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The Geometrical solution , of the Regular n-Polygons The Unsolved Ancient Greek Special Problems and Their Nature .

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Abstract : The Special Problems of E-geometry [47] consist the , ***Mould Quantization*** **, of Euclidean Geometry in it , to become → Monad , through mould of Space –Anti-space in itself ,** ***which is the Material Dipole in monad Structure*** **→ Linearly , through mould of Parallel Theorem [44- 45] ,** ***which are the equal distances between points of parallel and line*** **→ In Plane , through mould of Squaring the circle [46] ,** ***where two equal and perpendicular monads consist a Plane acquiring the common Plane - meter*** **,** *π*,**→ and in Space (volume) , through mould of the Duplication of the Cube [46] ,** ***where any two Unequal perpendicular monads acquire the common Space-meter*** $\sqrt[3]{2}$ **,** ***to be twice each other*** **.** [44-47] . Now is added the , *Stores of Quantization* , which is the Regular-Polygons Mechanism .
The Unification ***of Space and Energy becomes through*** **[STPL]** ***Geometrical Mould Mechanism , the minimum Energy-Quanta*** **, In monads →** ***Particles , Anti - particles , Bosons , Gravity –Force , Gravity -Field , Photons , Dark Matter , and Dark-Energy , consisting the Material Dipoles in inner monad Structures*** [39-41] .
Euclid's elements consist of assuming a small set of intuitively appealing axioms , proving many other propositions . Because nobody until [9] succeeded to prove the parallel postulate by means of pure geometric logic , many self consistent non - Euclidean geometries have been discovered , based on Definitions , Axioms or Postulates , in order that non of them contradicts any of the other postulates . It was proved in [39] that the only Space-Energy geometry is Euclidean , agreeing with the Physical reality, on AB Segment which is Electromagnetic field of the Quantized on $\overline{AB}$ Energy Space Vector , on the contrary to the General relativity of Space-time which is based on the rays of the non-Euclidean geometries . Euclidean geometry elucidated the definitions of geometry-content , i.e.{[for Point , Segment , Straight Line , Plane , Volume, Space [S] , Anti-space [AS] , Sub-space [SS] , Cave, The Space - Anti-Space Mechanism of the Six-Triple-Points -Line , that produces and transfers Points of Spaces , Anti-Spaces and Sub-Spaces in Gravity field [MFMF] , Particles]} and describes the Space-Energy vacuum beyond Plank´s length level [Gravity`s Length 3,969.10¯62 m] , reaching the absolute Point ≡ $L_v = e^{i.\left(\frac{N\pi}{2}\right)b=10^{-N}=-\infty} = 0$ m , which is nothing and the Absolute Primary Neutral space PNS .[43-46] .
In Mechanics , the Gravity-cave *Energy Volume quantity* [wr] is doubled and is Quantized in Planck`s-cave Space quantity (h/2π) = The Spin = 2.[wr] ^{3} → i.e. Energy Space quantity ,wr , is Quantized , *doubled* , and becomes the Space quantity **h/π** following Euclidean Space-mould of *Duplication of the cube*, in Sphere volume V=(4π/3).[wr]^3 following the *Squaring of the circle* , ***π*** , and in Sub-Space-Sphere volume $\sqrt[3]{2}$ **,** and the *Trisecting of the angle* .

Keywords : The Unsolved ancient - Greek Problems , The Nature of the Special E-Problems .
The solution of All Odd - Regular - Polygons , The Stores of Quantization .

CONTENT

Preface **:**

This article is the completion of the prior [44] and [45-47] . With pure Geometrical logic is presented the Algebraic and Geometric Solution , and the Construction of all the n-Regular Polygons of this very interested problem . A new method for *the Alternate Interior angles* , The Geometrical - Inversion , is presented as this issues for Right - Angles .In article [62B] is presented the new Geometrical Proof . The new article is based on the Geometrical logic with a short procession in Mechanics , without any presupposition to Geometric knowledge on coupler points .

The concept of , ***The Relation* ,** Mould ***, of Angles and Lengths* ,** is even today the main problem in science , Mechanics and Physics .Although the Mould existed in the Theory of Logarithm and in the Theory of Means this New Geometrical -Method is the Master key of Geometry and in Algebra and consequently to the Relation between Geometry and Nature , for their in between applications . The New Regular Polygons Mechanism , exhibits *The How and Where Work* (Energy → Kinetic or Dynamic) produced from any Removal , *is Stored* .

The Programming of the Methods is very simple and very interesting for Computer-Programmers .

In the next article [64] is prepared the Unification of Energy-monads , *The How Energy from Chaos Becomes the Spin of Discrete Energy Monads* , with Geometry-Monads , *in Black-Holes and Matter,* through the Material - Geometry – monads and the Geometrical Inversion .

1.. Definition of Quantization.

Quantization is the concept (*the Process*) that any, **Physical Quantity → [PQ]** of the objective reality (Matter, *Energy or Both)* is mapping the Continuous Analogous, *the points,* to only certain Discrete values. Quantization of Energy is done in Space-tanks, on the material points, tiny volumes and on points consisting the Equilibrium *, all the Opposite Twin ,* of Space Anti-space. [61]

In Geometry [PQ] are the Points , *the nothing* , only , transformed into Segments , Lines , Surfaces , Volumes and to any other Coordinate System such as (x,y,z) , (i , j , k) and which are all quantized .

Quantization of E-geometry is the way of Points to become as → (Segments, Anti-segments = Monads = Anti-monads), (Segments, Parallel-segments = Equal monads) , (Equal Segments and Perpendicular-segments = Plane Vectors) , (Un-equal Segments twice – Perpendicular -segments = The Space Vectors = Quaternion) .[46]

In Philosophy [PQ] are the concepts of Matter and of Spirit or Materialism and Idealism.

a).. Anaximander , claimed that non of the elements could be, *Arche* and proposed , *apeiron* , an infinitive substance from which all things are born and to which all will return.
b).. Archimedes , is very clear regarding the definitions, that they say nothing as to whether the things defined exist or not , but they only require to be understood . Existence is only postulated in the case where [PQ] are the Points to Segments (magnitudes = quantization process) . In geometry we assume Point , Segment , Line , Surface and Volume , without proving their existence , and the existence of everything else has to be proved .
The Euclid`s similar figures correspond to Eudoxus` theory of proportion .
c).. Zenon, claimed that , Belief in the existence of many things rather than , *only one thing* , leads to absurd conclusions and for *, Point and its constituents will be without magnitude* . Considering Points in space are a distinct place even if there are an infinity of points , defines the Presented in [44] idea of *Material Point* .
d).. Materialism or and Physicalism , is a form of philosophical monism and holds that matter (*without defining what this substance is*) is the fundamental substance in nature and that all phenomena , *including mental phenomes and consciousness* , are identical with material interactions by incorporating notions of Physics such as spacetime , physical energies and forces , dark matter and so on .
e).. Idealism , such as those of Hegel , *ipso facto* , is an argument against materialism (*the mind-independent properties can in turn be reduced to the subjective percepts*) as such the existence of matter can only be assumed from the apparent (*perceived*) stability of perceptions with no evidence in direct experience .
Matter and Energy are necessary to explain the physical world but incapable of explaining mind and so results , *dualism* . The Reason determined in itself and its relation to the world creates the very old question as , *what is the ultimate purpose of the world* ?.
f).. Hegel`s conceive for mind , *Idea* , defines that , mind is *Arche* and it is retuned to [PQ] the subjective percepts , while Materialism holds just the opposite .

In Physics [PQ] are The , Electrical charge , Energy , Light , Angular momentum , Matter which are all quantized on the microscopic level . They do not seem quantized in the macroscopic scale because the size of the steps between each possible value is so small .
a).. De Broglie found that , light and matter at subatomic level display characteristics of both waves and particles which move at specific speeds called Energy-levels .
b).. Max Planck found that , Energy and frequency of the Electromagnetic radiation is quantized as the relation $E = h.f$.
In Mechanics , *Kinematics* describes the motion while , *Dynamics* causes the motion.
c).. Bohr model *for Electrons in free-Atoms* is the Scaled Energy levels , *for Standing-Waves* is the constancy of Angular momentum , *for Centripetal-Force in electron orbit* , is the constancy of Electric Potential , *for the Electron orbit radii* , is the Energy level structure with the Associated electron wavelengths.
d).. Hesiod Hypothesis [PQ] is ***Chaos*** , i.e. *the Primary Point* from which is quantized to *Primary Anti-Point* . [From Chaos came forth **Erebus , *the Space Anti-space*** *,* and **Black Night** , ***The [STPL] Mechanism*** *,* but of Night were born **Aether** , ***The rest Gravity dipole Field connected by the Gravity***

Force , and **Day** , ***Particles Anti-particles*** **,** whom she conceived and **Bare** , *The* ***Equilibrium of Particles Anti-particles*** *, in Spaces Anti-spaces ,* from union in love with Erebus] . [43-46]

e).. Markos model for Physical Quantity → [PQ] is the Energy - Monad produced from Chaos ,which is the Zero - point 0 = ∅ ={⊕+⊖}= The Material-point = *The Quantum* = The Positive Space and the Negative Anti-Space , between Opposites = The equilibrium of opposite directions →← [58-61] In article is shown the How and Where this Physical Quantity is stored .

The Special Greek Problems .

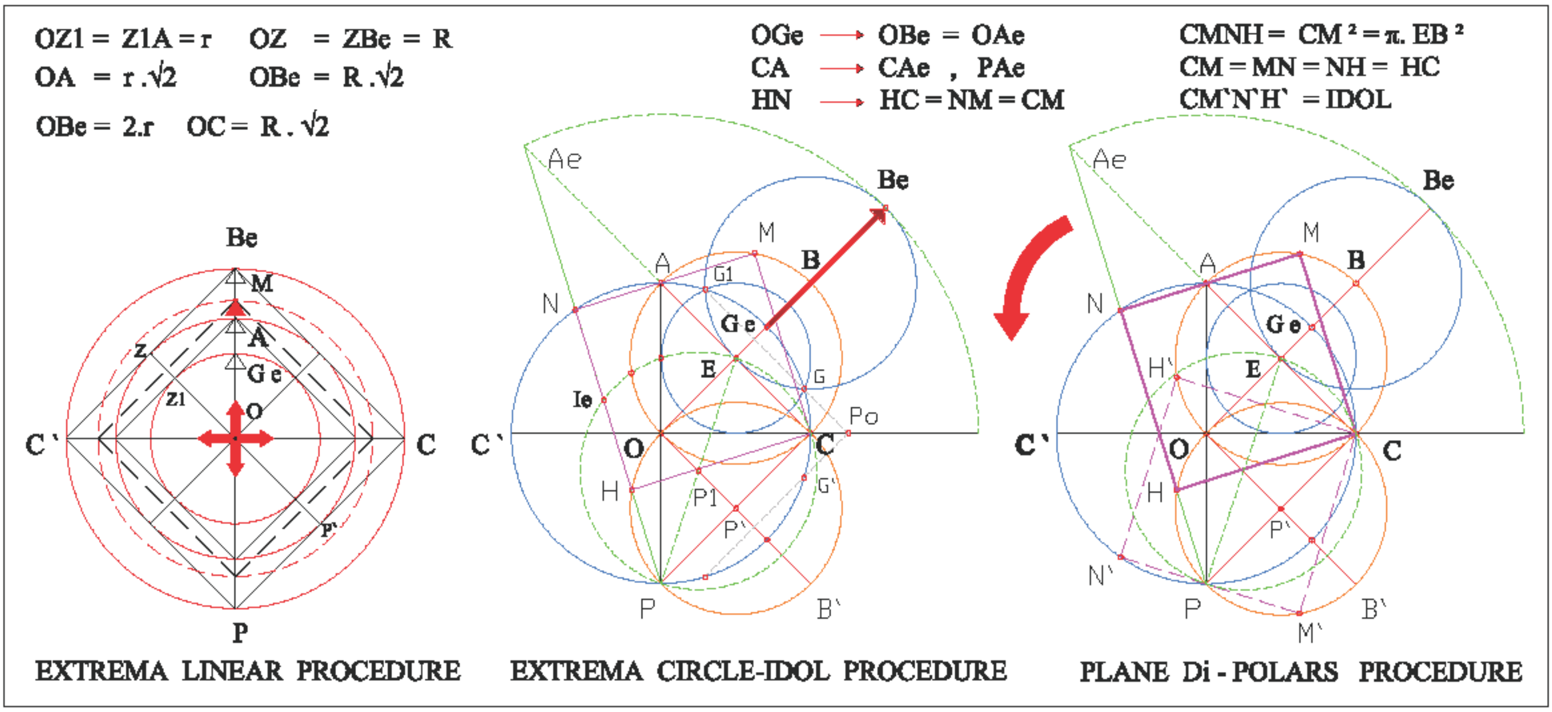
1.. The Squaring of the Circle .

