpha}{q} \cdot e_z^2 \int_0^{2\pi} \mathrm{co} \cdot \mathrm{si} \cdot d\varphi . \qquad (10a)$$

The integral in (10a) is a constant. In Appendix 3 it is shown that the value $\left(e_z^2 \right)$ is significant only at the center of the capacitor.

The energy flow, which propagates along the axis oz through the cross section of the condenser, is equal to

$$\overline{S_z} = \eta \iint_{r,\varphi} \left[s_z \cdot \mathrm{si} \cdot \mathrm{co} \right] dr \cdot d\varphi . \qquad (11)$$

Taking (9) into account, we obtain:

$$\overline{S_z} = -\frac{\varepsilon\omega}{\chi} \eta \iint_{r,\varphi} \left[\left(e_r^2 + \alpha e_\varphi^2 \right) \mathrm{si} \cdot \mathrm{co} \right] dr \cdot d\varphi \qquad (12)$$

or

$$\overline{S_z} = -\frac{\varepsilon\omega}{\chi} \eta \left(\int_r \left(e_r^2 + \alpha e_\varphi^2 \right)^2 dr \right) \left(\int_\varphi \mathrm{si} \cdot \mathrm{co} \cdot d\varphi \right) \qquad (13)$$

Both integrals in (13) are constants that do not depend on the coordinates z and t (as shown in [1, 2]). Consequently, **the energy flux of the electromagnetic wave is constant in time**. This flow is the active power $P = \overline{S_z}$, transmitted through the capacitor. This power does not depend on the design of the capacitor. The magnitude of the power does not depend on the intensities. There is only one parameter, which is not defined in the mathematical model of the wave - it is a parameter χ and power depends on it. More precisely, on the contrary, the power $P = \overline{S_z}$ determines the value of the parameter χ.

6. Radial wave

In the capacitor there is a wave along the radius with the intensities

$$H_r. = h_r(r) \cdot \cos(\alpha\varphi + \chi z + \omega t),$$

$$E_r. = e_r(r) \cdot \sin(\alpha\varphi + \chi z + \omega t)$$

- see (2.3) and (2.6). They correspond to the radial energy flux (5.10) considered above. It can be seen that these intensities are <u>shifted in phase by a quarter of a period</u>. In Appendix 3 shows the dependencies of these intensities and the energy flux on the radius. It can be seen that these tensions constitute a <u>longitudinal standing wave</u>, oscillating <u>along</u> the radius.

7. Voltage in the capacitor

The voltages in the solution found are determined to within a constant factor. For example, the intensity (2.8) should be written, taking into account (2.26) in the form:

$$E_z. = -A \cdot F_\alpha(r)\cos(\alpha\varphi + \chi z + \omega t), \quad (1)$$

where A is an indefinite constant for all the intensities.

We assume that the potential on the lower plate for $z = 0$ and some φ_o, r_o is zero, and the potential on the upper plate for $z = d$ and same φ_o, r_o is numerically equal to the voltage U across the capacitor. Then

$$U = -A \cdot F_\alpha(r_o)\cos(\alpha\varphi_o + \chi d + \omega t), \quad (2)$$

what can be used to determine the coefficient A. At some intermediate value z, the voltage for the same φ_o, r_o will be equal to

$$u(z) = -A \cdot F_\alpha(r_o)\cos(\alpha\varphi_o + \chi z + \omega t), \quad (3)$$

i.e. the voltage along the capacitor varies in function $\cos(\chi z)$.

8. Discussion

The proposed solution of the Maxwell equations for a capacitor under an alternating voltage is interpreted as an electromagnetic wave with three electric intensities and two magnetic intensities (there is no magnetic field directed along the axis of the capacitor). We note the following features of this wave:

1. Magnetic and electrical intensities on a certain coordinate axis r, φ, z are shifted in phase by a quarter of a period.

2. The vectors of electric and magnetic intensities are orthogonal.

3. The instantaneous (and not the average for a certain period) energy flow through the capacitor does not change in time, which corresponds to the law of conservation of energy.

4. The energy flow is equal to the active power transmitted through the capacitor.

5. The velocity of propagation of an electromagnetic wave is less than the velocity of light

6. This velocity decreases with transmitted power (in particular, in the absence of power, the velocity is zero and the wave becomes standing)

7. The wave propagates along the radii; the intensities vary as the Bessel function of the radius.

8. There is a longitudinal standing wave in which the intensities and the energy flux oscillate along the radius; the total value of the energy flux is zero.

