On the validity of quantum physics below the Planck length

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Abstract

The widely held expectation that quantum physics breaks down below the Planck length (10^{-33} cm) is brought into question. A possible experiment is suggested that might test its validity at a *sub-Planckian* length scale.

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1. Introduction

The Planck length 1.616×10^{-33} cm has been the subject of much speculation since it was introduced into physics by Max Planck over a century ago. It has long been assumed that this is the shortest possible distance at which quantum mechanical processes have any meaning, which is clearly unsettling since it implies that quantum physics breaks down at sub-Planckian length scales. It should be emphasized, however, that this assumption is so far only an extrapolated hypothesis unsupported by experimental evidence. Indeed, as we shall see below, it is possible, utilizing dimensional analysis, to formulate a viable system of absolute units, more *diminutive* than Planck's, whose *size* suggests that they may be the key to a deeper understanding of physical processes at *sub-Planckian* length scales.

We employ for this purpose a system of absolute units, similar to Planck's, that is based on the Newtonian gravitational constant (G), the velocity of

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light (c), and, instead of Planck's reduced "action" constant, $\hbar$, the charge on the electron (e), in the form

$$M_0 = \left(\frac{e^2}{G}\right)^{1/2} = 1.859 \times 10^{-6} \text{ g} \tag{1}$$

$$L_0 = \left(\frac{e^2 G}{c^4}\right)^{1/2} = 1.380 \times 10^{-34} \text{ cm} \tag{2}$$

$$T_0 = \left(\frac{e^2 G}{c^6}\right)^{1/2} = 4.605 \times 10^{-45} \text{ sec} \tag{3}$$

which, for convenience, are expressed in CGS units. It will be seen at once that the magnitude of the absolute unit of length, L_0, deriving from Eq. (2), is an order of magnitude *smaller* than the Planck length, which is clearly significant since this sub-Planckian length involves electric charge and spacetime inputs (e and c). Hence, it is at this length scale, 10^{-34} cm, that we might expect to find a connection between the elementary charge and the fabric of spacetime.

2. Charge quantization in sub-Planckian spacetime

We shall take Eq. (1) as our starting point by expressing the *gravitational mass* equivalent, M_0, of the *electrostatic potential energy*, in terms of Einstein's mass-energy relation

$$E_0 = M_0 c^2. \tag{4}$$

Let us now assume that the *fabric* of spacetime vibrates with an *intrinsic* vibrational energy $M_0 c^2$, and frequency $1/T_0$, which is the reciprocal of the period, T_0, deriving from Eq. (3). It will then be seen that these two quantities are linked by a *non-Planckian* constant that has the *same* dimensions as Planck's quantum of "action" $\hbar$. Its magnitude is given by

$$\frac{M_0 c^2}{T_0^{-1}} = 7.695 \times 10^{-30} \text{ erg } s \tag{5}$$

which implies that this non-Planckian quantum of "action" is an *intrinsic* property of the fabric of spacetime. Hence, if the elementary charge e is a manifestation of the quantization of the *intrinsic* vibrational energy of the fabric of spacetime it must of necessity emerge from Eq. (5). For this we need only equate the intrinsic vibrational energy M_0c^2 and the vibrational frequency $1/T_0$, denoted ν_0, in the form

$$M_0c^2 = j\nu_0 \tag{6}$$

where for simplicity of presentation we have denoted the "action" constant M_0c^2/T_0^{-1} by the symbol j. The corresponding wavelength, denoted by λ_0, can be readily determined from Eq. (6) in terms of the momentum,

$$\begin{aligned} \lambda_0 &= \frac{j}{M_0c} \\ &= 1.380 \times 10^{-34} \text{ cm.} \end{aligned} \tag{7}$$

The energy *per cycle* can then be expressed in the form

$$\begin{aligned} E_{pc} &= (j\nu_0)\lambda_0 \\ &= 2.306 \times 10^{-19} \text{ erg cm} \end{aligned} \tag{8}$$

which is seen as being quantitatively equal to e^2. It therefore follows that

$$\begin{aligned} e &= \sqrt{(j\nu_0)\lambda_0} \\ &= 4.802 \times 10^{-10} \text{ esu} \end{aligned} \tag{9}$$

in quantitative agreement with the experimental value from which it draws its justification. We have thus achieved an easily interpreted expression for the quantization of electric charge in terms of the quantization of the intrinsic vibrational energy of the fabric of spacetime at a *sub-Planckian* length scale.

3. Coupling quantization

Richard Feynman use to refer to the fundamental physical constant α, that characterizes the strength of the electromagnetic interaction, as a magic number (1/137.036) that comes to us with no understanding by man [1]. Max Born was more sanguine. He strongly believed that this number was a law of nature [2]. They were both, of course, referring to the dimensionless coupling constant of quantum electrodynamics; the so-called fine-structure constant, α, which has been a mystery since it was introduced into physics by Sommerfeld in 1916. Of paramount interest is the question of the *origin* and *numerical value* of α, which comes under the purview of this conceptualization of the elementary charge. The agreement of the result of Eq. (9) with experiment would seem not only to support the validity of the above considerations but also to offer a theoretical basis for understanding this empirically determined constant in terms of the physically meaningful relation

$$\alpha = \frac{j}{\hbar} = 0.007297 \tag{10}$$

where j is the proportionality constant linking the *intrinsic* vibrational energy of the fabric of spacetime to its frequency (which manifests as an *electric charge*), and $\hbar$ is the proportionality constant linking the vibrational energy of the electromagnetic field to its frequency (which manifests as a *photon*). Whence the *origin* and *numerical value* of the dimensionless number 1/137.036 that characterizes the coupling strength of an electric charge with the photon.

