Quantum equations in empty space using mutual energy and self-energy principle

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Abstract

For photon we have obtained the results that the waves of photon obey the mutual energy principle and self-energy principle. In this article we will extended the results for photon to other quantum. The mutual energy principle and self energy principle corresponding to the Schrödinger equation are introduced. The results are that a electron, for example, travel in the empty space from point A to point B, there are 4 different waves. The retarded wave started from point A to infinite big sphere. The advanced wave started from point B to infinite big sphere. The return waves corresponding to the above both waves. There are 5 different flow corresponding to these waves. The self-energy flow corresponding to the retarded wave, the self-energy flow corresponding to the advanced wave. The return flows corresponding to the above two return waves. The mutual energy flow of the retarded wave and the advanced wave. It is found that the the mutual energy flow is the energy flow or the charge intensity flow or electric current of the the electron. Hence the electron travel in the empty space is a complicated process and do not only obey one Schrödinger equation. This result can also extend to to Dirac equations.

Keyword: Poynting; Maxwell; photon; retarded wave; advanced wave; time-reversal; absorber; emitter; action-at-a-distance; Schrödinger; Dirac; electron;

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I. INTRODUCTION

Maxwell equations have retarded solution and advanced solution. Wheeler and Feynman have introduced the absorber theory which involved the advanced wave [1][2]. The absorber theory is based on the action-at-a-distance [5, 16, 18]. In classical electromagnetic field theory the advanced wave is applied on mutual energy theorem, which is contribution of W.J. Welch [19], S.R. Zhao [6, 20, 21]. J. Cramer further worked on the absorber theory and introduced the transactional interpretation for quantum mechanics[3, 4].

This author combined the absorber theory and the mutual energy theorem and introduced the concept that the photon energy is transferred by the mutual energy flow[9–15, 17]. And the further derived that mutual energy principle[7] and the self-energy principle[8]. The mutual energy principle says that the electromagnetic field and the field for photon all should satisfy mutual energy principle. The solution of the mutual energy principle is retarded wave and an advanced wave. Both wave satisfies Maxwell equations. Both waves must be synchronized.

The self-energy principle for photon tells that the self-energy are returned. Hence the self-energy flows do not contribute to any energy transfers.

This article will apply the concept of the mutual energy principle and self-energy principle to other quanta for example electron. First we do not consider the spin in electron, hence assume the electron satisfy Schrödinger equation.

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II. SCHRÖDINGER EQUATION FOR RETARDED AND ADVANCED WAVE

We assume the quantum for example electron runs in the empty space from point **a** to **b**. This electron must satisfy in the the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2+V(\mathbf{r},t)\right]\Psi(\mathbf{r},t) \qquad (1)$$

where $i=\sqrt{-1}$. $\Psi(\mathbf{r},t)$ is the wave function.

A. The retarded equation for point a

In empty space there is,

$$V(\mathbf{r},t)=0 \tag{2}$$

We have know that the wave $\Psi_a(\mathbf{r},t)$ is retarded wave started from point $\mathbf{a}$ and spread to the infinite big sphere. This wave satisfies,

$$i\hbar\frac{\partial}{\partial t}\Psi_a(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_a(\mathbf{r},t) \tag{3}$$

We do not know the exact wave should be, but we know that this wave should be a retarded wave, from the experience of photon we know that this wave should be look like the following

$$\Psi_a(\mathbf{r},t)=\frac{1}{|\mathbf{r}-\mathbf{a}|}\exp_T(i\omega(t-\frac{|\mathbf{r}-\mathbf{a}|}{\frac{\omega}{k}}+t_a)) \tag{4}$$

where $\frac{\omega}{k}=v$ is the speed of the particle.

$$\exp_T(i\tau)=\begin{cases}\exp(i\tau) & 0<\tau<2\pi\\ 0 & otherwise\end{cases} \tag{5}$$

Where t_a is a initial constant. We have assume the wave has been truncated with only one wave length. This may be not true, perhaps the wave has a life time more than one wave length. Since the frequency of electron is very high, for example if the electron have speed of $v=\frac{c}{10}$, where c is light speed. Then moment of the electron is around,

$$\begin{aligned}p=mv&=9*10^{-31}kilogram*(3*10^8meter*\frac{1}{10})\\&=2.7*10^{-23}[kg][m]/[s]\end{aligned} \tag{6}$$

The wave length of the electron is,

$$\lambda=\frac{h}{p}=\frac{6.62607004*10^{-34}[kg][m]^2/s}{2.7*10^{-23}[kg][m]/s}=2.4541*10^{-11}[m] \tag{7}$$

The frequency of the wave is,

$$\lambda f=v \tag{8}$$

$$f=\frac{v}{\lambda}=\frac{3*10^8[m]/[s]*0.1}{2.4541*10^{-11}}=1.22244407*10^{18} \tag{9}$$

Assume the period of the wave is

$$fT = 1 \tag{10}$$

$$T = \frac{1}{f} = 8.1803 * 10^{-19}[s] \tag{11}$$

if we assume the wave is only have a length of wave length, then the wave will appear in space with the $\lambda = 2.4541 * 10^{-11}[m]$. The wave can also have a life time $t = 8.1803 * 10^{-19}[s]$. This is very short wave.