Appendix 1

Denote by:

$$e_{r\varphi} = e_r + e_\varphi, \tag{1}$$

Suppose, that

$$e_{r\varphi} = e_r + e_\varphi = \vartheta(h_\varphi - h_r) \tag{2}$$

Let us find the sum of the equations (2.19, 2.20):

$$e_{r\varphi} g + \frac{e_z}{r}\alpha + e_z' = 0\,. \tag{3}$$

where

$$g = -\left(\chi + \frac{\mu\omega}{\vartheta}\right). \tag{4}$$

Let us find the sum of the equations (2.18, 2.21):

$$e_{\varphi r}' + \frac{e_{\varphi r}}{r}\cdot(1-\alpha) + \chi e_z = 0\,. \tag{5}$$

From (3) we find:

$$e_{r\varphi} = -\left(e_z' + \frac{e_z}{r}\alpha\right)\frac{1}{g}, \tag{6}$$

$$e'_{r\varphi} = -\left(e''_z + \frac{e'_z}{r}\alpha - \frac{e_z}{r^2}\alpha \right)\frac{1}{g}. \quad (7)$$

From (5-7) we find:

$$\left(e''_z + \frac{e'_z}{r}\alpha - \frac{e_z}{r^2}\alpha \right)\frac{1}{g} + \left(e'_z + \frac{e_z}{r}\alpha \right)\frac{1}{g}\frac{1}{r}\cdot(1-\alpha) - \chi e_z = 0, \quad (8)$$

or

$$\left(e''_z + \frac{e'_z}{r}\alpha - \frac{e_z}{r^2}\alpha \right) + \left(e'_z + \frac{e_z}{r}\alpha \right)\frac{(1-\alpha)}{r} + e_z q = 0, \quad (9)$$

where

$$q = -g\chi. \quad (10)$$

After simplifying (9), we obtain:

$$e''_z + \frac{e'_z}{r} + e_z\left(q - \frac{\alpha^2}{r^2} \right) = 0. \quad (11)$$

It will be shown below, that $q > 0$. Therefore (11) is the Bessel equation - see Appendix 2. Next we will denote this solution as $F_\alpha(r)$. So,

$$e_z(r) = F_\alpha(r), \quad (12)$$

$$e'_z(r) = \frac{d}{dr}F_\alpha(r), \quad (15)$$

From (2.21, 1) we find:

$$e'_\varphi + \frac{1}{r}e_\varphi(1+\alpha) - \frac{\alpha}{r}e_{r\varphi} = 0, \quad (16)$$

From (6, 16) we find:

$$e'_\varphi + \frac{1}{r}e_\varphi(1+\alpha) + \frac{\alpha}{r}\left(e'_z + \frac{e_z}{r}\alpha \right)\frac{1}{g} = 0, \quad (17)$$

Suppose, that

$$e_\varphi = K\left(\frac{e_z}{r} \right) \quad (18)$$

$$e'_\varphi = K\left(\frac{e'_z}{r} - \frac{e_z}{r^2} \right) \quad (19)$$

We substitute (18, 19) into (17) and find:

$$K\left(\frac{e'_z}{r} - \frac{e_z}{r^2} \right) + \frac{1}{r}K\left(\frac{e_z}{r} \right)(1+\alpha) + \frac{\alpha}{r}\left(e'_z + \frac{e_z}{r}\alpha \right)\frac{1}{g} = 0,$$

$$\frac{e_z}{r^2}\left(-K + K(1+\alpha) + \frac{\alpha^2}{g} \right) + \frac{e'_z}{r}\left(K + \frac{\alpha}{g} \right) = 0,$$

$$\left(\frac{e_z}{r^2}\alpha+\frac{e_z'}{r}\right)\left(K+\frac{\alpha}{g}\right)=0,$$

$$K=-\frac{\alpha}{g}. \tag{20}$$

So, from (18--20) we find:

$$e_\varphi=-\frac{\alpha}{g}\left(\frac{e_z}{r}\right), \tag{21}$$

$$e_\varphi'=-\frac{\alpha}{g}\left(\frac{e_z'}{r}-\frac{e_z}{r^2}\right). \tag{21a}$$

From (1, 6, 21) we find:

$$e_r=e_{r\varphi}-e_\varphi=-\left(e_z'+\frac{e_z}{r}\alpha\right)\frac{1}{g}+\frac{\alpha}{g}\left(\frac{e_z}{r}\right)=-e_z'\frac{1}{g}$$

or, taking into account (10),

$$e_r=-\frac{1}{g}e_z'=-\frac{\chi}{q}e_z'. \tag{22}$$

Consider equations (2.22-2.25). Subtracting (2.24) from (2.23), we find:

$$-(h_\varphi-h_r)\chi-\varepsilon\omega(e_r+e_\varphi)=0, \tag{23}$$

From (2, 23) we find:

$$\vartheta=-\frac{\chi}{\varepsilon\omega} \tag{24}$$

Then from (4, 24, 10) we obtain:

$$g=-\left(\chi-\frac{\mu\varepsilon\omega^2}{\chi}\right). \tag{24a}$$

$$q=\chi^2-\mu\varepsilon\omega^2. \tag{25}$$

Subtracting (2.22) from (2.25), we find:

$$\frac{h_\varphi-h_r}{r}+h_\varphi'-h_r'+\frac{h_r-h_\varphi}{r}\cdot\alpha-\varepsilon\omega e_z=0. \tag{26}$$

