

Question 201: A fractal image

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abstract

This note presents a fractal image for $f(z) = \ln(1 + g(z))$.

1. Introduction. Two integrals

$$\int_0^{\infty} (e^{-1/x^2} - e^{-2/x^2}) dx = \sqrt{\pi}(\sqrt{2}-1) \quad (1)$$

$$\int_0^1 \left(\sqrt{\frac{1}{\ln\left(\frac{1+x}{2}\right)}} - \sqrt{\frac{1}{\ln\left(\frac{1-x}{2}\right)}} \right) x dx = 2\sqrt{\pi}(\sqrt{2}-1) \quad (2)$$

2. The functions $f(z)$ and $g(z)$.

$$g(z) = \left(\sqrt{\frac{1}{\ln\left(\frac{1+z}{2}\right)}} - \sqrt{\frac{1}{\ln\left(\frac{1-z}{2}\right)}} \right) z \quad (3)$$

$$f(z) = \ln(1 + g(z)) \quad (4)$$

3. Fractals for $f(z)$.

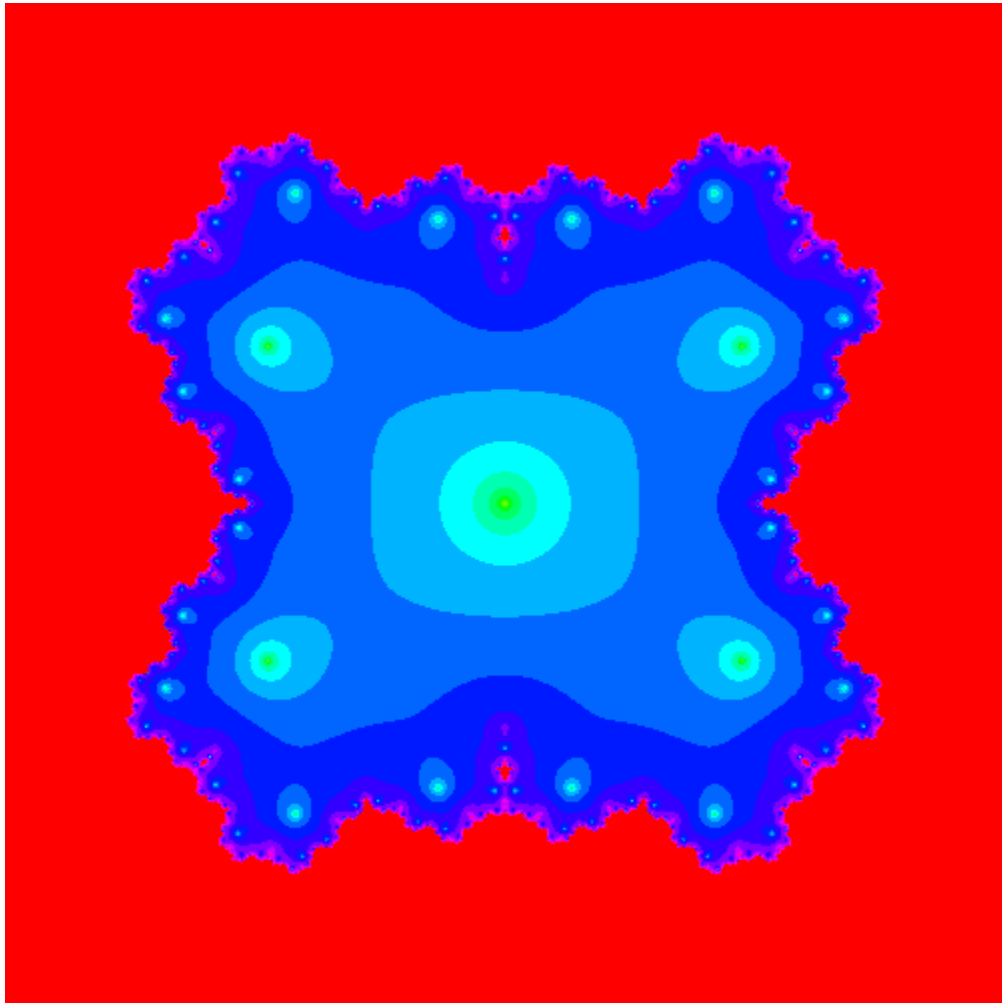


Figure 1.