We assume that the distance from point $\mathbf{a}$ to the origin point of the coordinates $\mathbf{r} = \mathbf{o}$ point is $|\mathbf{o} - \mathbf{a}| = l$, we assume when this retarded wave reach the point $\mathbf{o}$ the time is $t = 0$, hence we have,

$$(0 - \frac{|\mathbf{o} - \mathbf{a}|}{\frac{\omega}{k}} + t_a) = 0 \tag{12}$$

hence

$$t_a = \frac{l}{\frac{\omega}{k}} \tag{13}$$

$$\Psi_a(\mathbf{r}, t) = \frac{1}{|\mathbf{r} - \mathbf{a}|} \exp_T(j\omega(t - \frac{|\mathbf{r} - \mathbf{a}| - l}{\frac{\omega}{k}})) \tag{14}$$

This wave when $\mathbf{r} = \mathbf{a}$, $|\mathbf{r} - \mathbf{a}| = 0$

$$(t - \frac{0 - l}{\frac{\omega}{k}}) = 0 \tag{15}$$

$$t + \frac{l}{v} = 0 \tag{16}$$

$$t = -\frac{l}{v} \tag{17}$$

This means $t = -\frac{l}{v}$, the wave is at the $\mathbf{r} = \mathbf{a}$,

This wave when $\mathbf{r} = \mathbf{b}$,

$$|\mathbf{r} - \mathbf{a}| = |\mathbf{b} - \mathbf{a}| = 2l \tag{18}$$

$$(t - \frac{2l - l}{\frac{\omega}{k}}) = 0 \tag{19}$$

$$t = \frac{l}{v} \tag{20}$$

This means when $t = \frac{l}{v}$ it come to the point $\mathbf{b}$. We also $t = 0$, $\mathbf{r} = \mathbf{o}$

B. The advanced wave started from point b

According to the experience with photon, the retarded wave and the advanced wave satisfy the same Maxwell equations. This should be also true for other particles, hence here for the advanced wave it should also satisfy same Schrödinger equation (if Schrödinger equation cannot offer correct format of advanced wave, we believe at least the Dirac equation should be which will be discussed in section VI, here we assume Schrödinger equation is possible to described the advanced wave),

$$i\hbar\frac{\partial}{\partial t}\Psi_b(\mathbf{r},t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_b(\mathbf{r},t) \tag{21}$$

We have write τ as t. $\Psi_b(\mathbf{r},t)$ is the advanced wave starting from point $\mathbf{b}$.

$$\Psi_b(\mathbf{r},t) = \frac{1}{|\mathbf{r}-\mathbf{b}|}\exp_T(\omega j(t + \frac{|\mathbf{r}-\mathbf{b}|}{\frac{k}{\omega}} + t_b)) \tag{22}$$

We assume when $t = 0$ the advanced wave just pass the the origin point $\mathbf{r} = \mathbf{o}$ and

$$|\mathbf{o}-\mathbf{b}| = l \tag{23}$$

$$(0 + \frac{l}{\frac{k}{\omega}} + t_b) = 0 \tag{24}$$

hence we have

$$t_b = -\frac{l}{\frac{k}{\omega}} \tag{25}$$

$$\Psi_b(\mathbf{r},t) = \frac{1}{|\mathbf{r}-\mathbf{b}|}\exp_T(j\omega(t + \frac{|\mathbf{r}-\mathbf{b}| - l}{\frac{k}{\omega}}) \tag{26}$$

C. The advanced wave is synchronized with the retarded wave

Advanced wave and the retarded wave can be synchronized, this section we will show this. For the above advanced wave, when $\mathbf{r} = \mathbf{a}$,

$$|\mathbf{r}-\mathbf{b}| = 2l \tag{27}$$

$$(t + \frac{2l - l}{\frac{k}{\omega}}) = 0 \tag{28}$$

$$t = -\frac{l}{v} \tag{29}$$

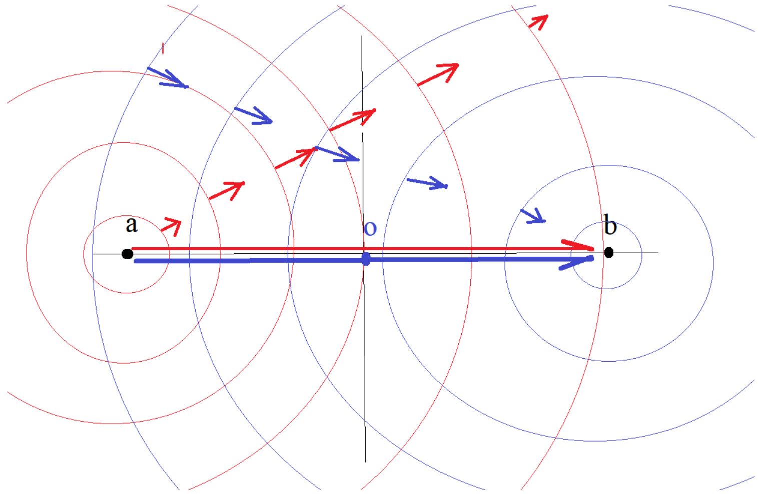

Figure 1. Retarded wave and the advanced wave of the particle, the particle move from point **a** to point **b**. In the time $t = 0$ the both wave reach the point $\mathbf{r} = \mathbf{o}$. The red wave is the retarded wave. And the blue wave is the advanced wave. The retarded wave is a divergent wave. The advanced wave is convergent wave.

