Prime Set Representation

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July 2017

1 Prime Set Representations of the Positive Even Integers

The *prime set representation* of any positive even integer is an infinite vector of positive integers which represent the residue classes of the even integer modulo the infinite set of odd prime moduli. The residue classes of a particular prime (let's say p_i) modulus may be represented by the integers $3, 5, 7, \dots, 2p_i + 1$. The p_i residue classes (and there are p_i of them) in turn may be represented by the consecutive integers $1, 2, 3, \dots, p_i$ in the same order.

For example, the prime set representation of 2 is:

$$\left(2, 3, 4, 6, 7, 9, 10, 12, 15, \dots, \frac{p_i + 1}{2}, \dots\right).$$

The above integers in the representation represent the residue classes of 2 modulo $3, 5, 7, \dots, p_i$, so that

$$2 \equiv 2\left(\frac{p_i + 1}{2}\right) + 1 \pmod{p_i}.$$

Obtain the prime set representation of 4 by simply adding a vector of 1s, $(1, 1, 1, \dots)$, to the prime set representation of 2 (column by column). The representation of 4 is:

$$\left(3, 4, 5, 7, 8, 10, 11, 13, 16, \dots, \frac{p_i + 3}{2}, \dots\right).$$

Obtain the prime set representations of 6, 8, 10, and 12 by adding a vector of 1s to each successive even number's representation. For 6 the representation is:

$$\left(1, 5, 6, 8, 9, 11, 12, 14, 17, \ldots, \frac{p_i + 5}{2}, \ldots\right).$$

For 8:

$$\left(2, 1, 7, 9, 10, 12, 13, 15, 18, \ldots, \frac{p_i + 7}{2}, \ldots\right).$$

For 10:

$$\left(3, 2, 1, 10, 11, 13, 14, 16, 19, \ldots, \frac{p_i + 9}{2}, \ldots\right).$$

For 12:

$$\left(1, 3, 2, 11, 12, 14, 15, 17, 20, \ldots, \frac{p_i + 11}{2}, \ldots\right).$$

And so the array of prime set representations of all the even integers is constructed. Notice the use of clock arithmetic in the applicable prime modulus column.

2 Using Prime Set Representations in the Well-defined Prime Set Representation Intervals

The *prime set representation intervals* are

$$\left[p_i^2 + 3, p_{i+1}^2 + 1\right],$$

where the p_i are the successive odd primes. The first few prime set representation intervals are:

$$\left[3^2+3, 5^2+1\right] \text{ or } [12,26]\,,$$
$$\left[5^2+3, 7^2+1\right] \text{ or } [28,50]\,,$$
$$\left[7^2+3, 11^2+1\right] \text{ or } [52,122]\,,$$
$$\left[11^2+3, 13^2+1\right] \text{ or } [124,170]\,.$$

When focusing on any particular prime set representation interval and the even numbers in that interval, it is sufficient to consider only the first i columns of each of the even numbers' prime set representations. For example, the number 88 is in the interval $[52, 122]$ so it is sufficient to consider only the first three columns (7 is the third odd prime) of its prime set representation. The first three columns of its prime set representation are:

$$3 \quad 1 \quad 5$$

Associated with the prime set representation of 88 is the *display* of its prime set representation. This looks like the following:

$$\begin{array}{ccc}
21 & & \\
18 & & \\
15 & 21 & \\
12 & 16 & \\
9 & 11 & 19 \\
6 & 6 & 12 \\
3 & 1 & 5
\end{array}$$

The above display is generated by adding multiples of 3 to the first column of the prime set representation of 88; multiples of 5 to the second column of the prime set representation of 88; and multiples of 7 to the third column of the prime set representation of 88. The largest number in each column is less than 22 because $(22)(2) + 1 > 44$.

The display provides valuable information about the even number 88. Writing the display again, but this time circling the numbers within the display that represent primes

$$
\begin{array}{ccc}
21 & & \\
18 & & \\
15 & 21 & \\
12 & 16 & \\
9 & 11 & 19 \\
6 & 6 & 12 \\
3 & 1 & 5
\end{array}
$$

and recognizing the numbers *not* in the display that represent primes (namely the numbers 2,8,14, and 20 which represent the primes 5,17,29, and 41) is tantamount to capturing all the Goldbach pairs of 88:

$$
\begin{aligned}
5 + 83 \\
17 + 71 \\
29 + 59 \\
41 + 47
\end{aligned}
$$

This routine is applicable to all even numbers in their respective prime set representation intervals. Notice also the absence of the numbers 4,7,10,13. These numbers represent the composite numbers 9,15,21,27. Their respective complements in 88 are also prime: 79,73,67,61.