The Plane Procedure Method . [45-46]

The property ,of Resemblance Ratio to be equal to 2 *on a Square , is transferred simultaneously by the equality of the two areas,* ***when*** *square is equal to the circle ,where that square is twice of the inscribed.*

This property becomes from the linear expansion in three spaces of the inscribed (O, OG_e) to the circumscribed (O ,OM) circle , in a circle (O, OA) as in . F.1-(1) .

1..*The Extrema method of Squaring the circle* F.1



(1) (2) (3)

F.1 → *The steps for Squaring any circle* [O,OA] or (E,EA = EC = EO) *on diameter* CA *through the – The Expanding of the Inscribed circle* O,OG_e→ *to the circle* O,OA *and to the circumscribed* O,OM and the *Four Polar O, A, C, P, Procedure method* :

In (1) *is Expanding Inscribed circle* O,OG_e → *to circle* O, OA *and to circumscribed* O,OM .

In (2) *The Inscribed square* CBAO *is Expanding to square* CMNH *and to circumscribed* CAC`P

In (3) *The Inscribed square* CBAO and its Idol CB`PO , Rotate through the pole C , Expand through Pole O on OB line , and Translate through pole P on PN chord . Extrema *Edge point* B_e of *circle* O,OB_e Rotate *to* A_e *point* , forming *extrema square* CMNH = NH² = π.EA² .

The Plane Procedure method :

It is consisted of two equal and perpendicular vectors CA , CP , ***the Mechanism*** , where CA = CP and CA ⊥ CP , *such , so that the Work produced is zero and this because each area is zero* , with the three conjugate Poles A , C , P related to central O , with the three Pole-lines CA ,CP ,AP and the three perpendicular Anti -Pole-lines OB , OB\`, OC , and is ***Converting the Rectilinear motion in* (1) *, on the Mechanism , to Four - Polar Expanding rotational motion.***

The formulated Five Conjugate circles with diameters → CA = OB , CP = OB\` , $EB_e$ = OB , PC = OB\` , $P_oG_1 = P_oG`_1$ = CA and also the circumscribed circle on them ← define ***A System of infinite Changable Squares from*** → the Inscribed CBAO to → CMNH and to → the Circumscribed CAC \`P , ***through the Four - Poles of rotation .***

***The Geometrical construction* : F.2**

1.. Let E be the center , and CA is the diameter of any circle (E , EA = EC) .

2.. Draw CP = CA perpendicular at point C and also the equal diameter circle (P\`, P\`C = P\`O).

3.. From mid-point O of hypotynuse AP as center , Draw the circle (O, OA = OP = OC) and complete squares , OCBA , OCB\`P .
On perpendicular diameters OB , OB \` and from points B , B\` draw the circles , (B, BE = Be) , (B\`, B\`P\`) intersecting (O, OA) = (O, OP) circle at double points [G ,$G_1$] , [G\`, $G`_1$] respectively, and OB , OB\` produced at points B_e , $B`_e$, respectively.

4.. Draw on the symmetrical to OC axis , lines GG_1 and $GG`_1$ intersecting OC axis at point P_o .

5.. Draw the edge circle (O, OB_e) intersecting CA produced at point Ae and draw PA_e line intersecting the circles , (O, OA) , (P\`, P\`P) at points N - H , respectively.

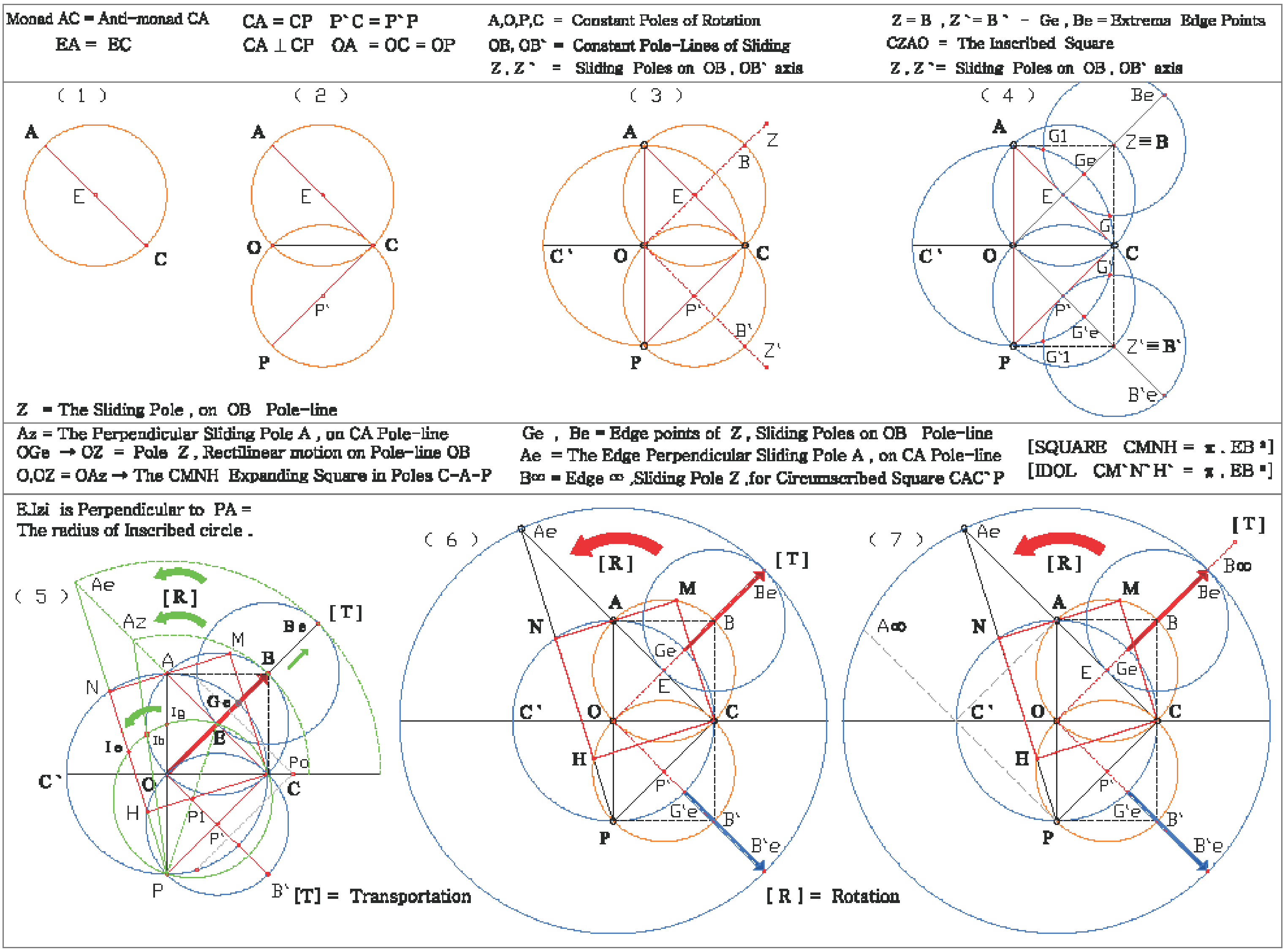
6.. Draw line NA produced intersecting the circle (E , EA) at point M and draw Segments CM , CH and complete quatrilateral CMNH ,calling it the *Space = the System* .
Draw line CM \` and line M \`P produced intersecting circle (O,OA) at point N\` and line AN \` intersecting circle (E, EA) at point H \`, and complete quatrilateral CM\`N\`H\`, calling it *The Anti-space = Idol = Anti – System* . P_1

7.. Draw the circle (P_1, P_1E) of diameter PE intersecting OA at point I_g ,and (E,EA) circle at point I_b

A.. ***Show*** that quadrilaterals CMNH , CM\`N\`H\` are Squares .

B.. ***Show*** that it is an Extrema Mechanism , on Four Poles where , ***The Two dimensional Space (the Plane) is Quantized to a System of infinite Squares*** → CBAO → CMNH → CAC\`P, and to **CMNH *square of side* CM = HN ,** *where holds* $CM^2 = CH^2 = \pi . EA^2 = \pi . EO^2$

C.. ***Show*** that , in circle (E , EA = EC = EO = EB) the ***Inscribed square CBAO*** , ***the square CMNH*** which is equal to the circle , and ***the Circumscribed square CAC\`P*** , Obey , ***Rotation of Squares*** through pole **P** , ***Translation of circle*** (E , EO) on **OB** Diagonal ,and ***Expansion*** in **CA** Segment.



F.2 → *The steps for Squaring the circle* (E, EA = EC) *on diameter* CA *through Plane Procedure Mechanism*