For this wave, when $\mathbf{r} = \mathbf{b}$

$$|\mathbf{r} - \mathbf{b}| = |\mathbf{b} - \mathbf{b}| = 0 \tag{30}$$

$$(t + \frac{0 - l}{\frac{k}{\omega}}) = 0 \tag{31}$$

$$t = \frac{l}{v} \tag{32}$$

We have evaluated that the wave retarded $\Psi_a(\mathbf{r}, t)$ and the advanced wave $\Psi_b(\mathbf{r}, t)$ are reach the points **a**, **o**, **b** at time $t = -\frac{l}{v}$, $t = 0$, and $t = \frac{l}{v}$. Hence these two waves are synchronized at this 3 points. Actually the wave are synchronized at the who line from point **a** to **b**.

This way the wave $\Psi_b(\mathbf{r}, t)$is said synchronized with $\Psi_a(\mathbf{r}, t)$. We look the wave on the conect line between **a** and **b**. That means that on this line when the retarded wave just started from point **a** the advanced wave also reached the point **a**, When the retarded wave reach the point **o** the advanced wave also reached the point **o**. When the retarded wave reach the point **b** the advanced wave also reach the point **b**. We can see the Figure 1 about the synchronization of the two waves. It is clear the most energy flow are go through the region close to the line between **a** to **b**.

III. MUTUAL ENERGY FLOW

A. The mutual energy flow from a to b

Using Ψ_b^* multiply Eq(3) from right we have

$$(i\hbar\frac{\partial}{\partial t}\Psi_a)\Psi_b^* = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_a\Psi_b^* \tag{33}$$

Using Ψ_a multiply the complex conjugate of the Eq(21) from the left, we have

$$-i\hbar\Psi_a\frac{\partial}{\partial t}\Psi_b^* = \Psi_a\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_b^* \tag{34}$$

Subtract the Eq.(34) from Eq.(33) we obtain

$$(i\hbar\frac{\partial}{\partial t}\Psi_a)\Psi_b^* + i\hbar\Psi_a\frac{\partial}{\partial t}\Psi_b^*$$

$$= \frac{-\hbar^2}{2\mu}(\nabla^2\Psi_a\Psi_b^* - \Psi_a\nabla^2\Psi_b^*) \tag{35}$$

or

$$\frac{\partial}{\partial t}(\Psi_a\Psi_b^*) = -\frac{\hbar}{2\mu i}\nabla\cdot(\nabla\Psi_a\Psi_b^* - \Psi_a\nabla\Psi_b^*) \tag{36}$$

or

$$\frac{\partial}{\partial t}(\rho_{ab}) = -\nabla\cdot\mathbf{J}_{ab} \tag{37}$$

where

$$\rho_{ab} = \Psi_a\Psi_b^* \tag{38}$$

$$\mathbf{J}_{ab} = \frac{\hbar}{2\mu i}(\nabla\Psi_a\Psi_b^* - \Psi_a\nabla\Psi_b^*) \tag{39}$$

The above formula are mutual energy flow principle. $\mathbf{J}_{ab}$ are mutual energy flow.

$$\frac{d}{dt}\iiint\limits_V \rho_{ab}dV = -\oiint\limits_\Gamma \cdot J_{ab}\hat{n}d\Gamma \tag{40}$$

This flow is not a divergence flow $\mathbf{J}_{ab}$. It is a point to point converged flow. This can be proved similar to the photon as following, assume Γ is big sphere, the radius of the big sphere is infinity. Assume the wave $\Psi_a(\mathbf{r},t)$ is a short time wave. In the time $t_{a0}=0$ the wave is at the place of point **a**. afterwards the wave begin to spread out. When the wave reached the big sphere surface Γ, it happened at a future time

$$t_a = \frac{R}{v} \tag{41}$$

, where R is the radius of the sphere.

The advanced wave started at the time when the retarded wave reached the point **b**, which is the time $t_{b0} = \frac{2l}{v}$, where $2l$ is the distance from point **a** to point **b**.

v is the speed of the wave. The advanced wave $\Psi_b(\mathbf{r}, t)$ reach the big sphere is at the past time

$$t_b = \frac{2l}{v} - \frac{R}{v} \tag{42}$$

. We have assume

$$2l \ll R \tag{43}$$

Since the retarded wave come to the big sphere in the future, the advanced wave come to the big sphere in the past. The retarded wave and the advanced wave are not nonzero in the same time at the big sphere Γ, hence

$$\nabla\Psi_a\Psi_b^* - \Psi_a\nabla\Psi_b^* = 0 \tag{44}$$

at the sphere Γ. The $\mathbf{J}_{ab}$ has no any flux go out the big sphere Γ.

$$\int_{-\infty}^{\infty} \oiint_{\Gamma} \mathbf{J}_{ab} \cdot \hat{n} d\Gamma dt = 0 \tag{45}$$

This means that mutual energy flow $\mathbf{J}_{ab}$ do not go outside our universe. Inside the volume V their is only the two sources for the charges at **a** and **b** hence the flow can only started from **a** to **b**. The flow $\mathbf{J}_{ab}$ is very thin in the two ends point **a** and **b**. The flow $\mathbf{J}_{ab}$ are very thick in the middle between the two points **a** and **b**. The flow will has the same flux integral with time in any surface between the two point **a** and **b**. If the particle is a electron, this flow $\mathbf{J}_{ab}$ is the current. This flow is the electron itself.