4. Possible experiment

To decide unequivocally if vibratory phenomena in sub-Planckian spacetime are governed by this newly derived *non-Planckian* "action" constant, one must determine if the perturbations generated in the curvature of the fabric of spacetime, by accelerated masses, are also a function of this non-Planckian constant. More succinctly, one must perform an experiment that can differentiate this "action" constant from Planck's. One such possibility is suggested by Eq. (10). It will be observed that the non-Planckian "action" constant, j, is quantatively *smaller* than Planck's reduced "action" constant, $\hbar$, by a factor of 0.007297. It should therefore be possible to differentiate between these two elementary "actions" from the displacement *amplitude* produced by a gravitational wave when it interacts with a resonant-mass. For a given bandwidth the best solution for measuring such an effect is a spherically shaped detector. In addition to maximizing gravitational wave absorption, spherical detectors are omnidirectional, which means that they have the same sensitivity in any direction of observation. As a result, only a single detector is needed to determine the direction and polarizations of the incoming wave. One of the two more innovative of these detectors is the Schenberg resonant-mass telescope in Brazil [3], which is designed to sense multipole modes of vibration. When fully operational it will provide information regarding a wave's amplitude, polarization, and direction of source. The detector program, which we shall presently exploit, uses an 1150 Kg spherical resonant-mass made of CuAl (6%) alloy, and has a *resonance* frequency ν of 3200 Hz. We may then express the energy, E, for a single quantum of excitation, as a function of Planck's constant, $\hbar$, in the form

$$E = \hbar\omega \tag{11}$$

where $\hbar = 1.054 \times 10^{-34} J \cdot s$ and ω is the *angular* frequency ($2\pi\nu$). We can then profit from the fact that the *vibrational* energy induced in the spherical mass by a gravitational wave can be converted into a value for the *actual* displacement of the sphere by making use of the relation between amplitude x, energy E, and the total mass M for a harmonic oscillator, in the familar form

$$E = 1/2 M\omega^2 x^2. \tag{12}$$

It is then possible, using Eq. (12), to calculate the displacement caused by a single quantum of excitation by putting energy $= \hbar\omega$, and substituting the designated values,

$$\begin{aligned} x &= \left(\frac{2\hbar}{M\omega}\right)^{1/2} \\ &\simeq 3.02 \times 10^{-21} m \end{aligned} \tag{13}$$

which corresponds to the detector's *quantum limit* (expressed in meters, instead of as a strain dL/L, for purely practical reasons). A comparison with the *derived* non-Planckian "action" constant, j, is then possible by putting energy $= j\omega$, and, once again, substituting the designated values,

$$\begin{aligned} x &= \left(\frac{2j}{M\omega}\right)^{1/2} \\ &\simeq 2.58 \times 10^{-22} m \end{aligned} \tag{14}$$

where $j = 7.695 \times 10^{-37} J \cdot s$. It will thus be seen that if Eq. (14) corresponds to reality the resulting displacement will be *smaller* than the detector's quantum limit by a factor of 0.0854, which is simply the square root of the ratio of these two elementary "action" constants, $j/\hbar$.

A similar spherical detector known as MiniGRAIL is presently under development in the Netherlands [4]. It consists of a 1400 Kg spherical test mass, made of CuAl (6%) alloy, which has a resonance frequency ν of 3000

Hz. In order to facilitate a comparison of these two elementary "actions" in the experimental arena we shall utilize the same procedure as before by substituting the designated value for the spherical test mass and angular frequency ω in Eqs. (13) and (14), respectively,

$$\begin{aligned} x &= \left(\frac{2\hbar}{M\omega}\right)^{1/2} \\ &\simeq 2.82 \times 10^{-21} m \end{aligned} \tag{15}$$

and

$$\begin{aligned} x &= \left(\frac{2j}{M\omega}\right)^{1/2} \\ &\simeq 2.41 \times 10^{-22} m \end{aligned} \tag{16}$$

which is consistent with the Schenberg results.

To determine unequivocally which of these two "action" constants corresponds to reality will require ultra-high sensitive measurements that extend beyond the detector's quantum limit. Fortunately, such measurements are now possible utilizing quantum *squeezing* technology. It is anticipated that these spherical detectors will be highly sensitive in the 2–4kHz range, suitable for detecting gravitational waves from neutron star instabilities and small black hole mergers.

5. Discussion

A system of absolute units, based on the elementary charge e, was used to investigate length scales below the Planck length (10^{-33} cm). It was found that the quantization of electric charge can be explained, in a fundamentally consistent manner, as a manifestation of the quantization of the *intrinsic* vibrational energy of the fabric of spacetime by a *non-Planckian* quantum of "action" at a *sub-Planckian* length scale of 10^{-34} cm. It was shown that the properties of the elementary processes, imposed by this more diminutive system of absolute units, makes the formulation of this conceptualization of the elementary charge appear almost unavoidable, particularly the non-Planckian "action" constant j, which is quantitatively *smaller* than Planck's reduced "action" constant, $\hbar$, by a factor of 0.007297; recognizable as the dimensionless coupling constant of quantum electrodynamics, α, for which we now have a theoretical understanding in the context of this conceptualization of the elementary charge.

Guided by the foregoing considerations we are as good as forced to conclude that these two "action" constants mark the boundary between different appropriate descriptions of the physical world, which is fundamentally reassuring since in addition to underscoring the dynamic role of the fabric of spacetime it relegates to the quantum the primary role of describing physical phenomena.

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