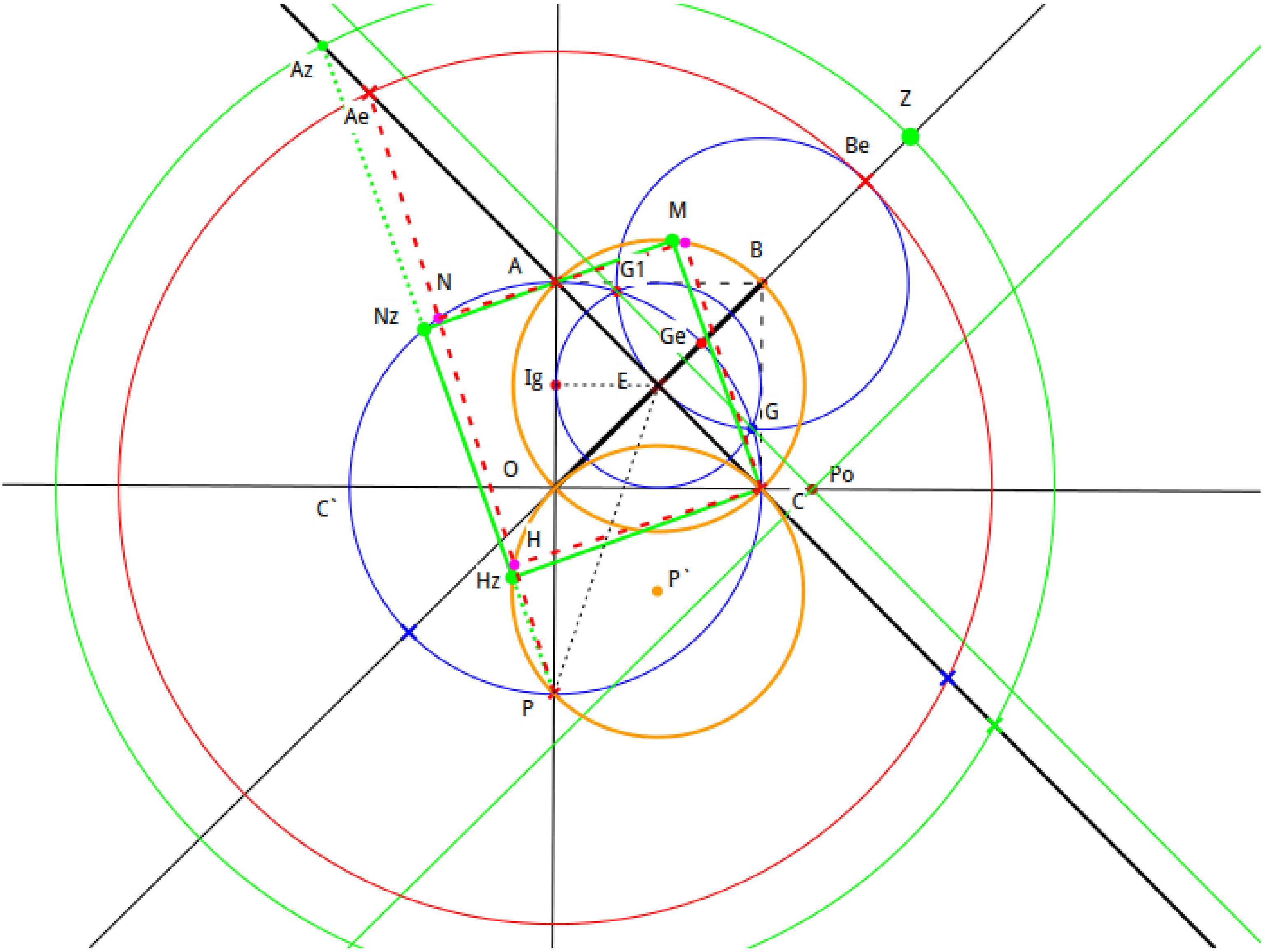
1.. Draw on any Orthogonal - System OA ⊥ OC , the circle (O , OA = OC) such that intersects the system at points P , C` respectively .

2.. Draw (E , EA = EC) circle on CA hypotynousa , intersecting OE line at point B , and from point B draw the circle (B ,BE = BB_e) and draw on CP hypotynousa circle (P`, P `C = P`P)

3.. Draw circle (O , OB_e) intersecting CA line produced at points at point A_e , and Draw A_eP intersecting (O , OA) circle at point N , and (P` , P ` P) circle at point H .

4.. Draw NA produced at point M on (E , EA) circle , and join chord MC on circle .

5.. Square CMNH is equal to the circle (E , EA) and issues → $\pi . CE^2 = CM . CH$



F.2-A → ***A Presentation of the Quadrature Method on Dr. Geo-Machine Macro - constructions .***
The Inscribed Square CBAO , *with Pole-line* AOP , *rotates through Pole* P , ***to the*** → *Circle - Square* CMNH *with Pole-line* NHP , ***and to the*** → *Circumscribed Square* CAC\`P , *with Pole - line* C\`PP ≡ C\`P , *of the circle* E , EO = EC **.**

The limiting Position of circle **(E , EB) to (B , BE = BB_e)** defines B_e point , and $OB_e = OA_e$ radius , such that CMNH Square be equal to π . OA ² .
The Initial relation Position $CE^2 = EB.EO = EO^2 = \frac{(CA)^2}{4}$ becomes → $\frac{(CN)^2}{4} = \pi . \frac{(CA)^2}{4}$,
for all Squares $C\,M_zN_zH_z$ on circles of Expanding radius OG_e to OB , to OB_e and to OZ .
This has a Special-reason for square CE^2 to become equal to number π .

Analysis :

In (**1**) - F.2 , Radius EA = EC and the unique circle (E , EA) of Segment AC , where AC , CA is The monad the Anti-monad.

In (**2**) - F.2 , Since circles (E , EA) , (P\`, P\`P) are symmetrical to OC axis (line) then are equal (*conjugate*) and since they are Perpendicular so , → No work is executed for any motion ← .

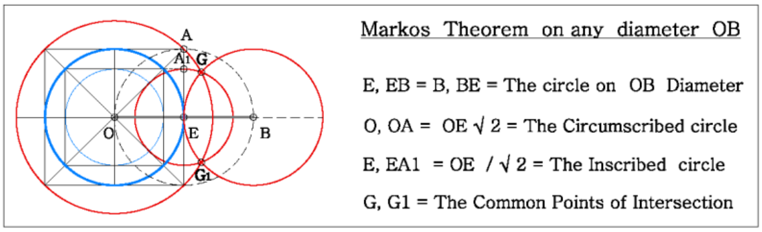
In (**3**) Points A ,C, P and O are the constant ***Poles*** of Rotation , and OB ,OB\` , OC – C A , CP , AP the Six , ***Pole*** and ***Anti – Pole*** , lines , of sliding points Z , Z \`, *and* A_Z , $A`_Z$ , while CA , CP are the constant Pole – lines { PA ,$PA_e$ ,$PA_Z$ , PC \`} , of Rotation through pole P .

In (**4**) Circles (E, EO) , (P\`, P\`O) on diameters OB , OB\` follow, *My Theorem of the three circles on any Diameters on a circle* ,where the pair of points G , $G_1$ and G\`, $G`_1$ consist a Fix and Constant system of lines $GG_1$ and $G`G`_1$.
When Points Z , Z \` coincide with the Fix points B , B\` and thus forming the inscribed Square CBAO or CZAO , *(this is because point* Z *is at point* A).
The PA , *Pole-line* , rotates through pole P where G_e , B_e , are the Edge points of the sliding poles on this Rectilinear - Rotating System .

In (**5**) When point Point Z≡B , Z\`≡B\` *on lines* OB,OB\`, then points $A_Z$ , $A`_Z$, are the Sliding points while CA , CP , are the constant Pole – lines { PA, PA_Z , PA_e, PC\`}, of Rotation through pole P.
Sliding points Z , Z \`, $A_Z$ , $A`_Z$, are forming Squares CMNH , CM\`N\`H\` , *and this as in Proof* [A-B] *below* , *where* PN , AN\` are the *Pole-lines* rotating through poles P, A , and diamesus HM passes through O . The circles (E , EO) , (P\`, P\`O) on diameters OB , OB\`, *blue color*, follow also , *my Theorem of the Diameters on a circle which follows* .

In (**6**) , Sliding poles Z , Z \` being at Edge point $G_e$≡ Z *formulates* CBAO *Inscribed square* , at Edge point $B_e$ , $B_e$≡ Z *formulates* CMNH *equal square to that of circle* and , at Edge point B∞ , *formulates* CAC\`P square , which is the *Circumscribed square*.

In (**7**) , are holding → CBAO *the Inscribed square* , CMNH , *The equal to the* (E , EO = P\`O) *Circle - square* , and CAC\`P *the Circumscribed square* .



F.3. → *Markos Theorem , on any OB diameter on a circle* .

Theorem : [F.1-(2)] , F.3

On each diameter **OEB** of any circle (**E , E B**) we draw,
*1.. **the circumscribed circle*** (O , OA = OE .√2) *at the edge point* **O** *as center ,*
*2.. **the inscribed circle*** (E ,OE/ √ 2 = OA/ 2 = EG) *at the mid-point* **E** *as center ,*
*3.. **the circle*** (B , BE = B , B_e) = (E , EO) *at the edge point* B *as center ,*

Then the three circles *pass through the common points* G , G_1 *, and the symmetrical to* OB *point* G_1 *forming an axis perpendicular to* OB , which has the Properties of the circles , where *the tangent from* *point B* to the circle (O , OA = OC) is constant and equal to 2.EB [2] , and has to do with , *Resemblance Ratio equal to 2* . Circle is squared on this Geometric Procedure by Rotation ,Expantion and Translation.

The Common-Proofs [A-B-C] :

In F.1-(2) , F.2-(5) ,
Angle < CHP = 90° because is inscribed on the diameter CP of the circle (P΄, P΄P) .
The supplementary angle < CHN =180 – 90 = 90° . Angle < PNA = PNM = 90° because is inscribed on the diameter AP of the circle (O , OA) and Angle < CMA = 90 ° because is inscribed on the diameter CA of the circle (E , EA = EC) .
The upper three angles of the quadrilateral CHMN are of a sum of 90+90+90 = 270 , and from the total of 360° , the angle < MCH = 360 – 270 = 90° , ***Therefore shape CMNH is rightangled*** and exists CM ⊥ CH .
Since also CM ⊥ CH and CA ⊥ CP therefore angle < MCA = HCP .
The rightangled triangles CAM , CPH are equal because have hypotynousa CA = CP and also angles < CMA = CHP = 90° , < MCA = HCP , **therefore side CH = CM** , and **Because CH = CM , the rechtangle CMNH is Square .** The same for Square CM`N`H` . (ο.ε.δ),(q.e.d) .

This is the General proof of the squares on this Mechanism without any assumptions .
From the equal triangles COH , CBM angle < CHO = CHM = 45◦ because lie on CO chord ,
and so points H,O,M lie on line HM *i.e.*
On CA line , Any segment* PA → PA $_z$ → PA $_e$ → PC` = CA** , ***drawn from Pole , P ,*** *beginning from* A *to* ∞ , is intersecting the *circumscribed* (O,OA) *circle* , and the *circle* ( P`, P`P = *P`C = EO = EC*) at the points N , H , and ***Formulates Squares CBAO , CMNH , $CM_zN_zH_z$, CAC`P respectively , which are , ***The Inscribed , In-between , Circumscribed Squares , of circle* ( O,OE )** = ( E , EO = EB ) = ( P, P`O).
Since angles < CA_zP , HCP have their sides CA_z ⊥ CP , A_zP ⊥ CH_z perpendicular each other , then are equal so angle < $PA_zC = PCH_z$, *and so point* A_z *, is common to circle* O,OZ , *Pole-line* CA , *and Pole-axis* PN , where the perpendicular to CM .

Since PE is diameter on (P_1 , P_1 P) circle , therefore triangle E. I_g.P is right-angled and segment ,EI_g, perpendicular to OA and equal to OE/√2 = OA/2 , the radius of the Inscribed circle . Since also point , I_g, lies on PA , therefore moves on (P_1 ,P_1 P) circle and point A on CA Pole-line , and so point B is on the same circle as A_z , while point B moves on circle E , EB .