The above formula also means that

$$\int_{-\infty}^{\infty} \oiint_{S_i} \mathbf{J}_{ab} \cdot \hat{n}_{abi} dS = const, \qquad i = 1, 2, ..n \tag{46}$$

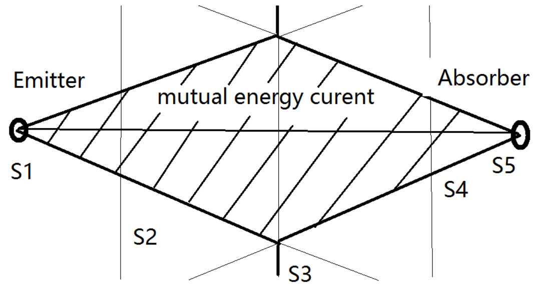

Figure 2. The mutual energy flow theorem tell us the time integral of the mutual energy flow $\mathbf{J}_{ab}$ will be same at any surface S_i where $i = 1, 2, 3, 4, 5$, between the two point **a** and **b**.

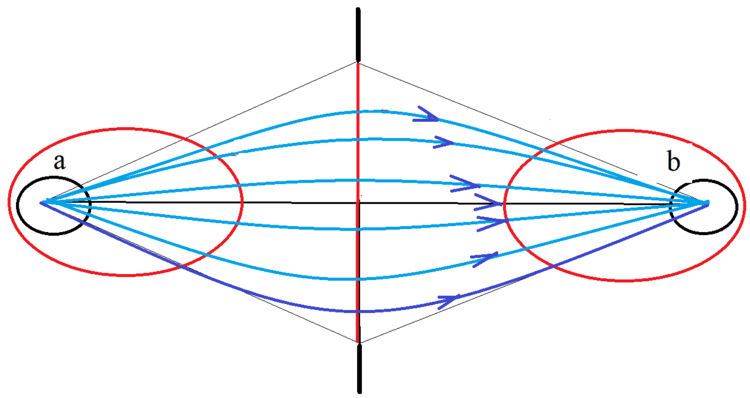

Figure 3. The mutual energy flow between the two point **a** and **b**. Assume there is a partition board. This wave is quasi-plane wave.

See Figure 2, where $\hat{n}_{abi}$ is unit vector of the surface S_i, the direction of $\hat{n}_{abi}$ is from **a** to **b**. This can be referred as the mutual energy flow theorem, The time integral of the total flux of the flows in any different surface S_i are same. This is same as the photon situation.

Assume there is a partition board. The mutual energy flow between point **a** and **b**, see Figure 3. If there are double slits on the partition board, it is no any problem for this kind of mutual energy flow to go through the two slits.

Since the mutual energy flow go through the double flits in the same time, and the wave at two end points **a** and **b** looks like particle, and at the middle between two end points **a** and **b** the mutual energy flow looks like wave. This explains the particle and wave duality for all particle includes electron.

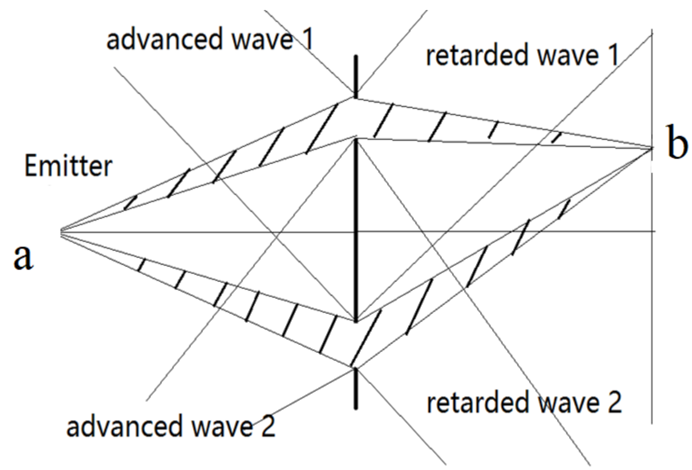

Figure 4. Assume there is a partition board which is put between the point **a** and **b**. Double slits are opened on the partition board which allow the particle to go through. The ship of the the mutual energy flow for the double slits are shown.

IV. SELF ENERGY FLOW

We also know that for the retarded wave started from point **a** there is,

$$\frac{\partial}{\partial t}(\rho_a) = -\nabla \cdot \mathbf{J}_a \tag{47}$$

For the advanced wave started from point B there is

$$\frac{\partial}{\partial t}(\rho_b) = -\nabla \cdot \mathbf{J}_b \tag{48}$$

where

$$\mathbf{J}_a = \frac{\hbar}{2\mu i}(\nabla \Psi_a \Psi_a^* - \Psi_a \nabla \Psi_a^*) \tag{49}$$

$$\mathbf{J}_b = \frac{\hbar}{2\mu i}(\nabla \Psi_b \Psi_b^* - \Psi_b \nabla \Psi_b^*) \tag{50}$$

$\mathbf{J}_a$ is the so called probability current of retarded wave Ψ_a which is a current sends energy from point **a** to infinite big sphere.