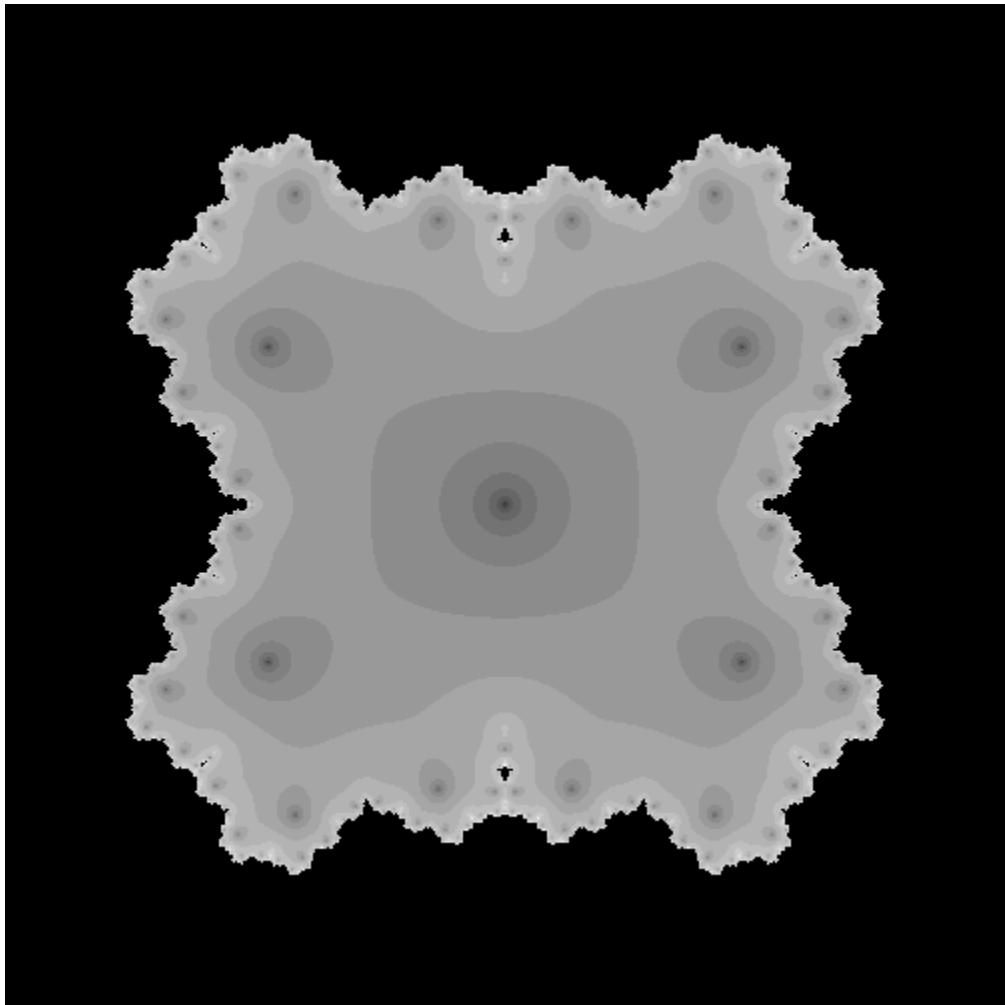


Figure 2.