B.. Proof (**1**) : **F**.2-(5) , F.2-A

(**1**) Any Point **Z** , which moves on diameter OB produced , Beginning from Edge-point $\mathbf{G_e}$ of the first circle , Passing from center **B** of the second circle , Passing from Edge-point $\mathbf{B_e}$ of the third circle , and Ending to infinite ∞ **, →** ***Creates on the three circles*** (O,OA) , (E,EO) , (B,BE) , with their centers on the diameter OB , the ***Changeable moving Squares***

a)..The Inscribed **CBAO** , when point Z ≡ G_e and center point O ,
b)..The In-between $\mathbf{CM_zN_zH_z}$ when point Z ≡ B and center point E ,
c)..The Extrema **CMNH** , when point Z ≡ B_e and center point B ,
d)..The Circumscribed **CAC`P** . when point Z ≡ B_∞ and center point ∞ ,

(**2**). Through the four constant Poles A,C,P – O of the *Plane Procedure Mechanism* , Squares *Rotate* through P , the Sides and Diamesus Slide *on* OB as Squares , Anti-Squares .Point **Z** moving from Edge points G_e (*forming Inscribed square* CBAO) , to in-between points G_e - B_e(*forming squares* $CM_zN_zH_z$) , to Extrema point B_e (***forming square*** CMNH ***equal to the circle***) , and to B_e - ∞ .
(**3**). Point I_g , belongs to the Inscribed circle (E,EO) and is Rotating , *expanding* , Inscribed Edge poind on (P_1 , P_1 P) circle to I_g, I_b, I_e and to → P point . The other two , *Sliding* , Edge moving points B , A

slide on OB , CA , Pole-lines respectively .In Initial square COAB and rightangled triangle COB the side CE squared is $CE^2 = EB.EO = [\sqrt{2}CB/2] . [\sqrt{2}CB/2] = CB^2 /2$. In Edge square CMNH and rightangled triangle CHM the side CN/2 squared is $CE_e^2 = E_eM. E_eH. = [\sqrt{2}CM/2] . [\sqrt{2}CM/2] = CM^2 /2$. In Infinite square CAC`P and rightangled triangle CPA the side CC`/2 = CO squared is $CO^2 = OA.OP = [\sqrt{2}CA/2] . [\sqrt{2}CA/2] = CA^2 /2$ .From above relations and since CE=OE ,CE_e = (HM/2),CO=CC`/2 ***then*** ,

$OE^2 = CB^2/2 = 2.CE^2 / 2 = [2/2] . CE^2 = k . CE^2$, where k = [2/2] = 1

$CE_e^2 = CM^2 /2 = k.(CB^2 /2)$ where $k = CM^2/ CB^2 = CM^2/ 2CE^2$

$CO^2 = CA^2 /2 = 2 . [CB^2 /2] = 2.CE^2 = k . CE^2$, where k = [2/2/2] = 2

A – Proof (**2**) : **F**.2-(5),F.2-A

Since BC ⊥ CO , the tangent from point B to the circle (O , OA) is equal to :

$\mathbf{BC^2} = BO^2 – OC^2 = (2. EB)^2 – (EB .\sqrt{2})^2 = 2. EB^2 = (2.EB).EB$ **= (2.BG) . BG** and since 2.BG = BG_1 then **BC^2 = BG . BG_1 ,** where point G_1 lies on the circumscribed circle , and this means that BG produced intersects circle (O, OA) at a point G_1 twice as much as BG . Since E is the mid-point of BO and also G midpoint of BG_1 , **so EG** is the diamesus of the two sides BO, BG_1 of the triangle BOG_1 and equal to 1/2 of radius OG_1 = OC , *the base* , and since the radius of the inscribed circle is half (½) of the circumscribed radius **then the circle (E , EB / √2 = OA/2) passes through point G .** Because BC is perpendicular to the radius OC of the circumscribed circle , ***so BC is tangent and*** **equal to BC^2 = 2 . EB^2 , i.e.** ***the above relation .***

Proofs F.(**2) : (5-6**) :

Following again prior A-B common proof ,

Angle < CHP = 90° because is inscribed on the diameter CP of the circle (P′,P′P) . The supplementary angle < CHN = 180 – 90 = 90° . Angle < PNA = PNM = 90° because is inscribed on the diameter AP of the circle (O , OA) and Angle < CMA = 90 ° because is inscribed on the diameter CA of the circle (E , EA = EC) . The upper three angles of the quadrilateral CHMN are of a sum of 90+90+90 = 270 , and from the total of 360° , the angle < MCH = 360 –270 = 90°, therefore shape CMNH is rightangled and exists CM ⊥ CH .

Since also CM ⊥ CH and CA ⊥ CP therefore angle < MCA = HCP .

The rightangled triangles CAM , CPH are equal because have hypotynousa CA = CP and also angles < CMA = CHP = 90°, < MCA= HCP and side CH = CM therefore , rechtangle CMNH is Square on **CA,CP** Mechanism , through the three constant Poles C,A,P of rotation . The same for square CM`N`H` .From the equal triangles COH , CBM angle < CHO = CHM = 45◦ then points H,O,M lie on line HM .i.e. Diagonal **HM** of squares CMNH on Mechanism passes through central Pole O.

The two equal and perpendicular vectors CA , CP , ***which is the Plane Mechanism*** **, of these Changable Squares through the two constant Poles C, P of rotation , is converting** ***the Circular motion to Four - Polar Rotational motion , and as linear motion through points*** **O , A .**

Transferring the above property to [F.2 –(5)] then when point Z moves on OB line → Point A_Z moves on CA and → PA_Z Segment rotates through point P , defining on circle (P_1 , P_1 P = P_1 E) ,

the Idol , [the points I_Z on circles O,OA = The Circumscribed P`P`O = The Circle] ,and points H,N such that shapes → CHNM are all Squares between the Inscribed and Circumscribed circle . i.e.

Archimedes trial , *The Central – Expansion of the Inscribed to the Circumscribed circle* **,**

is altered to the equivalent as , Polar and Axial motion on this Plane Mechanism .

The areas of above circles are →

Area of Inscribed $= \frac{1}{2} \pi.OE^2 = \frac{1}{2} \pi.\frac{CB^2}{2} = \pi.\frac{CB^2}{4} = [\frac{k\pi}{4}].CB^2$

Area of Circle $= 1 \pi.OE^2 = 1 \pi.\frac{CM^2}{2} = k\pi.\frac{CB^2}{4} = [\frac{k\pi}{4}].CB^2$

Area of Circumscribed $= 2 \pi.OE^2 = 2 \pi.\frac{CA^2}{2} = 2k\pi.\frac{CB^2}{4} = [\frac{k\pi}{2}].CB^2$

and those of corresponding squares , then one square of *Plane Mechanism* is equal to the circle , but which one ??.

→That square which is formed in Extrema Case of The Plane Mechanism **:**

The radius of the inscribed circle is AB/2 and equal to the perpendicular distance between center E and OA , so any circle of EP diameter passes through the edge-point (I_g) , and point (I_b) is the Edge common point of the two circles .G_e,
The Common Edge –Point of the three circles is (I_e) belongs to the Edge point Be of circle (B,BE = BB_e) , so exists ,

Case	:	[1]	[2]	[3]	[4]
Point	**Z** at →	G_e	B	B_e	B_∞
Point	**A** at →	A	A(I)	A_e	A_∞
Point	**Ig** at →	I_g	$I_Z = I_b$	I_e	P
		↓	↓	↓	↓
Square		CBAO ,	$CM_iN_iH_i$,	CMNH ,	CAC`P

i.e. Square CMNH of case [3] is equal to the circle , and $\mathbf{CM^2 = CH^2 = \pi . EA^2 = \pi . EO^2}$

On the three Circles (E,EO) , (P_1, P_1,P) , (O , OZ) *and Lines* OB,CA *exists* → F.2 - (5)

a)..Circle (O,OZ = OG_e) is Expanding to → (O,OZ = OB_e) Circumscribed circle , for the Inscribed CBAO square ,

b).. Point A , to → (A - A_Z) is The Expanding Pole-line A - A_Z for the In-between $CM_ZN_ZH_Z$ square ,

c).. Circle (P_1, P_1I_g) is Expanding to → (P_1, P_1I_b) Inscribed circle (E , E.I_g) to I_b and I_e point.

d).. Circle (O,OB → O B_∞, Pole-lines (A –A A_e→ A_∞) and (P –P I_e = PP → P) , for CAC`P square , Point N on (O,OA) , belongs to Circumscribed circle Point $I_e$ , on circle with diameter , PE , belongs to the Inscribed circle ( E , $EI_g$ = EG) Point H , on (P`,P`O) , belongs to the Circle.

i.e. ***It was found a Mechanism where the Linearly Expanding Squares →*** **CBAO – CMNH – CAC`P** ***, and circles →*** **(P_1 , P_1E) – (B , BE) – (O,OA)** *,which are between the Inscribed and Circumscribed ones* ***, are Polarly – Expanded as Four – Polar Squares .***
The problem is in two dimensions determining an edge square between the inscribed and the circumscribed circle . A quick measure for radius r = 2694 m gives side of square 4775 m and π = 3,1416048 → 11/10/2015

The Segments CM = CM ` , is the Plane Procedure Quantization of radius EC = EO = CP` in Euclidean Geometry , through this Mould , *the Mechanism* **.**
The Plane Procedure Method is called so , because it is in two dimensions → CA ⊥CP , as this happens also in , Cube mould ***, for the three dimensions of the spaces , which is a Geometrical machine for constructing Squares and Anti -Squares and that one equal to the circle .***
This is the Plane Quantization of , E - Geometry , i.e. The Area of square CMNH is equal to that of one of the five conjugate circles , or $\mathbf{CM^2 = \pi . CE^2}$ ***, and System with number π tobe a constant .***

***Remarks* :**

Since Monads AC = ds = 0 → ∞ are simultaneously (*actual infinity*) and (potential infinity) in Complex number form , *this defines that the infinity exists also between all points which are not coinciding* , and **ds** comprises any two edge points with imaginary part , for where this property differs between the infinite points between edges .This property of monads shows the link between Space and Energy which Energy is ***between*** the points and Space ***on*** points.
In plane and on solids , energy is spread as the Electromagnetic field in surface .
The position and the distance of points , can be calculated between the points and so to ***perform independent Operations*** (Divergence , Gradient , Curl , Laplacian) on points .

This is the Vector relation of Monads , ds = CA ,(or , as Complex Numbers in their general form w = a + b. i = *discrete and continuous* **) , and which is the Dual Nature of Segments = monads in Plane, tobe discrete and continuous). Their monad – meter in Plane ,** and in two dimensions **is CM** , *the analogous length* , **in the above Mechanism of the Squaring the circle with monad the diameter of the circle .** ***Monad is*** **ds** = CA= OB , *the diameter of the circle* (E ,EA) *with* CBAO *Square* , *on the Expanding* by Transportation and Rotation ***Mechanism*** *which is* → {Circumscribed circle (O,OA) – Inscribed circle (E , EG = E I_g) - Circle (B, BE) }← ***In extended moving System*** → {OB Pole-line – CA Pole-line – Circle (P_1 , P_1B = P_1. I_g) } , and ***is quantized*** to CMNH square.

The Plane Ratio square of Segments* – CE , CM -** ***is constant and Linear , and for any Segment **CN / 2** ***on circle*** **in Square CMNH** ***exists another one*** **CE** ***such that ,***

$$\rightarrow \ EC^2 / (CN/2)^2 = k = \text{constant} \ \leftarrow$$

i.e. the Square Analogy of the Heights in any rectangle triangle **COB** ***is linear to Extrema Semi - segments*** **(CN/2)** ***or to*** **(CA/2) ,** ***or the mapping of the continuous analog segment CE to the discrete segment (CN/2) .***

***The Physical notion of* Quadrature :**

The exact Numeric Magnitude of number ,π, may be found only by numeric calculations.[44]
All magnitudes exist on the < ***Plane Formation Mechanism of the first dimentional unit*** **AB** > as geometrical elements consisting , ***the Steady Formulation ,*** (The Plane System of the Isosceles Right-angle triangle ACP with the three Circles on the sides) and ***the moving and Changeable Formulation of the twin , System-Image ,*** (This Plane Perpendicular System of Squares , Anti-squares is such that *, the Work produced in a between closed area to be equal to zero*) .
Starting from this logic of correlation upon Unit , we can control *Resemblance Ratio* and construct all Regular Polygons on the unit Circle as this is shown in the case of squares .
On this **System** of these three circles F.3 (The Plane Procedure Mechanism which is a Constant System) is created also , a *continues* and , a *not continues* Symmetrical Formation , the changeable System of the Regular Polygons , and the **Image** (Changeable System of Regular anti-Polygons) the **Idol** ,as much this in **Space** and also in **Time ,** and was proved that in this Constant System , *the Rectilinear motion of the Changeable Formation is Transformed into a twin and Symmetrically axial - centrifugal Pole rotation (this is the motion on System) .*
The conservation of the Total Impulse and Momentum , as well as the conservation of the Total Energy in this Constant System with all properties included , exists in this Empty Space of the un-dimensional point Units of mechanism.