$\mathbf{J}_b$ is the so called probability current of advanced wave Ψ_b which is a current send energy from point b to infinite big sphere. Since this is advanced wave the energy current is at reversal direction. The energy flux is go from infinite big sphere Γ to the point **b**.

It should notice here, in this article we do not call $\mathbf{J}_a$ and $\mathbf{J}_b$ probability current instead we call them self-energy flows. The reason will be cleared at section VII.

We know that

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \mathbf{J}_a \cdot \hat{n} d\Gamma dt = const \tag{51}$$

The wave started from point **a** is retarded wave and hence this part of energy is at a future time to reach the the big sphere Γ.

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \cdot \mathbf{J}_b \hat{n} d\Gamma dt = -const \tag{52}$$

The negative symbol on the left of the above formula "−" is because this is a advanced wave, hence the result is a negative constant. The wave started from point **b** is advanced wave, this is part of energy will at past time to reach the big sphere. Unless our universe at the infinite big sphere is connected from future to the past, the energy send form point **a** can be received by the point **b**. Otherwise the retarded flow $\mathbf{J}_a$ from **a** will lose some energy in a future time at infinite big sphere Γ. The advanced flow $\mathbf{J}_b$ started from **b** will receive some energy in the past time at the infinite big sphere Γ. All these are not possible. This violate the energy conservation law. Our solution for this is described in the following section.

V. THE RETURN WAVES

A. The equation of the return wave

Advanced wave is obtained by a time reversal transform **R** which is defined by

$$\mathbf{R}\Psi(\mathbf{r}, \mathrm{t}) = \Psi_{\mathrm{r}}(\mathbf{r}, -\mathrm{t}), \tag{53}$$

Assume the Schrödinger equation is,

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[\frac{-\hbar^2}{2\mu} \nabla^2 + V(\mathbf{r}, t) \right] \Psi(\mathbf{r}, t) \tag{54}$$

In empty space there is,

$$V(\mathbf{r}, t) = 0 \tag{55}$$

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi(\mathbf{r},t) \tag{56}$$

The returned wave corresponding retarded wave are,

$$i\hbar\frac{\partial}{\partial t}\Psi_r(\mathbf{r},-t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_r(\mathbf{r},-t) \tag{57}$$

or

$$-i\hbar\frac{\partial}{\partial(-t)}\Psi_r(\mathbf{r},-t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_r(\mathbf{r},-t) \tag{58}$$

Let $-t=\tau$

$$-i\hbar\frac{\partial}{\partial(\tau)}\Psi_r(\mathbf{r},\tau)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_r(\mathbf{r},\tau) \tag{59}$$

We also know that $\Psi^*(\mathbf{r},\tau)$ also satisfy

$$-i\hbar\frac{\partial}{\partial(\tau)}\Psi^*(\mathbf{r},\tau)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi^*(\mathbf{r},\tau) \tag{60}$$

Compare the above formula we have the flowing results,

$$\Psi_r(\mathbf{r},\tau)=\Psi^*(\mathbf{r},\tau) \tag{61}$$

The return wave is just the conjugate wave. The return wave can be obtained from the original wave by change the sign before the items of $\frac{\partial}{\partial t}$.

B. The flow of the return waves

According discussion in the end of last section, we assume there are return waves for $\mathbf{J}_a$ and $\mathbf{J}_b$. The return wave for $\mathbf{J}_a$ is a wave from infinite big sphere at future time to the point $\mathbf{a}$. The return wave for $\mathbf{J}_b$ is a wave start from infinite big sphere at a past time to the point $\mathbf{b}$.

Hence for a quantum travel from $\mathbf{a}$ to $\mathbf{b}$ there 4 different waves, and 5 flows:

(1) retarded wave started from point $\mathbf{a}$, which is referred as $\mathbf{J}_a$

(2) advanced wave started from point $\mathbf{b}$, which is referred as $\mathbf{J}_b$

(3) return wave for (1), which is referred as $\mathbf{J}_{ar}$

(4) return wave for (2), which is referred as $\mathbf{J}_{br}$

The return wave for (1) satisfy

$$-i\hbar\frac{\partial}{\partial t}\Psi_{ar}(\mathbf{r},t)=\left[\frac{-\hat{\hbar^2}}{2\mu}\nabla^2\right]\Psi_{ar}(\mathbf{r},t) \tag{62}$$

It has the same equation with conjugate wave. The advanced wave is send from point **a**, in the $t=now$ to the time $t=past$. The returned wave Ψ_{ar} is from start from big sphere at time $t=future$ to the point **a** at time $t=now$.