3 Acceptance of the Prime Set Representation Model Guarantees the Infinitude of the Twin Primes

Prime set representation theory unmasks the apparent behavior in the relative numbers of primes separated by an even number. When primes differ by 2, this is a twin prime pair. The theory uses the prime set representation of even numbers and their displays to locate primes and pairs of primes. The theory posits the following:

- An even number $2n$ (in the interval $\left[p_i^2+3, p_{i+1}^2+1\right]$) free of a 1 in the first i-columns of its prime set representation signals that $2n-3$ is a prime.
- An even number $2n$ in the same interval free of a 1 and 2 in its representation signals that $2n-3$ and $2n-5$ are primes.
- An even number with a prime set representation free of a 1 and 3 signals that $2n-3$ and $2n-7$ are primes.
- An even number with the *display* of its prime set representation free of a 1 and 4 signals that $2n-3$ and $2n-9$ are primes.
- $\vdots$
- An even number with the display of its prime set representation free of a 1 and a k signals that $2n-3$ and $2n-(2k+1)$ are primes.

  For example,

$$\frac{1}{3}\cdot\frac{3}{5}\cdot\frac{5}{7}\cdot\frac{9}{11}\cdot\ \ldots\ \cdot\frac{p_i-2}{p_i}$$

is a valid measure of the number of even numbers in the interval $\left[p_i^2+3, p_{i+1}^2+1\right]$ free of 1 and 2 in the first i columns of their prime set representations.

$$\frac{1}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}\cdot\frac{9}{11}\cdot\ \ldots\ \cdot\frac{p_i-2}{p_i}$$

is a valid measure of the number of even numbers free of 1 and 36 in the first i columns of the displays of their representations.

A comparison of the two measures shows:

$$\frac{\text{free of 1 and 36}}{\text{free of 1 and 2}}=\frac{4\cdot 6}{3\cdot 5}=\frac{8}{5}.$$

This ratio agrees with the ratio conjectured and shown in a table in Polya's "Heuristic Reasoning in the Theory of Numbers". All other tabular ratios in the paper may be similarly confirmed. They are no longer conjectures; they are guarantees of the prime set representation model.

Furthermore, prime set representation theory guarantees a relationship between the number of primes in the prime set representation intervals and the number of twin primes in the prime set representation intervals.

$$\frac{2}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}\cdot\frac{10}{11}\cdot\ \ldots\ \cdot\frac{p_i-1}{p_i}$$

is a valid measure of the number of even numbers in the interval $\left[p_i^2+3, p_{i+1}^2+1\right]$ free of a 1 in the first i columns of their prime set representations.

A comparison of the measures of primes and twin primes is:

$$\frac{\text{free of 1}}{\text{free of 1 and 2}} = \frac{2\cdot 4\cdot 6\cdot 10\cdot\ \ldots\ \cdot(p_i-1)}{1\cdot 3\cdot 5\cdot 9\cdot\ \ldots\ \cdot(p_i-2)}.$$

The number of primes is infinite. If the number of twin primes were finite, the above relationship would not be true (in the sense of a true model). Therefore, the number of twin primes is infinite.

When the measure of the number of even numbers free of 1 and 2 in the first i columns of their prime set representations is applied to the number of even numbers in the prime set representation interval, the beautiful approximation formula

$$\frac{1}{3}\cdot\frac{3}{5}\cdot\frac{5}{7}\cdot\ \ldots\ \cdot\frac{p_i-2}{p_i}\cdot\frac{p_{i+1}^2-p_i^2}{2}$$

results for the number of twin primes in the prime set representation interval $\left[p_i^2+3, p_{i+1}^2+1\right]$.