All the forgoing referred can be shown (maybe presented) with a Ruler and a Compass , or can be seen , live , on any Personal Computer . The method is presented on Dr.Geo machine .
The theorem of *Hermit-Lindeman* that number , pi , is not algebraic , is based on the theory of Constructible numbers and number fields (*on number analysis*) and not on the < *Euclidean Geometrical origin-Logic on unit elements basis* >

The mathematical reasoning (*the Method*) is based on the restrictions imposed to seek the solution < i.e. *with a ruler and a compass* > .
By extending Euclid logic of Units on the Unit circle *to unknown and now proved Geometrical unit elements* ,thus the settled age-old question for the unsolved problems is now approached and continuously standing solved . All Mathematical interpretation and the relative Philosophical reflections based on the theory of the non -solvability must properly revised .

Application in Physics **:**

From math theory of Elasticity , Cauchy equations of ***Stresses*** in three dimensions are ,

$\frac{\partial \sigma x}{\partial x} + \frac{\partial \tau yx}{\partial y} + \frac{\partial \tau zx}{\partial z} + X = 0$ $\frac{\partial \tau xy}{\partial x} + \frac{\partial \sigma y}{\partial y} + \frac{\partial \tau xy}{\partial z} + Y = 0$ $\frac{\partial \tau xz}{\partial x} + \frac{\partial \tau yz}{\partial y} + \frac{\partial \sigma z}{\partial z} + Z = 0$ where are ,

σx ,σy ,σz = Principal stresses in x,y,z axis , τxy,τxz,τyz = shear-stresses in xy,xz,yz Plane ,
X,Y,Z = The components of external forces and of ***Strain*** , $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$, $\frac{\partial}{\partial x}\frac{\partial v}{\partial y} = 0$, $\frac{\partial}{\partial x}\frac{\partial w}{\partial z} = 0$
where u = u (y,z) → are Deformation components , *the displacements* , in y,z axis .
v = c **x** z = the Rotation on z , axis
w = - c **x** y Anti-rotation in y axis .

Applying above equations on an orthogonal section of a solid , then exist the differential equations of equilibrium , and for the boundary conditions is found that , the Stress function is satisfying equations ,

$$\frac{\partial \tau yx}{\partial y} + \frac{\partial \tau zx}{\partial z} = \frac{\partial \gamma yx}{\partial y} + \frac{\partial \gamma zx}{\partial z} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial x}\frac{\partial w}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots (1)$$

and the boundary conditions on solid`s surface , $\frac{\partial u}{\partial y} dz - \frac{\partial u}{\partial z} .dy + y.dy + z.dz = 0$ ……..(2)
where , γxy , γxz , γyz = the slip components where is , $\gamma xy = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$.

Equations show that the resultant shear-stress at the boundary is directed along the tangent to the boundary and that , the Stress function u =u(yz) must be constant along the boundary of the cross section . i.e. each cross section on **x**, axis is rotated as a disk in its plane , from which points follow relation u = u(yz) and since stress function are constant , then from equation (2) y.dy + z.dz = 0 or $y^2 + z^2$ = constant , meaning that , ***a Cross-section under Stress stays Plane only in circle circumference , or a Plane Space , under Energy Stress , remains Flat only when the Plane becomes a circle ,***
i.e . follows the Plane Mould which is the squaring of the circle.
The same is seen in Laplace`s equation $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} \equiv \nabla^2 u = 0$ which is termed a harmonic function.

Placing $\nabla^2 u = 0$ in both parts of the equation of the circle , becomes Identity and $\nabla^2 u.(y^2+z^2) = \nabla^2 u.(c)$,

or any Monad = Quaternion *, consisted of the real part* ***the Plane Space , and under Energy Stress*** *the imaginary part* ***, remains in Flat only when the Plane becomes a circle , i.e. the Energy-Space discrete continuum follows extrema E-geometry Mould ,π, which is the squaring of the circle***.
If Potential Energy is zero then vector $\bar{\tau}$ is on the surface indicating the conjugate function. [49].

In Electricity , when an electric current flows through a conductor , then a transverse circular

Electromagnetic field is produced around itself following the vector – cross - product Plane mould ,π.

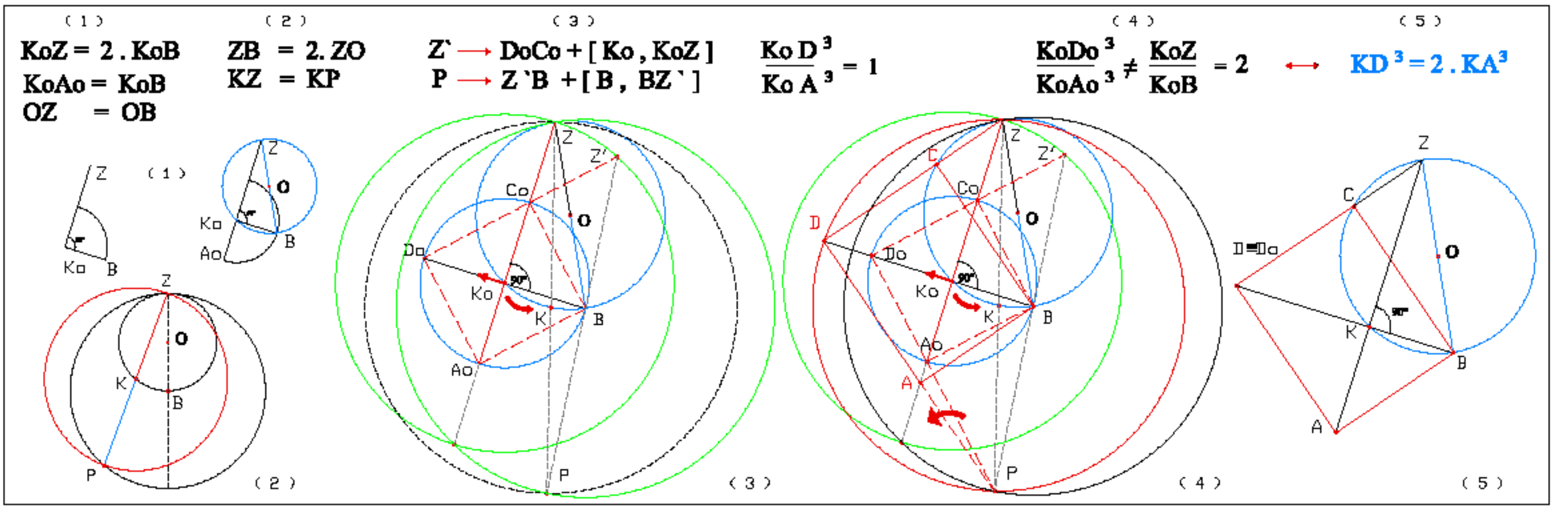
Because , the n^{th} - degree - equations are the vertices of the n-polygon in circle so , π , is their mould .

2.. The Duplication of the Cube , Or the Problem of the two Mean Proportionals , The Delian Problem.

The Extrema method for the Duplication of the cube ? [44-45]

This problem is in three dimensions as this first was set by Archytas proposed by determining a certain point as the intersection of three surfaces , *a right cone* , *a cylinder* , *a tore or anchoring with inner diameter nil* . Because of the three master - meters *where there is holding the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (continuous analogy) in all Spaces* , the solution of this problem , *as well as that of squaring the circle* , is linearly transformed .

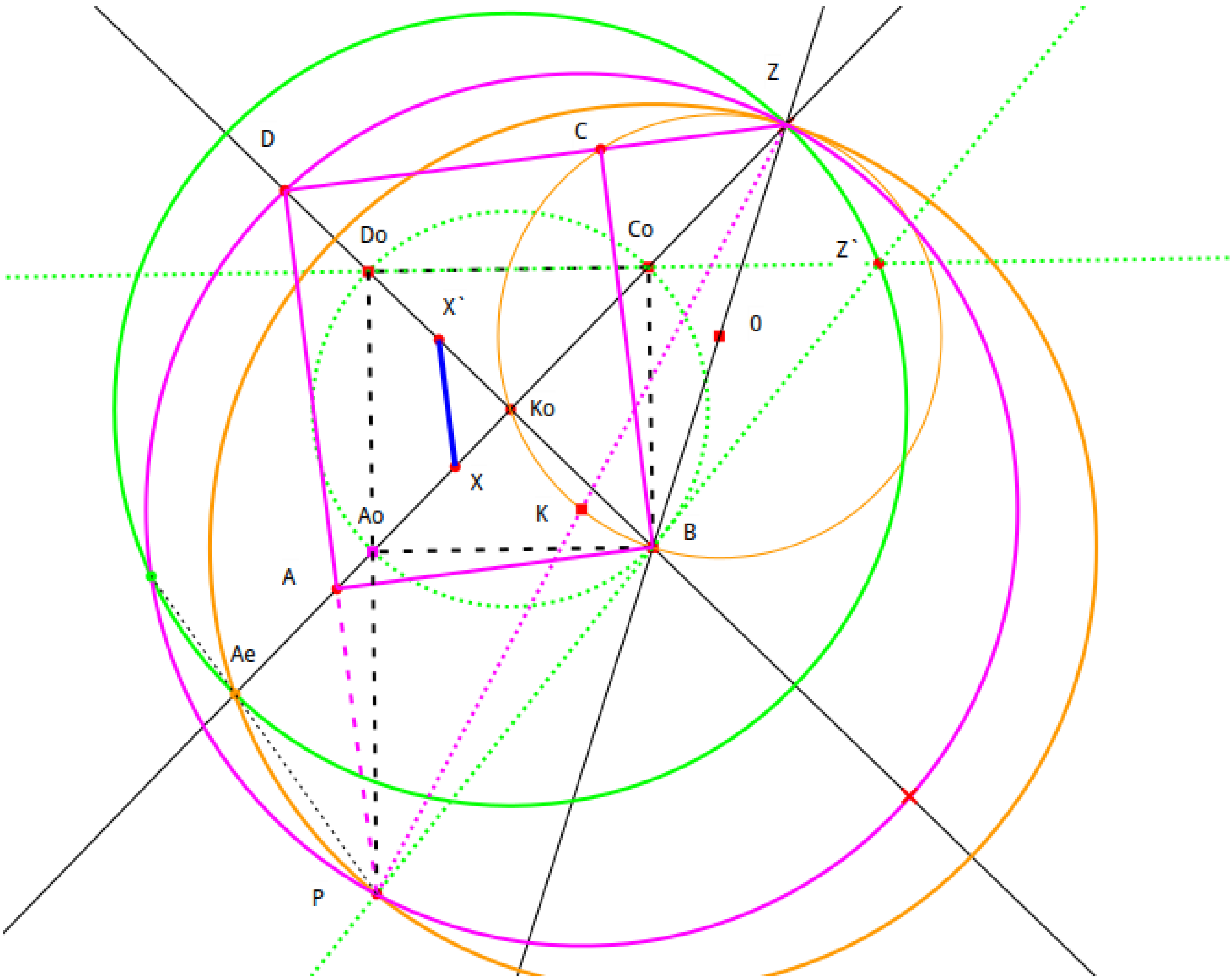
The solution is based on the known two locus of a linear motion of a point .
The geometrical construction Step – By – Step in F-4 :
The Presentation of the method on Dr-Geo machine for macro constructions in F.4 - A.