The return wave for (2) satisfy

$$-i\hbar\frac{\partial}{\partial t}\Psi_{br}(\mathbf{r},t)=\left[\frac{-\hat{\hbar^2}}{2\mu}\nabla^2\right]\Psi_{br}(\mathbf{r},t) \tag{63}$$

It has the same equation with the complex conjugate wave. The retarded wave from now to the future. $\Psi_{br}(\mathbf{r},t)$ is from big sphere at time $t=past$ to the point **b** at time $t=now$. The two return flow can be defined as following,

$$\mathbf{J}_{ar}=\frac{\hbar}{2\mu i}(\nabla\Psi_{ar}\Psi_{ar}^*-\Psi_{ar}\nabla\Psi_{ar}^*)$$

$$=\frac{\hbar}{2\mu i}(\nabla\Psi_a^*\Psi_a-\Psi_a^*\nabla\Psi_a)$$

$$=-\frac{\hbar}{2\mu i}(\Psi_a^*\nabla\Psi_a-\nabla\Psi_a^*\Psi_a)$$

$$=-\mathbf{J}_a \tag{64}$$

Hence we have,

$$\mathbf{J}_a+\mathbf{J}_{ar}=0 \tag{65}$$

Similarly we also have,

$$\mathbf{J}_b+\mathbf{J}_{br}=0 \tag{66}$$

We assume that the wave Ψ_{br} and Ψ_{ar} can not interfere. If it can interfere the mutual energy flow $\mathbf{J}_{ab}$ will be canceled also and that is not what we hope. The above two formula tells us the $\mathbf{J}_a$ is canceled by $\mathbf{J}_{ar}$ and $\mathbf{J}_b$ is canceled by $\mathbf{J}_{br}$ hence the self-energy flows have no contribution to the energy flow from point **a** to the point **b**.

The energy flow with the mutual energy flow and the return wave is show in the Figure 5. We do not show in the picture of self-energy flows $\mathbf{J}_a$ and $\mathbf{J}_b$. Which helps to create the

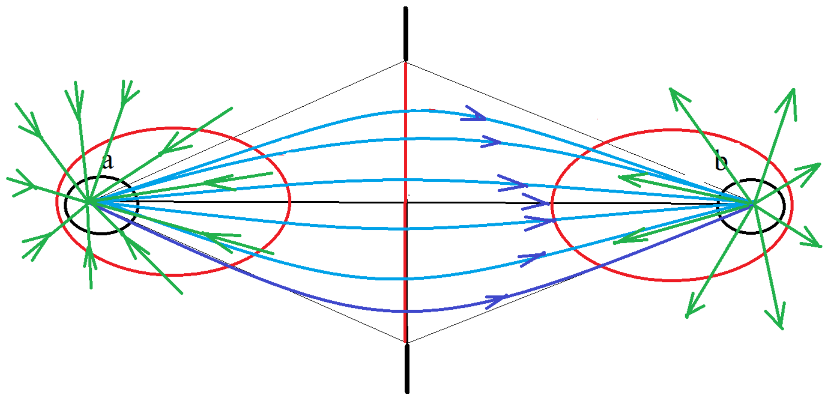

Figure 5. The mutual energy flow between the two point **a** and **b**. Assume there is a partition board. This wave looks like a quasi-plane wave. In the point of **a**, there is a return wave, the direction of energy flow of this return wave is point to the point **a**. In the point **b** there is a return wave, the direction of energy flow is starts from the point **b**. The return waves are show with green.

mutual energy flow $\mathbf{J}_{ab}$. In the figure we have only shown only 3 flows which are $\mathbf{J}_{ar}$, $\mathbf{J}_{br}$, $\mathbf{J}_{ab}$. Actually there are 5 flows: $\mathbf{J}_a$,$\mathbf{J}_{ar}$, $\mathbf{J}_b$,$\mathbf{J}_{br}$, $\mathbf{J}_{ab}$.

We have to assume that Ψ_{ar} do not interfere with Ψ_a and Ψ_b and Ψ_{br}, Ψ_{br} do not interfere with Ψ_a and Ψ_b and Ψ_{ar}. Ψ_{ar}, Ψ_{br} and Ψ_a, Ψ_b are different fields they satisfies different equations.

VI. IN CASE OF DIRAC EQUATION

A. Dirac equation

We have know that the Dirac equation can be written as

$$\frac{1}{c}\frac{\partial\psi_\mu}{\partial t}+\boldsymbol{\alpha}_{\mu\nu}\cdot\frac{\partial\psi_v}{\partial\mathbf{x}}+\frac{imc}{\hbar}\beta_{\mu\nu}\psi_v=0 \tag{67}$$

$$\mu,\nu=1,2,3,4... \tag{68}$$

Where $\boldsymbol{\alpha}=[\alpha_x,\alpha_y,\alpha_z]$, The components of α_x is $\alpha_{x\mu\nu}.\beta$ is a no dimension unit constant. $i=\sqrt{-1}$. m is the mass of the quantum. And

$$\boldsymbol{\alpha}^{\dagger} \equiv [\boldsymbol{\alpha}^{*}]^{T} = \alpha \tag{69}$$

$$\beta^{\dagger} \equiv [\beta^{*}]^{T} = \beta \tag{70}$$

$$\alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 = I$$

$$\frac{\boldsymbol{\beta_{\sigma\mu}}}{c}\frac{\partial \psi_{\mu}}{\partial t} + \beta_{\sigma\mu}\boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_{v}}{\partial \mathbf{x}} + \frac{imc}{\hbar}\psi_{\sigma} = 0$$

or

$$\frac{\boldsymbol{\gamma^{0}_{\sigma v}}}{c}\frac{\partial \psi_{v}}{\partial t} + \gamma^{\mathbf{1}}_{\sigma\mathbf{v}} \cdot \frac{\partial \psi_{v}}{\partial x} + \gamma^{\mathbf{2}}_{\sigma\mathbf{v}} \cdot \frac{\partial \psi_{v}}{\partial y} + \gamma^{\mathbf{3}}_{\sigma\mathbf{v}} \cdot \frac{\partial \psi_{v}}{\partial z} + \frac{imc}{\hbar}\psi_{\sigma} = 0$$