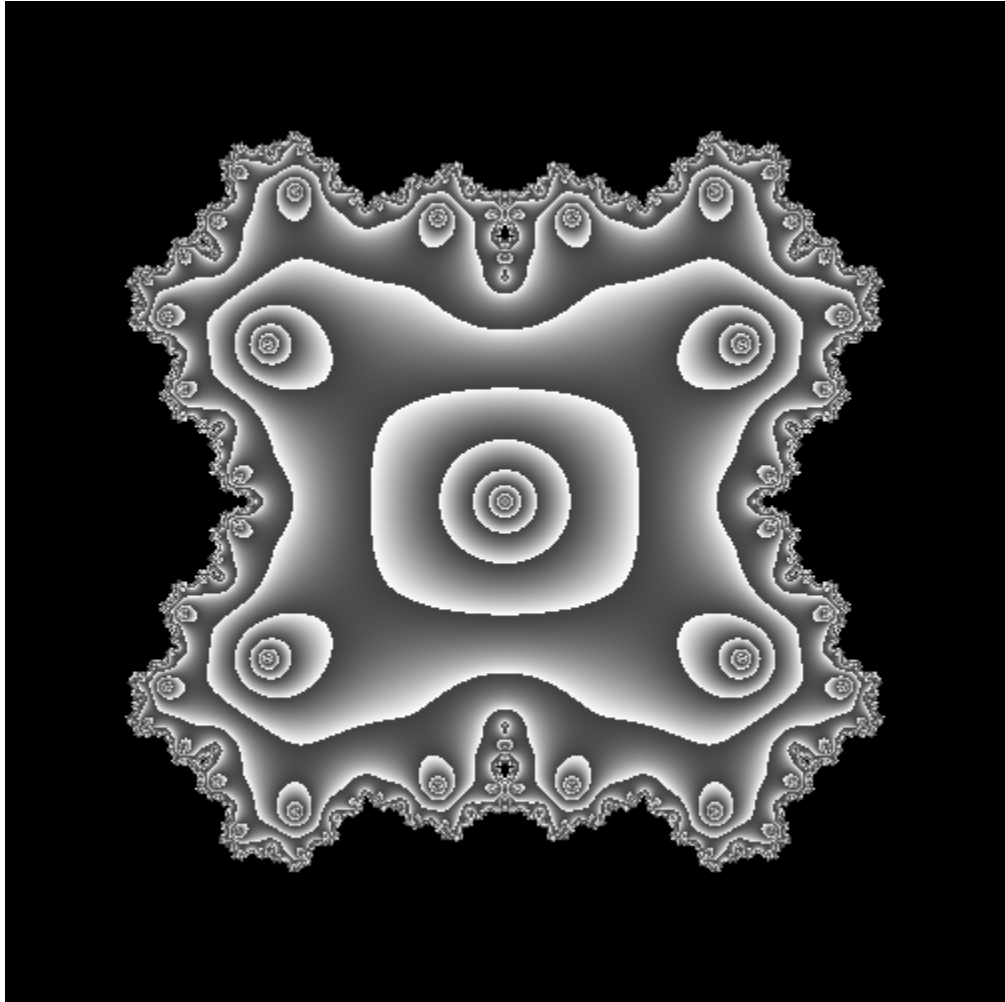


Figure 3.