(1) (2) (3) (4) (5)

F.4.. → *The Mechanical Extrema Constant Poles* Z , K , P *of rotation in any circumcircle of triangle* ZKoB

1.. Draw on any Orthogonal - System $K_oZ \perp K_oB$, Segment $K_oZ = 2. K_oB$ and on BZ as hypotynousa the circle (O , OB = OZ) .
2.. Draw on K_oZ produced $K_oA_o = K_o B$ and form the square $B C_oD_oA_o$, .
3.. Draw the circles (K_o , K_o Z) , (B , BZ) which are intersected at points Z , A_e , and $D_o C_o$ produced at point Z ` , and $D_o A_o$ produced at point P .
4.. Draw on ZP as diameter the circle (K , KZ = KP) intersecting K_oD_o produced at point D and join DZ , DP intersecting the circle (O ,OZ) and line K_oA_o produced at point A .
5.. On Rectangle BCDA , the Cube of Segment K_oD is twice the Cube of Segment KoA and , exists $K_o D^3 = 2. K_o A^3$



F4-A. → ***A Presentation of the Dublication Method on Dr.Geo - Machine Macro - constructions***

$BC_oD_oA_o$, Is the initial Basic Quadrilateral ,*square* , on K_oZ , K_oB Extrema - lines mechanism . BCDA is the In-between Quadrilateral , on (K,KZ) Extrema-circle , and on K_oZ - K_oB Extrema lines of common poles Z , P , mechanism . *The Initial Quadrilateral* $BC_oD_oA_o$, *with Pole- lines* D_oA_o P - D_oC_o Z `, ***rotates*** *through Pole* P *and the moveable Pole* Z ` *on* Z`Z *arc* , ***to the*** → *Extreme Quadrilateral* BCDA *through Pole-lines* DAP - DCZ *with point* D_o , *sliding on* BK_oD_o *Pole-line* .

The Final Position of the Rotation – Translation is Quadrilateral BCDA *where* $K_oD^3 = 2. K_oA^3$

2.1. The Processus of The Duplication of Cube : F. 4 , F4 – A

1..Draw Line segment $K_o Z$ tobe perpendicular to its half segment K_oB or as $K_oZ = 2. K_oB \perp K_oB$ and the circle (O, BZ / 2) of diameter BZ . Line -segment ZK_o produced to $K_oA_o = K_oB$ (*or and* $K_oX_o \neq K_oB$) is forming the Isosceles right-angled triangle A_oK_oB .

2.. Draw segments BC_o , A_oD_o equal to $B A_o$ and be perpendicular to A_oB such that points C_o ,D_o meet the circle (K_o , K_oB) in points C_o , D_o , respectively, and thus forming the inscribed square $B C_oD_oA_o$. Draw circle (K_o , K_o Z) intersecting line D_oC_o produced at point Z ` and draw the circle ( B , BZ ) intersecting diameter Z `B , produced at point P (*the constant Pole*) .

3.. Draw line ZP intersecting (O, OZ) circle at point K , and draw the circle (K , KZ) intersecting line BD_o produced at point D . Draw line DZ intersecting (O, OZ) circle at point C and Complete Rectangle CBAD on the diamesus BD .

Show that this is an Extrema Mechanism on where ,

***The Three dimensional Space* KoA** → *is Quantized to* K_oD as → $K_oD^3 = 2. K_oA^3$.

Analysis :

In (**1**) - F.4 , $K_oZ = 2. K_oB$ and $K_o A_o= K_oB$, $K_oB \perp K_oZ$ and $K_oZ / K_oB = 2$.

In (**2**) Circle (B, BZ) with radius twice of circle (O, OZ) is ***the extrema*** case where circles with radius KZ = KP are formulated and are the locus of all moving circles on arc BK as in F4-(2) , F.5

In (**3**) Inscribed square $B C_oD_oA_o$. passes through middle point of K_oZ so $C_oK_o = C_o Z$ and since angle < $ZC_oO = 90^o$, then segment $OC_o // BK_o$ and $BK_o = 2.OC_o$.

Since radius OB of circle (O,OB = OZ) is ½ of radius OZ of circle (B,BZ =2.BO) then , **D** , is is ***Extrema*** case where circle (O,OZ) is the ***locus of the centers*** of all circles (K_o, K_oZ) , (B, BZ) moving on arc , K_oB, as this was proved in F.5.

All circles ***centered on this locus*** are common to circle (K_o, K_oZ) and (B , BZ) separately.

The only case of being together is the common point of these circles which is their common point P , where then → ***centered circle exists on the Extrema edge , ZP diameter.***

In (**4**) , F4-(4) Initial square $A_oBC_oD_o$, ***Expands and Rotates*** through point B , while segment D_oC_o limits to DC , where ***extrema poin***t Z ` moves to Z . Simultaneously , the circle of radius $K_oZ$ moves to circle of radius BZ on the locus of ½ chord $K_oB$ . Since angle < Z `D_oA_o P is always 90^o so , exists on the diameter Z`P of circle ( B, BZ `) and is the limit point of chord D_oA_o of the rotated square $BC_oD_oA_o$, and not surpassing the common point Z .

Rectangle $BA_oD_o C_o$ in angle < PD_oZ ` is expanded to Rectangle BADC in angle < PDZ by existing on the two limit circles ( B , BZ`= BP) and (K_o, K_o Z) and point D_o by sliding to D .

On arc K_oB of these limits is ***centered circle on ZP diameter ,*** i.e. ***Extrema*** happens to →

the common Pole of rotation through a constant circle centered ***on K_oB arc*** , and since point Do is the intersection of circle (K_o ,$K_oB = K_oD_o$) which limit to D , therefore the intersection of the common circle (K , KZ = KP) and line K_oD_o denotes that extrema point , where the expanding line D_oC_oZ` with leverarm D_oA_oP ***is rotating through Pole P***, and limits to line DCZ , ***and Point P is the common Pole of all circles on arc , K_oB, for the Expanding and simultaneously Rotating Rectangles.***

In (**5**) rectangle BCDA formulates the two right-angled perpendicular triangles

ADZ , ADB which solve the problem.

Segments K_oD , $K_oA_o= K_oB$ are the two Quantized magnitudes in Space (volume) such that Euclidean Geometry Quantization becomes through the *Mould of Doubling of the Cube* **.**
[*This is the Space Quantization of E-Geometry i.e. The cube of Segment K_oD is the double magnitude of K_oA cube , or monad $K_oD^3 = 2$ times the monad K_oA^3*] . About Poles in [5] .

The Proof : F.4. (3)-(4)-(5) .

1.. Since $K_oZ = 2. K_oB$ then $(K_oZ / K_oB) = 2$, and since angle < $ZK_oB = 90^o$ then BZ is the diameter of circle (O,OZ) and angle < $ZK_oB = 90^o$ on diameter ZB

2.. Since angle < $ZK_oA_o = 180^o$ and angle < $ZK_oB = 90^o$ therefore angle < $BK_oA_o = 90^o$ also .

3.. Since $BK_o \perp ZK_o$ then K_o is the midpoint of chord on circle (K_o, K_oB) which passes through Rectangle (*square*) B $A_oD_oC_o$. Since angle < ZDP = 90^o (*because exists on diameter ZP*) and since also angle < BCZ = 90^o (*because exists on diameter ZB*) therefore triangle BCD is right-angled and BD is the diameter .
Since Expanding Rectangles B $A_oD_oC_o$, BADC rotate through Pole , **P**, then points A_o , A lie on circles with BD_o , BD diameter , therefore point D is common to BD_o line and (K, KZ = KP) circle , and BCDA is Rectangle . F.4-(2) i.e. Rectangle BCDA possess $AK_o \perp BD$ and DCZ a line passing through point Z .

4.. From right angle triangles ADZ , ADB we have ,

On triangle Δ ADZ → $KD^2 = KA . KZ$ … (a)
On triangle Δ ADB → $KA^2 = KD . KB$ … (b)

and by division (a) / (b) then →

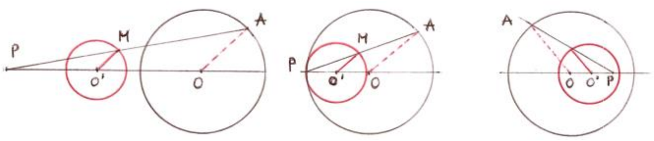
$$\frac{KD^2 = KA.KZ}{KA^2 = KD.KB} = \left|\frac{KD^2}{KA^2}\right| = \left|\frac{KA.KZ}{KD.KB}\right| \text{ or } \left|\frac{KD^3}{KA^3}\right| = \left|\frac{KZ}{KB}\right| = 2$$ (ο.ε.δ),(q.e.d)

i.e. → **$K_oD^3 = 2 . K_oA^3$,** ***which is the Duplication of the Cube*** .

In terms of Mechanics , ***Spaces Mould happen through ,*** *Mould of Doubling the Cube* ***, where for any monad*** **ds =K_oA** ***analogous to*** **K_oA_o*, the Volume or The cube of segment*** **K_oD** ***is double the volume of*** **K_oA** ***cube , or monad*** **$KD^3 = 2. K_oA^3$.** ***This is one of the basic Geometrical Euclidean Geometry Moulds , which create the METERS of monads → where*** **Linear** ***is the Segment*** **MA_1 ,** **Plane** ***is the square*** **CMNH** ***equal to the circle and in*** **Space** ***, is volume*** **$K_oD^3 = KD^3$** ***in all Spaces , Anti-spaces and Sub -spaces of monads = Segments ← i.e***

The Expanding square **B $A_oD_oC_o$** ***is Quantized to*** **BADC** ***Rectangle by*** **Translation** ***to point*** **Z`*, and by*** **Rotation** ***, through point*** **P** ***(the Pole of rotation) to point*** **Z .**

The Constructing relation between segments $K_o X$, $K_o A$ is → $(K_oX)^2 = (K_oA)^2.(XX_1 /AD)$
such that $X X_1$ // AD , as in Fig.6 (4), F7.(3). *All comments are left to the readers* , 30 / 8 / 2015.