Hence the Dirac equation also can be written as,

$$\gamma^{\mu}_{\sigma v}\frac{\partial}{\partial x^{\mu}}\psi_{v} + \frac{imc}{\hbar}\psi_{\sigma} = 0$$

$$\mu, \nu = 1, 2, 3, 4...$$

B. Mutual energy flow corresponding to Dirac equation

Take complex conjugate to the Eq.(67) , we have,

$$\frac{1}{c}\frac{\partial \psi_{\mu}^{*}}{\partial t} + [\boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_{v}}{\partial \mathbf{x}}]^{*} - \frac{imc}{\hbar}[\beta_{\mu\nu}\psi_{v}]^{*} = 0$$

or

$$\frac{1}{c}\frac{\partial \psi_{\mu}^{*}}{\partial t} + \cdot[\frac{\partial \psi_{v}}{\partial \mathbf{x}}]^{*}\boldsymbol{\alpha}^{*\mathrm{T}}_{\nu\mu} - \frac{imc}{\hbar}{\psi_{v}}^{*}[\beta_{\mu\nu}]^{*T} = 0$$

Considering Eq.(70) we have,

$$\frac{1}{c}\frac{\partial \psi_{\mu}^{*}}{\partial t} + \cdot[\frac{\partial \psi_{v}}{\partial \mathbf{x}}]^{*}\boldsymbol{\alpha}_{\nu\mu} - \frac{imc}{\hbar}[\psi_{v}]^{\dagger}\beta_{\nu\mu} = 0$$

or

$$\frac{1}{c}\frac{\partial \psi^{\dagger}}{\partial t} + [\frac{\partial \psi}{\partial \mathbf{x}}]^{\dagger}\boldsymbol{\alpha} - \frac{imc}{\hbar}[\psi]^{\dagger}\beta = 0$$

Assume ϕ is also a wave function similar to ψ. We useϕ left multiply to the above formula we get:

$$\frac{1}{c}\frac{\partial\psi^\dagger}{\partial t}\phi+[\frac{\partial\psi}{\partial\mathbf{x}}]^\dagger\boldsymbol{\alpha}\phi-\frac{imc}{\hbar}[\psi]^\dagger\beta\phi=0$$

In the similar way we can obtains,

$$\frac{1}{c}\phi^\dagger\frac{\partial\psi}{\partial t}+\phi^\dagger\boldsymbol{\alpha}[\frac{\partial\psi}{\partial\mathbf{x}}]+\frac{imc}{\hbar}\phi^\dagger\beta[\psi]=0$$

Add the two formula together we have,

$$\frac{1}{c}(\frac{\partial\psi^\dagger}{\partial t}\phi+\phi^\dagger\frac{\partial\psi}{\partial t})+[\frac{\partial\psi}{\partial\mathbf{x}}]^\dagger\boldsymbol{\alpha}\phi+\phi^\dagger\boldsymbol{\alpha}[\frac{\partial\psi}{\partial\mathbf{x}}]=0$$

or

$$\frac{1}{c}\frac{\partial}{\partial t}(\psi^\dagger\phi)+\frac{\partial}{\partial\mathbf{x}}(\psi^\dagger\boldsymbol{\alpha}\phi)=0$$

Write

$$\rho=\psi^\dagger\phi$$

$$\mathbf{J}=c\psi^\dagger\boldsymbol{\alpha}\phi$$

we have

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial\mathbf{x}}\cdot\mathbf{J}=0$$

or

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{J}=0$$

We have know that the retarded wave and advanced wave corresponding to Dirac equation all satisfy the same equation. Assume ψ is the retarded wave send from point **a**, and ϕ is the advanced wave send from point **b**. **J** will be the mutual energy flow of the wave function $\psi$and ϕ. similar to the last section the mutual energy flow cannot go to outside of the infinite big sphere. Hence the mutual energy flow theorem should be also established for **J** which is corresponding to the wave function of Dirac.

C. The self energy flow of the Dirac equation

Similarly we can have the self energy flow for the retarded wave send from **a**,

$$\rho_\psi = \psi^\dagger \psi$$

$$\mathbf{J}_\psi = c\psi^\dagger \boldsymbol{\alpha} \psi$$

and for the advanced wave send from **b**,

$$\rho_\phi = \phi^\dagger \phi$$

$$\mathbf{J}_\phi = c\phi^\dagger \boldsymbol{\alpha} \phi$$

D. For the return wave of Dirac waves

Considering the return operator Eq.(53) we have the return wave equation,

$$-\frac{1}{c}\frac{\partial \psi_\mu^r}{\partial t} + \boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_v^r}{\partial \mathbf{x}} + jmc\beta_{\mu\nu}\psi_v^r = 0 \tag{71}$$