4 The Failure of the Display of the Prime Set Representation as a Prime Number Generator Guarantees the Truth of the Goldbach Conjecture

The display of the prime set representation of 88 does not contain all odd primes less than 44. Consequently, 88 has the four aforementioned Goldbach pairs. By definition, the display of the prime set representation of 88 only utilizes the first three numbers of the prime set representation of 88, and therefore, has only three columns. The numbers 2, 8, 14, and 20 that represent the primes 5, 17, 29, and 41 and do not occur in the display of 88, naturally occur in the prime set representation of 88 in columns to the right of the first three columns of the representation. See the display of the prime set representation of 88 shown together with the extended prime set representation of 88.

Table 1: The Display of the Prime Set Representation of 88
and the Extended Prime Set Representation of 88

21																					
18																					
15	21																				
12	16																				
9	11	19																			
6	6	12																			
3	1	5	5	11	1	15	9	29	28	25	23	22	20	17	14	13	10	8	7	4	2

Columns beyond the first i columns of the prime set representation of the even number $2n$ in the prime set representation interval $\left[p_i^2+3, p_{i+1}^2+1\right]$ are always necessary to capture all primes less than n (i.e., the representation of them) just as can be seen in the display and extension of 88. Even numbers with greater and greater numbers of columns in their i-column representations (i.e., even numbers in the interval $\left[p_i^2+3, p_{i+1}^2+1\right]$) are literally riddled with *holes*, if by a hole is meant the non-appearance of $\frac{p-1}{2}(p \leq n)$ in the display of the even number $2n$ in the interval $\left[p_i^2+3, p_{i+1}^2+1\right]$ (Here, p is an odd prime).

Analysis of the display of the prime set representation of an even number $2n$ in the prime set representation interval $\left[p_i^2+3, p_{i+1}^2+1\right]$ makes possible meaningful estimation formulas for the number of Goldbach pairs of $2n$.

For displays having i columns, the formula is:

$$\left(\prod_{j=1}^{i} \frac{p_j-2}{p_j-1}\right) \cdot \frac{n}{\ln n}$$

or when any of the $p_j \mid n$, the formula is:

$$\left(\prod_{j=1}^{i} \frac{p_j - 2}{p_j - 1}\right) \cdot \left(\prod_{p_j \mid n} \frac{p_j - 1}{p_j - 2}\right) \cdot \frac{n}{\ln n}$$

where $p_1 = 3, p_2 = 5, p_3 = 7, \ldots, p_i =$ the largest prime less than $\sqrt{2n-2}$ and $2n$ is in the interval $\left[p_i^2 + 3, p_{i+1}^2 + 1\right]$.

5 Applying Prime Set Representation Theory to Primes of the Form $x^2 + 1$

There are $\frac{p_{i+1} - p_i}{2}$ even numbers of the form $x^2 + 4$ in the interval $\left[p_i^2 + 3, p_{i+1}^2 + 1\right]$. If any of these even numbers has no 1 in its prime set representation, then the associated integer $x^2 + 1$ is prime. Columns of the prime set representation of $x^2 + 4$ headed by primes of the form $4k - 1$ will never contain a 1; columns of the prime set representation headed by primes of the form $4k + 1$ will not contain a 1 approximately $\frac{p_i - 1}{p_i} \cdot 100\%$ of the time (here the p_i is the prime having the form $4k + 1$). The expected number of primes of the form $x^2 + 1$ in the interval $\left[p_i^2 + 3, p_{i+1}^2 + 1\right]$ is:

$$\left(\prod_{j=1}^{i} \frac{p_j - 1}{p_j}\right) \cdot \frac{p_{i+1} - p_i}{2}$$

where the p_js are primes of the form $4k + 1$ and $p_i < \sqrt{2n-2}$.

6 The Prime Set Representation of the Integer 1

The focus has been the prime set representations of the even integers. Take a look at the prime set representation of the integer 1:

$$\begin{array}{l} \underline{3}\ \underline{5}\ \underline{7}\ \underline{11}\ \underline{13}\ \underline{17}\ \underline{19}\ \underline{23}\ \underline{29} \ldots \underline{p_i} \ldots \\ 3\ 5\ 7\ 11\ 13\ 17\ 19\ 23\ 29 \ldots p_i \ldots \end{array}$$

It contains all the odd primes, and, when added component-by-component to the prime set representation of any other integer, acts like zero. This is one of the secrets of the prime set representation.