From (2, 26) we find:

$$\frac{e_{r\varphi}}{\vartheta r}+\frac{e_{r\varphi}'}{\vartheta}-\frac{e_{r\varphi}}{\vartheta r}\cdot\alpha-\varepsilon\omega e_z=0 \tag{27}$$

or

$$\frac{e_{r\varphi}}{r}(1-\alpha)+e_{r\varphi}'-\vartheta\varepsilon\omega e_z=0. \tag{28}$$

From (24, 28) we find:

$$\frac{e_{r\varphi}}{r}(1-\alpha)+e'_{r\varphi}+\chi e_z=0. \qquad (29)$$

Equation (29) coincides with (5) This means that the assumptions made are satisfied.

From (2) we find:

$$h_\varphi=\frac{e_{r\varphi}}{\vartheta}+h_r \qquad (30)$$

From (2.22, 30) we find:

$$\frac{h_r}{r}+h'_r+\frac{\alpha}{r}\left(\frac{e_{r\varphi}}{\vartheta}+h_r\right)=0, \qquad (31)$$

or

$$-\vartheta h'_r-\vartheta h_r\frac{1+\alpha}{r}-\frac{\alpha}{r}e_{r\varphi}=0, \qquad (32)$$

Сравнивая (32) и (16), замечаем, что

$$-\vartheta h_r=e_\varphi \qquad (33)$$

From (33, 24) we find:

$$h_r=-\frac{e_\varphi}{\vartheta}=e_\varphi\frac{\varepsilon\omega}{\chi} \qquad (34)$$

From (30, 34, 1) we find:

$$h_\varphi=\frac{e_{r\varphi}}{\vartheta}+h_r=\frac{e_{r\varphi}}{\vartheta}-\frac{e_\varphi}{\vartheta}=\frac{e_r}{\vartheta}$$

or, taking into account (24, 22),

$$h_\varphi=-e_r\frac{\varepsilon\omega}{\chi}=\frac{\varepsilon\omega}{q}e'_z. \qquad (35)$$

Consider the equation (2.20)

$$e_r(r)\chi-e'_z(r)+\mu\omega h_\varphi(r)=0$$

and paste in it (35, 22). Then we get:

$$e_r(r)\chi-ge_r(r)-\frac{\mu\varepsilon\omega^2}{\chi}e_r(r)=0 \qquad (36)$$

or

$$\chi-g-\frac{\mu\varepsilon\omega^2}{\chi}=0 \qquad (37)$$

or, taking into account (24a),

$$\chi-\left(\chi-\frac{\mu\varepsilon\omega^2}{\chi}\right)-\frac{\mu\varepsilon\omega^2}{\chi}=0. \qquad (38)$$

Thus, equation (2.20) becomes an identity, what was to be shown.

Appendix 2.

We know the Bessel equation, which has the following form:

$$y''+\frac{y'}{x}+y\left(1-\frac{\nu^2}{x^2}\right)=0, \qquad (1)$$

where ν is the order of the equation. Denote by $Z_\nu(y)$ the general integral of the Bessel equation of order. It is shown in [4, p. 403] that an equation of the form

$$y''+\frac{a}{x}y'+y\cdot\left(bx^m+\frac{c}{x^2}\right)=0. \qquad (2)$$

can be transformed into an equation of the form (1), where $Z_\nu(y)$ and order ν is determined through the parameters $a,\ b,\ m,\ c$.

In particular, equation (11) from Appendix 1 is transformed into an equation of the form (1) by the following substitution:

$$a=1,\ b=q,\ m=0,\ c=-\alpha^2,\ \nu=\frac{1}{2}\left(\sqrt{-4\left(-\alpha^2\right)}\right)=\alpha. \qquad (3)$$

Thus, the solution of equation (11)

$$e_z(r)=F_\alpha(r)=Z_\alpha\left(r\sqrt{q}\right). \qquad (4)$$

Because the

$$\frac{d}{dy}Z_\nu(y)=\frac{1}{2}\left(Z_{\nu-1}(y)-Z_{\nu+1}(y)\right), \qquad (5)$$

then

$$e_z'(r)=\frac{1}{2}\left(Z_{\alpha-1}\left(r\sqrt{q}\right)-Z_{\alpha+1}\left(r\sqrt{q}\right)\right). \qquad (6)$$

Appendix 3.