$$\mu, \nu = 1, 2, 3, 4... \tag{72}$$

We obtained the return wave by change the sign before the items of $\frac{\partial}{\partial t}$. superscript r in ψ^r means the return wave. We also can assume this return wave do not interfere with the original retarded and the advanced Dirac waves. This two return waves corresponding to the retarded wave and the advanced wave also do not interfere. We have

$$\rho_\psi^r = \psi^{r\dagger} \psi^r$$

$$\mathbf{J}_\psi^r = c\psi^{r\dagger} \boldsymbol{\alpha} \psi^r$$

similarly we can have the return wave for the advanced wave, and hence,

$$\rho_\phi^r = \phi^{r\dagger} \phi^r$$

$$\mathbf{J}_\psi^r = c\phi^{r\dagger} \boldsymbol{\alpha} \phi^r$$

And the return flow of the self energy flow should cancel the original self energy flow,

$$\mathbf{J}_\psi + \mathbf{J}_\psi^r = 0$$

$$\mathbf{J}_\phi + \mathbf{J}_\phi^r = 0$$

Hence in the empty space, the quantum from point **a** move to point **b** is down by the mutual energy flow $\mathbf{J} = c\psi^\dagger \boldsymbol{\alpha} \phi$.

VII. SUMMARY

For a quantum for example an electron, it travel from point **a** to point **b** in the empty space, there are 4 different waves instead one Schrödinger/Dirac wave. The 4 waves are retarded wave sends from **a** go to the big sphere surface Γ. The advanced wave sends from **b** and go to the big sphere surface Γ, the return wave for the retarded wave and the return wave for the advanced wave. Between point **a** and point **b** there is mutual energy flow $\mathbf{J}_{ab}$ which is transfer the energy or amount of charge from point **a** to point **b**. This flow is from point to point and do not diffused. This flow is very thin in the two ends and hence in it looks like a particle. This flow is very thick in the middle between the points **a** and **b**, and hence it looks a wave. In the middle if there are double slits. This flow will go through the two slits in the same time. This explained the duality of the quantum or particle.

The self-energy flow for $\mathbf{J}_a$ and $\mathbf{J}_b$ do not transfer and energy or amount of charge. We can think they are canceled by the return flow $\mathbf{J}_{ar}$ and $\mathbf{J}_{br}$. It is important to say that, the above flows $\mathbf{J}_{ab}$, $\mathbf{J}_a$, $\mathbf{J}_b$, $\mathbf{J}_{ar}$,$\mathbf{J}_{br}$ are all physics flow with energy or amount of the charge and they are not the probability flows.

We know the the electromagnetic field has sources which is electric current. We assume there are also some sources we do not know for the wave $\Psi_a(\mathbf{r}, t)$ and $\Psi_b(\mathbf{r}, t)$ which is stayed at the point **a** and point **b**. The source at point **a** can randomly sends the retarded wave. The source at **b** randomly send advanced wave. Point **b** is the target, actually on the place close to **b** there are thousands points similar to point **b** for example: $\mathbf{b}_1$, $\mathbf{b}_2$, ... $\mathbf{b}_n$... they all randomly send the advanced waves.

The probability come from the source of the retarded wave starts at point **a** and the source of the advanced wave at point **b**, they are synchronized concurrently, the mutual energy flow $\mathbf{J}_{ab}$ is produced. The retarded wave $\Psi_a(\mathbf{r}, t)$ is a random events, the advanced wave $\Psi_b(\mathbf{r},t)$ is also a random events, the two random events just meet together is also a random events. This leads to the position of the particle has be received with the probability. We do not know exactly which advanced wave started at points $\mathbf{b}_1$, $\mathbf{b}_2$, ... $\mathbf{b}_n$... will finally synchronized with the retarded wave $\Psi_a(\mathbf{r},t)$.

This can be referred as the the interpretation with the mutual energy principle for the quantum mechanics. This interpretation is enhancive transactional interpretation of John Cramer.

If the retarded wave flow $\Psi_a(\mathbf{r},t)$ cannot meet a advanced wave which is synchronized to the retarded wave $\Psi_a(\mathbf{r},t)$. This retarded wave flow $\mathbf{J}_a$ just returned through the corresponding return wave $\mathbf{J}_{ar}$. If it meet the advanced wave $\Psi_b(\mathbf{r},t)$ which is synchronized with the retarded wave $\Psi_a(\mathbf{r},t)$, the mutual energy flow $\mathbf{J}_{ab}$ is produced. After the $\mathbf{J}_{ab}$, there is the return flow $\mathbf{J}_{ar}$. Hence no matter the mutual energy flow is produced or not the self-energy flow $\mathbf{J}_a$ always returned through $\mathbf{J}_{ar}$. For the advanced wave, the similar things also happens. No mater $\mathbf{J}_{ab}$ is produced or not, there is $\mathbf{J}_{br}$ to cancel $\mathbf{J}_b$. Hence the self-energy flows do not transfer the energy and also do not lose the energy at infinite big sphere Γ.

VIII. CONCLUSION

We have introduced mutual energy principle and self-energy principle for photon and electromagnetic fields. In this article we applied the concept of the mutual energy principle and self-energy principle to other particles for example electron. We use Schrödinger and Dirac equation to study this problems. We study the problem in the empty space, but this should be easy to extended where the potential $V(\mathbf{r},t)$ do not vanish.

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