(1) (2) (3)

F.5. → *For any point* **A** *on , and* **P** *Out-On-In circle* [O, OA] *and* O`P = O`O, *exists* O`M = OA / 2 .[16]

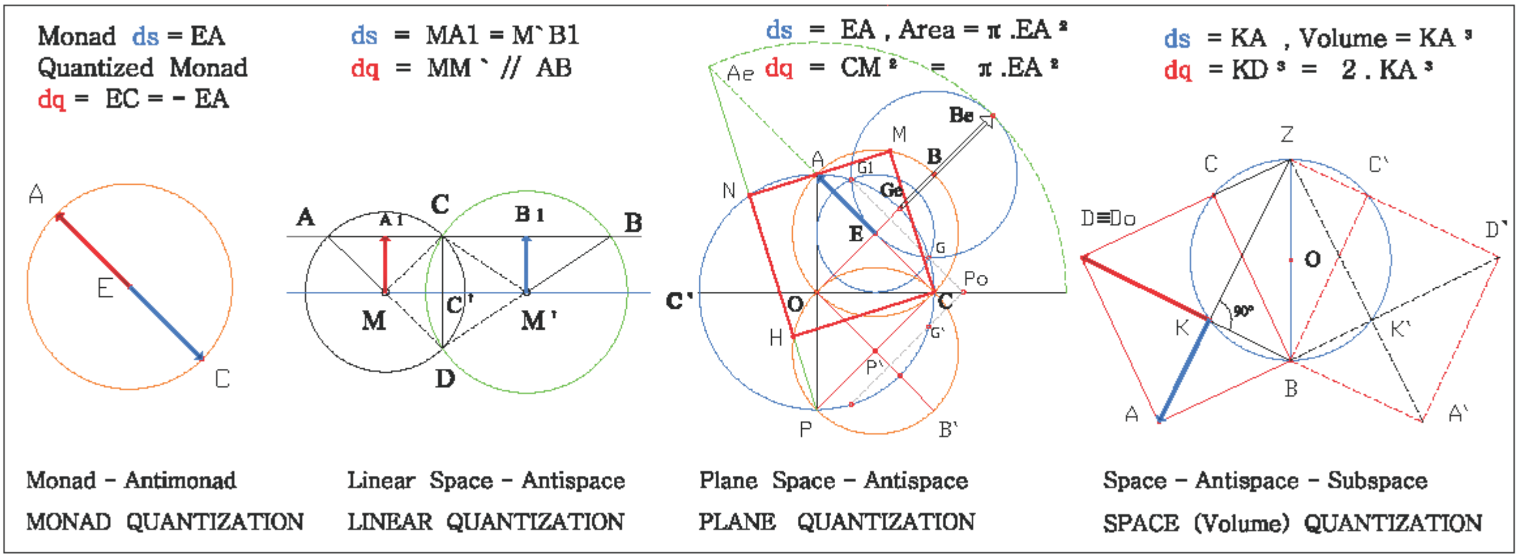
2.2 The Quantization of E-Geometry, { Points, Segments, Lines, Planes, and the Volumes } , to its moulds F-6 .

Quantization of E-geometry is the Way of Points to become as a → (Segments , Anti-segments = Monads = Anti-monads) , (Segments , Parallel-segments = Equal monads) , (Equal Segments and Perpendicular - segments = Plane Vectors) , (Non-equal Segments and twice-Perpendicular-segments = The Space Vectors = Quaternion) , by defining the mould of quantization .

The three Ways of quantization *are* → ***for Monads*** = The Material points , the Mould is the *Cycloidal Curl* Electromagnetic field , ***for Lines*** the Mould is that of *Parallel Theorem with the least constant distance* , ***for Plane*** the Mould is the *Squaring of the circle* ,π, and , ***for Space*** is the Mould of the *Duplication of cube* $\sqrt[3]{2}$. All methods in , F- 6 below .

In [61] The Glue-Bond pair of opposites [⊖ ⊕] ,creates rotation with angular velocity w = v/r , and velocity $v = w.r = \frac{2\pi}{T} = 2\pi r.f = [\frac{\sigma}{2}].(1+\sqrt{5})$, frequency $f = \frac{(1+\sqrt{5}\,]).\sigma}{4\pi r}$, Period $T = \frac{4\pi r}{\sigma(1+\sqrt{5})}$ where ± σ are the two Centripetal F_p and Centrifugal F_f forces .

Odd and Even number of opposites , on a Regular Polygon , defines the Quality of Energy- monad .



(1) (2) (3) (4)

F.6. → *Quantization for Point E , for Linear* ds = MA_1 *, for Plane ,π, Space (volume)* $\sqrt[3]{2}$.
Moulds for E-geometry Quantization are , **of** *monad* EA *to Anti-monad* EC – **of** AB *line to Parallel line* MM`- **of** AE *Radius to the* CM side of Square **of** KA *Segment* to KD Cube *Segment* .

***The numeric METERS of Quantization of any material monad* ds = AB *are as* →**
In any point A , ***happens through*** *Mould in itself* (The material point as a → ± dipole) in [43]

In monad ds = AC , ***happens through*** *Mould in itself for two points* (The material dipole in inner monad Structure as the Electromagnetic Cycloidal field which equilibrium in dipole by the Anti-Cycloidal field as in [43]).

For monad ds = EA the quantized and Anti-monad is dq = EC = ± EA

Remark 1: *The two opposite signs of monads EA , EC represent the two Symmetrical equilibrium monads of Space-Antispace , the Geometrical dipole* AC *on points* A,C *which consist space* AC *as in* F6 - (1)

Linearly , ***happens through*** *Mould of Parallel Theorem* ***, where for any point M not on ds = ± AB , the Segment*** $\mathbf{MA_1}$ ***= Segment*** $\mathbf{M`B_1}$ ***= Constant .*** F6 - (1-2)

Remark 2 : *The two opposite signs of monads represent the two Symmetrical monads in the Geometrical machine of the equal and Parallel monads* [MM`//AB *where* $MA_1 \perp AB$ , $M`B_1 \perp AB$ and $MA_1 = M`B_1$ ] *which are* → *The Monad* $MA_1$ *– Antimonad* $M`B_1$ *, or* → *The Inner monad* MA1 *Structure –The Inner Anti monad structure* $M`B_1 = - MA_1$ = Idle *, and* **{** The ***Space = line AB , Anti-space = the Parallel line MM` = constant*** **}**.

The Parallel Axiom is no-more Axiom because this has been proved as a Theorem [9-32-38-44].

Plainly , ***happens through*** *Mould of Squaring of the circle* ***, where for any monad ds = CA = CP , the Area of square CMNH is equal to that of one of the five conjugate circles and π = constant , or as*** **CM ² = π . CE ² .**
On monad ds = EA = EC , the Area = π.EC² and the quantized Anti-monad dq = CM ² = ± π. EC² and this because are perpendicular and produce Zero Work . F6-(3)
Remark 3 :

The two opposite signs represent the two Symmetrical squares in Geometrical machine of the equal and perpendicular monads as , [CA⊥ CP *, and CA = CP*] *, which are →*
The Square CMNH – Antisquare CM`N`H`, or → The Space – Idol = Anti-Space .

In Mechanics this propety of monads is very useful in Work area , where the two perpendicular vectors produce Zero Work . {***Space = square CMNH , Anti-space = Anti-square CM`N`H`*****}.**

In three dimensional Space , ***happens through*** *Mould Doubling of the Cube* ***, where for any monad ds = KA , the Volume or , The cube of a segment*** **KD** ***is the double the volume of*** **KA** ***cube , or monad*** **KD ³ = 2.KA ³ .**

On monad ds = KA the Volume = KA³ and the quantized Anti-monad , dq **= KD ³ = ± 2. KA³ .** F6-(4)

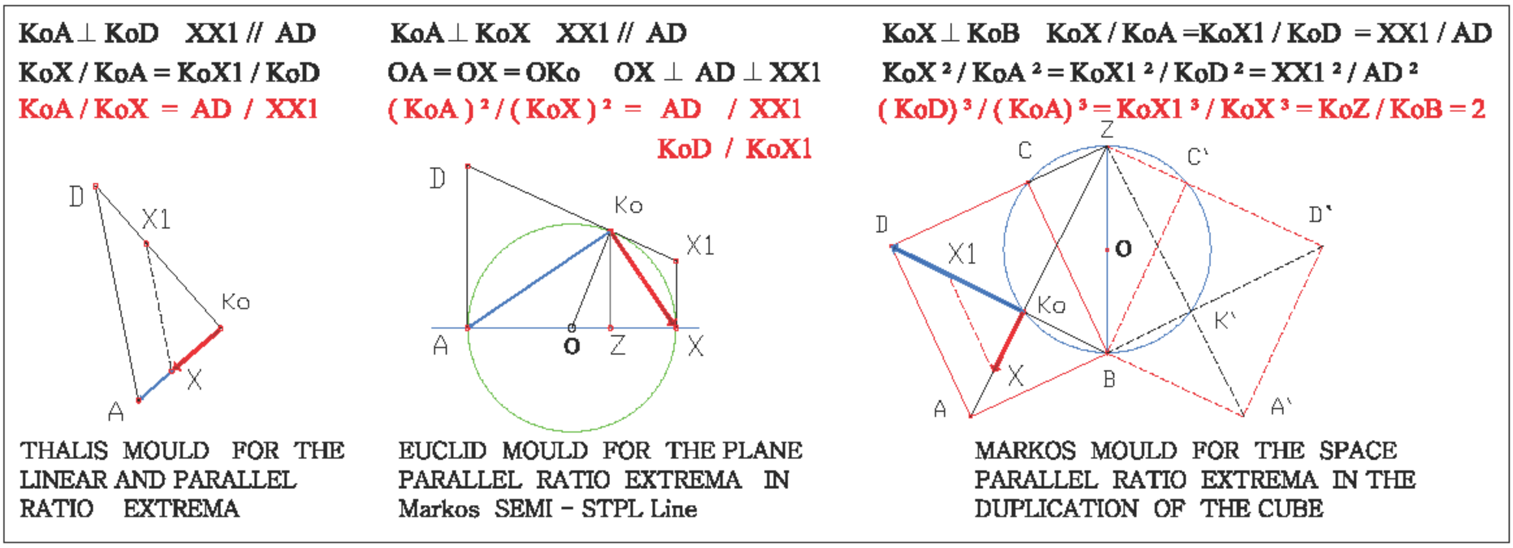
Remark 4 :
The two opposite signs represent the two Symmetrical Volumes in Geometrical machine of triangles [Δ ADZ ⊥ Δ ADB] *,which are → The cube of a segment KD is the double the volume of KA cube – The Anti-cube of a segment K`D` is the double the Anti-volume of K`A` cube* **,** *Monad ds = KA , the Volume = KA³ and the quantized Anti-monad dq = KD ³ = ± 2. KA³* **.**

{The ***Space = the cube*** **KA ³ ,** The ***Anti - Space = the Anti - Cube*** **KD ³ } .**

In Mechanics this property of Material monads is very useful in the Interactions of the Electromagnetic Systems where Work of two perpendicular vectors is Zero .

{ *Space = Volume of KA , Anti-space = Anti –Volume of KD, and this in applied to Dark-matter , Dark - Energy in Physics* } . [43]

Radiation of Energy is enclosed in a cavity of the tiny energy volume λ , (which is the cycloidal wavelength of monad) with perfect and absolute reflecting boundaries where this cavity may become infinite in every direction and thus getting in maxima cases (the edge limits) the properties of radiation in free space .



. (1) (2) (3)

F.7. → *The Thales , Euclid , Markos Mould , for the* ***Linear – Plane - Space*** *, Extrema Ratio Meters*

The electromagnetic vibrations in this volume is analogous to vibrations of an Elastic body (Photo-elastic stresses in an elastic material [18]) in this tiny volume , and thus Fringes are a superposition of these standing (*stationary*) vibrations .[41]

Above are analytically shown , the Moulds (The three basic Geometrical Machines) of Euclidean Geometry which create the METERS of monads i.e.

Linearly is the Segment **MA_1** ***, In Plane the square*** **CMNH** ***, and in Space is volume*** **KD^3** ***in all Spaces , Anti-spaces and Sub-spaces .***
This is the Euclidean Geometry Quantization in points to its constituents , i.e. the

1.. ***METER*** *of Point* A ***is the Material Point A , the ,***

2.. ***METER*** *of line* ***is the discrete Segment*** **ds = AB =** ***monad = constant , the***

3.. ***METER*** *of Plane* ***is that of circle ,number π , on Segment = monad , which is the Square equal to the area of the circle , and the***

4.. ***METER*** *of Volume* ***is that of Cube*** *$\sqrt[3]{2}$* ***, of any Segment = monad , which is the Double Cube of Segment and Thus is the measuring of the Spaces , Anti-spaces and Sub-spaces in this cosmos .***

5.. In Physics , ***METER*** *of* Mass is the Reaction of Matter , *anything material* , against Motion , *the contrast Inertia of matter against kinetic effects , and it is a number only without any other Physical meaning* . [39-40]
The meter of mass during a Parallel -Translation is a constant magnitude for every Body ,while for Moment of Inertia during a Rotational - motion is not , except it is referred to the same axis of the Body . markos 11/9/2015 **.**

2.3 The Three Master - Meters in One ,

for E-geometry Quantization , F-7

Master - meter is the linear relation of the Ratio , (*continuous analogy*) of geometrical magnitudes , of all Spaces and Anti-spaces in any monad .This is so because of the , extrema - ratio - meters .