4. Fractals for $\ln(1+zg(z))$.

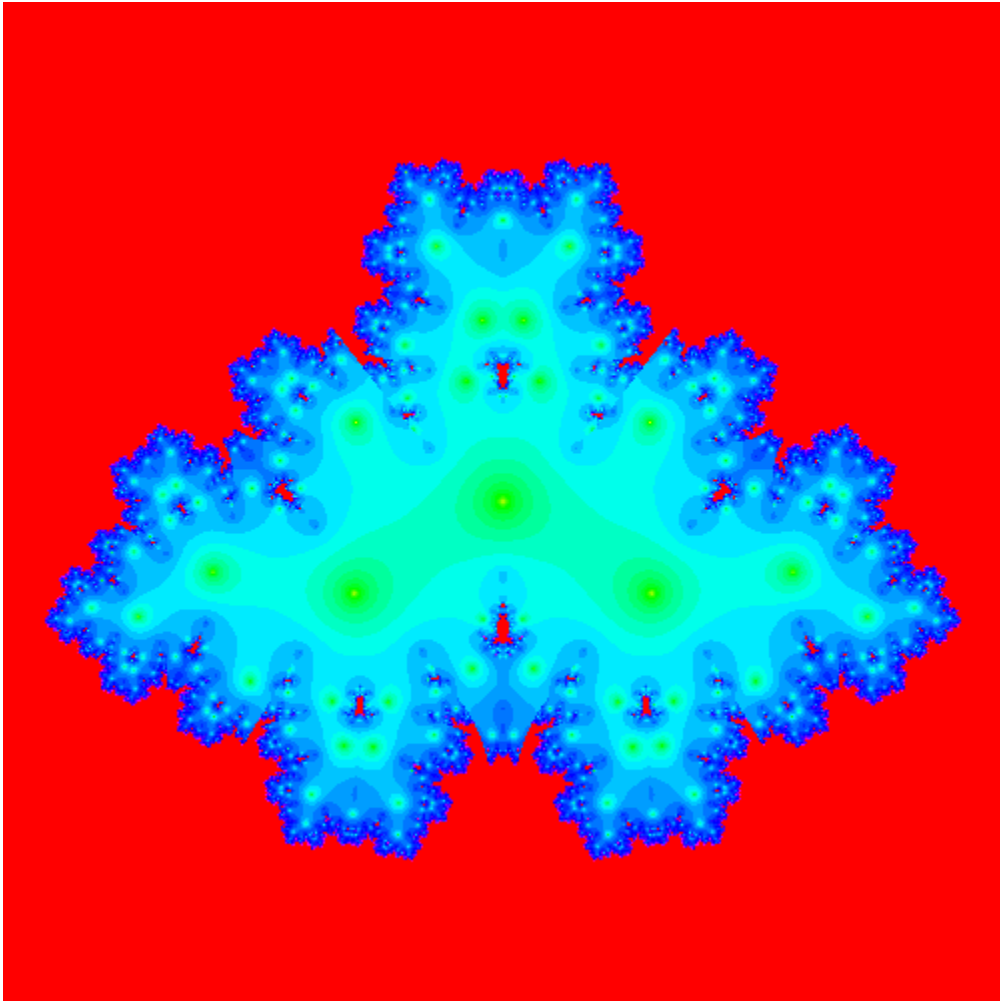


Figure 4.

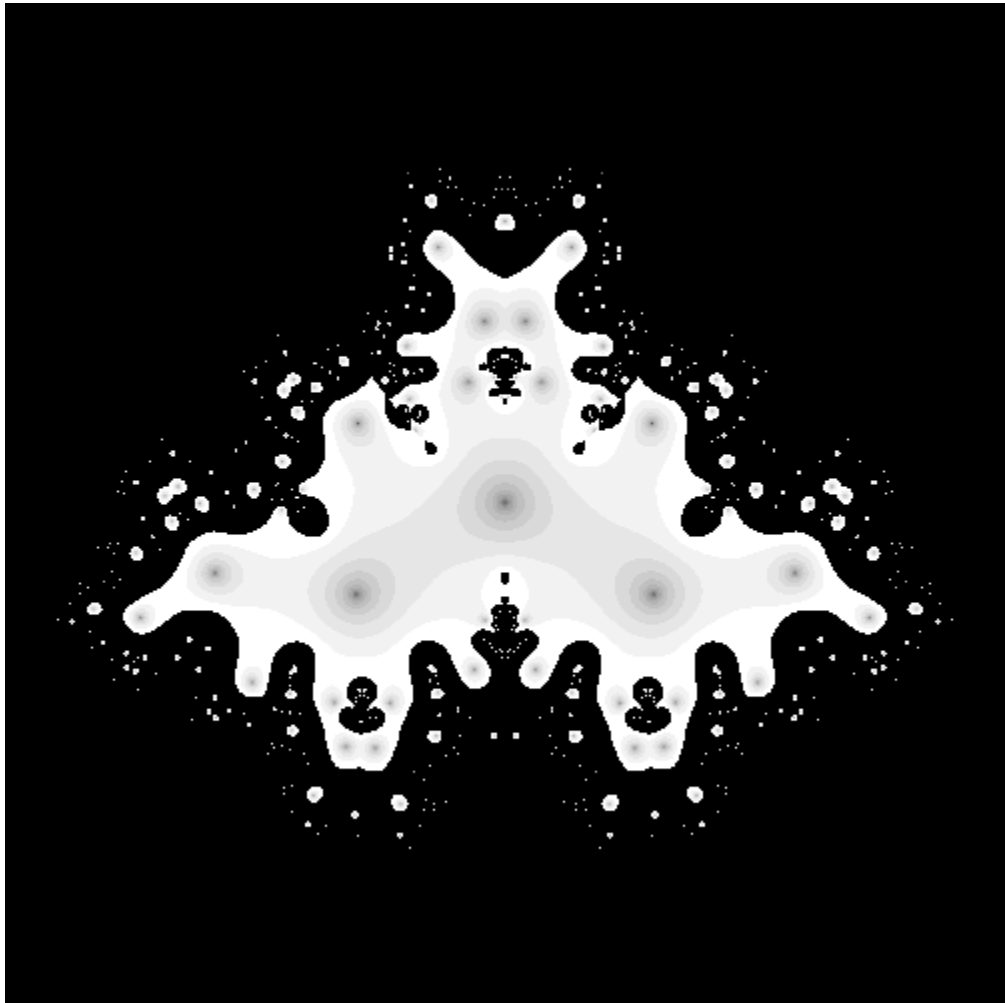


Figure 5.



Figure 6.

References

1. Devaney, R.L.: A First Course in Chaotic Dynamics Systems. westview Press.1992.
2. Milnor, J.: Dynamics in One Complex Variable. Introductory Lectures, Vieweg, 1999.