Рассмотрим уравнение Бесселя

$$y''+\frac{y'}{r}+y\left(1-\frac{1}{r^2}\right)=0, \qquad (1)$$

и функцию вида

$$\Phi(r) = y(r) \cdot y'(r) \cdot r\,. \qquad (2)$$

In Fig. 1 shows graphs of

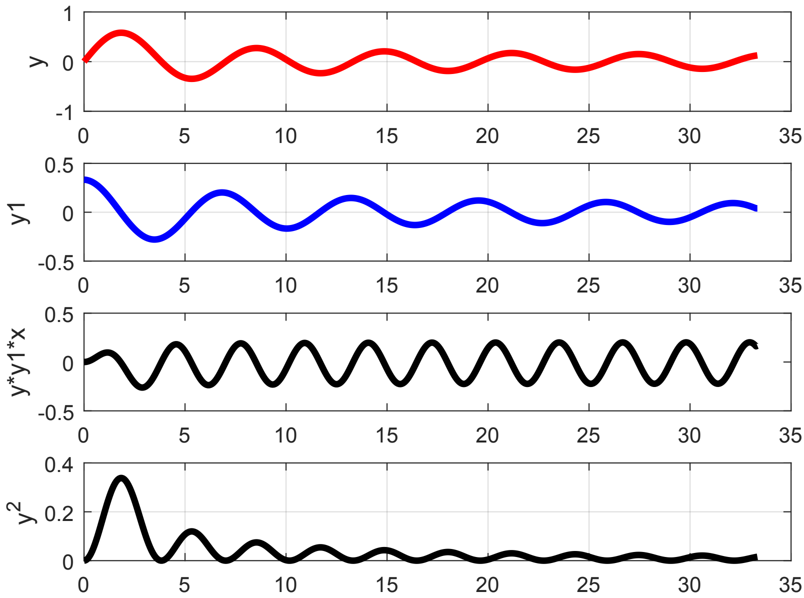
- Bessel function y,
- a derivative y' of this function,
- function $\Phi(r)$,
- function y^2.

It can be seen that the function $\Phi(r)$ is a periodic function.

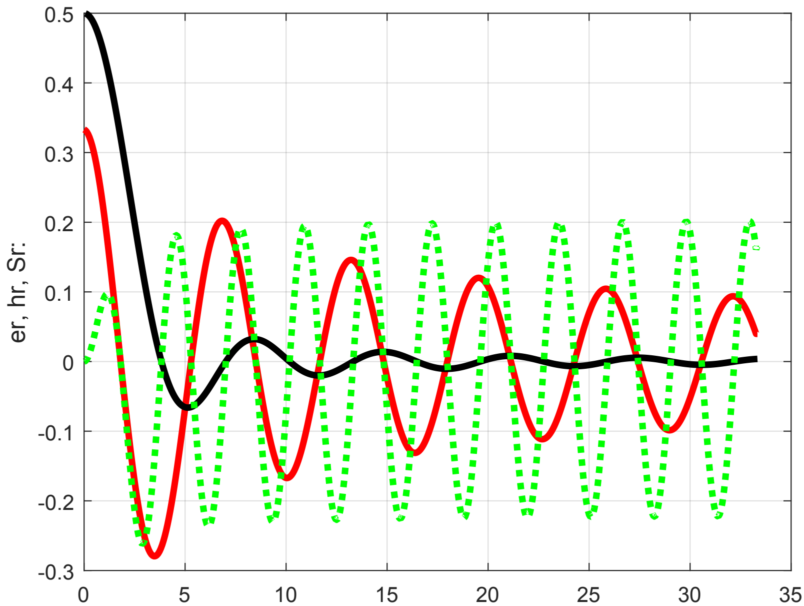
In Fig. 2 shows graphs of

- a derivative y', which is proportional to the intensity $e_r(r)$ - see (2.28, 2.27) and a solid curve with a large amplitude,
- a function y/r that is proportional to the intensity $h_r(r)$ - see (2.30, 2.29, 2.26) and a solid curve with a small amplitude approaching the axis
- a function $\Phi(r)$ that is proportional to the energy flux along the radius $\overline{S_r}$ - see (5.10) and the dotted curve.

- производной от функции Бесселя, которая пропорциональна напряженности $e_r(r)$ - см. (2.28, 2.27) и сплошную кривую с большой амплитудой,
- функции y/r, которая пропорциональна напряженности $h_r(r)$ - см. (2.30, 2.29, 2.26) и сплошную кривую с малой амплитудой, приближающуюся к оси
- функции $\Phi(r)$, которая пропорциональна потоку энергии по радиусу $\overline{S_r}$ - см. (5.10) и пунктирную кривую.



Bes.m, fig. 1



Bes.m, fig. 2

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