Saying **master-meters** , we mean That the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (*continuous analogy*) in all Spaces , *in one in two in three dimensions*, as this happens to the Compatible Coordinate Systems as these are the Rectangular [x,y,z] , [i,j,k] , the Cylindrical and Spherical -Polar . The position and the distance of points can be then calculated between the points , and thus to ***perform independent Operations*** (Divergence , Gradient , Curl , Laplacian) on points only .This property issues on material points and monads .

This is permitted because , Space is quaternion and is composed of Stationary quantities , the position $\bar{r}(t)$ and the kinematic quantities , the velocity → $\bar{v} = dr/dt$ and acceleration → $\bar{a} = d\bar{v}/dt = d^2r/dt^2$. Kinematic quantities are also the tiny Energy volume caves **(*cycloid is length ,λ, the* Space *of velocity*** $\bar{v}$ ***, and*** $\bar{a}$ *consist in gravity`s field* ***the infinite*** **Energy *dipole Tanks*** *in where energy* is conserved) . In this way all operations on edge points are possible and applicable .

Remarks :

In F7-(1) ,The Linear Ratio , *for Vectors* , begins from the same Common point K_0 , of the two concurring and Non-equal , Concentrical and Co-parallel Direction monads $K_0X – K_0A$ and becomes K_0X_1- K_0D .

In F7-(2) ,The Linear Ratio , *for Plane* , begins from the same Common point K_0, of the two Non-equal , Concentrical and Co-perpendicular Direction monads.

Proof :

Segment $K_0A \perp K_0X$ because triangle AK_0X is rightangled triangle and $K_0Z \perp AX$. Radius $OK_0 = OA = OX$. Since DA , X_1X are also perpendicular to AX , therefore $K_0Z // X_1X // DA$. According to Thales theorem ratio $(ZA/ZX) = (K_0D/K_0X_1)$ and since tangent $DA = DK_0$ and $X_1K_0 = X_1X$ then $AZ / ZX = DA/ XX_1$. From Pythagorean theorem (Lemma 6)→ $K_0A^2 / K_0X^2 = (AZ/ZX) = (DA/XX_1) = (K_0D / K_0X_1)$ **i.e.**
The ratio of the two squares K_0A^2 , K_0X^2 are proportional to line segments K_0D , K_0X_1) . (o.ε.δ).

In F7-(3) ,The Linear Ratio , *for Volume* , begins from the same Common point K_0 , of the two

Non-equal , Concentrical and Co-perpendicular Direction monads.

In (1) → Segment $K_0A \perp K_0D$, Ratio $K_0X / K_0A = K_0X_1 / K_0D$, and Linearly (*in one dimension*) the Ratio of $K_0A / K_0X = AD / XX_1$, i.e. in Thales linear mould [XX_1 // AD] ,

Linear Ratio of Segments $\mathbf{XX_1}$ **, AD** ***is , constant and Linear , and it is the Master key Analogy of the two Segments , monads .***

In (2) → Segment $K_0A \perp K_0X$, $OK_0 = OA = OX$ and since OX_1 , OD are diameters of the two circles then $K_0D = AD$, $K_0X_1 = XX_1$, and Linearly (*in one dimension*) the Ratio of K_0A / K_0X

$= AD / XX_1$, in Plane (*in two dimensions*) the Ratio $[K_0A]^2 / [K_0X]^2 = AD / XX_1$, i.e.
in Euclid`s Plane mould [$K_0A \perp K_0X$] ,

The Plane Ratio square of Segments **–** $\mathbf{K_0A , K_0X}$ **-** ***is constant and Linear , and for any Segment*** $\mathbf{K_0X}$ ***on circle*** **(O,O** $\mathbf{K_0}$**)** ***exists another one*** $\mathbf{K_0A}$ ***such that ,***

$$\rightarrow \mathbf{K_0A^2 / K_0X^2 = AD / XX_1 = K_0D / K_0X_1} \leftarrow$$

i.e. the Square Analogy of the sides in any rectangle triangle* $\mathbf{A\,K_0\,X}$ *is linear to Extrema Semi-segments* $\mathbf{AD}$ *,* $\mathbf{X\,X_1}$ *or to* $\mathbf{K_0\,D}$ *,* $\mathbf{K_0X_1}$ *monads , or the mapping of the continuous analog segment* $\mathbf{K_0\,X}$ *to the discrete segment* $\mathbf{K_0\,A}$ *.

In (3) → Segment $K_0B \perp K_0X$, O K_0 = OB = OZ and since $X\,X_1$ // AD , then K_0A / K_0D = K_0X / K_0X_1 = AD / $X\,X_1$, and Linearly (*in one dimension*) the Ratio of K_0A / K_0X = AD / $X\,X_1$ and in Space (*Volume*) (*in three dimensions*) the Ratio [K_0 A] ³ / [K_0 D] ³ = [K_0X / K_0X_1] ³ = ½ .

i.e. in Euclid`s Plane mould [K_0A // K_0X , K_0D // K_0X_1] , ***Volume Ratio of volume Segments***

– $\mathbf{K_0\,A}$ **,** $\mathbf{K_0\,D}$ **- ,** ***is constant and Linear , and for any Segment*** $\mathbf{K_0\,X}$ ***exists another one*** $\mathbf{K_0X_1}$

such that → $\mathbf{(K_0X_1)^3 / (K_0\,X)^3 = 2}$ ← ***i.e. the Duplication of the cube.***

In F-7 , The ***three*** dimensional Space [K_0 A ⊥ K_0 D ⊥ K_0 X…] , where $X\,X_1$ // AD , The ***two*** dimensional Space [$K_0A \perp K_0X$] , where $X\,X_1$ // AD , The ***one*** dimensional Space [$X\,X_1$ // AD] , where $X\,X_1$ // AD , is constant and Linearly Quantized in each dimension.

i.e. ***All dimensions of Monads coexist linearly in Segments – monads and separately*** **(*they are the units of the three dimensional axis* x,y,z - i ,j, k -)** ***and consequently in all Volumes , Planes , Lines , Segments , and Points of Euclidean geometry, which are all the one point only and which is nothing.*** For more in [49-51] . 25/9/2015

At the beginning of the article it was referred to Geometers scarcity from which instigated to republish this article and to locate the weakness of prooving these Axioms which created the Non - Euclid geometries and which deviated GR in Space-time confinement . Now is more referred ,

a). There is not any Paradoxes of the infinite because is clearly defined what is a Point and what is a Segment .

b). ***The Algebra of constructible numbers and number Fields is an Absurd theory*** based on groundless Axioms as the fields are , and with directed non-Euclid orientations which must be properly revised .

c). ***The Algebra of Transcental numbers has been devised to postpone the Pure geometrical thought*** , which is the base of all sciences , by changing the base - field of the geometrical solutions to Algebra as base . The Pythagorians discovered the existence of the incommensurable of the diagonal of a square in relation to its side without giving up the base of it , which is the geometrical logic.

d). All theories concerning ***the Unsolvability of the Special Greek problems are based on Cantor`s shady proof*** , < *that the totality of All algebraic numbers is denumerable* > and not edifyed on the geometrical basic logic which is the foundations of all Algebra .

The problem of Doubling the cube F.4-A , as that of the Trisection of any angle F.11-A , is a Kinematic Mechanical problem with moveable Poles , and could not be seen differently , while Quadrature F.2-A with constant Poles of rotation and the proposed Geometrical solutions are all clearly exposed to the critic of the readers .

All trials for Squaring the circle are shown in [44] and the set questions will be answerd on the Changeable System of the two Expanding squares ,*Translation* [T] *and Rotation* [R] . The solution of Squaring the circle using the Plane Procedure method is now presented in F.1,2 , and consists an , *Overthrow* , to all existing theories in Geometry , Physics and Philosophy .

e). Geometry is the base of all sciences and it is the reflective logic from the objective reality and which is nature .

***The Physical notion of Duplication* :**

This problem follows , The three dimensional dialectic logic of ancient Greek , Αναξίμανδρος , [« τό μή Ον , Ον γίγνεσθαι » The Non-existent Exists when is done , ‘ The Non - existent becomes and never is] , *where* ***the geometrical magnitudes*** *, have a linear relation* (the continuous analogy on Segments) *in all Spaces as , in one in two in three dimensions , as this happens to the Compatible Coordinate Systems .*

The Structure of Euclidean geometry is such [8] that it is a Compact Logic where ***Non - Existent*** is found everywhere , and ***Existence*** *, monads* , is found and is done everywhere .

In Euclidean geometry points do not exist , but their position and correlation is doing geometry. The universe cannot be created , because it is continuously becoming and never is . [9]

According to Euclidean geometry ,and since the position of points (*empty Space*) creates the geometry and Spaces , Zenon Paradox is the first concept of Quantization . [15]
In terms of Mechanics , Spaces Mould happen through *,Mould of Doubling the Cube , where for any monad* ds = KoA *and analogous to* KoD *, the Volume or The cube of segment* KoD *is the double the volume of* KoA *cube , or monad* $KoD^3 = 2.KoA^3$. *This is one of the basic Geometrical Euclidean Geometry Moulds , which create the METERS of monads which →* ***Linear*** *is the Segment* ds = MA1 *,* ***Plane*** *is ,π, the square* CMNH *equal to the circle , and in* ***Space*** *is* $\sqrt[3]{2}$ *volume* KoD^3*, in all Spaces , Anti-spaces and Sub -spaces of monads ← i.e. The Expanding square* BAoDoCo *is Quantized to* BADC *Rectangle by* Translation *to point* Z `*, and by* Rotation *through point* P , (*the Pole of rotation)* .
The Constructing relation between any segments KoX , KoA is →
$(KoX)^3 = (KoA)^3 .(XX1 / AD)$ as in F.7

***Application in Physics* :**

The Electromagnetic waves are able to transmit Energy through a vacuum (empty space) by storing their energy vector in an Standing Transverse Electromagnetic dipole wave , and so considered completely particle like , and in the transverse interference pattern tobe considered as completely wave , so **the Same Quantity of Energy** is as ,

Energy $I_d = \frac{\rho\pi^2c^3}{2\lambda^2}[\varepsilon E^2+\mu H^2]$ ***in volume*** $V = [\frac{4(w^2r^2)^3}{3\pi}]$ having mass → ***Particle Energy***

$I_d= (\frac{\rho.c}{2}).(wA_o)^2$ ***in Interference pattern*** as → ***Wave***

This is the Wave-Particle duality unifying the classical Electromagnetic field and the quantum particle of light .Angular momentum of particles is → Spin = $\frac{\mathbf{E}}{\mathbf{w}} = [\pm\bar{v}.s^2] / w = (r.s^2) = w^2r^3 = [wr]^3$ and , as Spin = $\frac{\mathbf{h}}{\boldsymbol{\pi}}$ **= 2.[wr]³ , or *Energy Space quantity* wr , *is doubled and becomes the Space quantity*** $\frac{\mathbf{h}}{\boldsymbol{\pi}}$
The above relation of Spin shows the deep relation between Mechanics and E-geometry , *where in the tiny* ***Gravity-cave*** of $r =10^{-62}$ m , *the Energy -Volume-quantity* [wr] in cave , ***is doubled and is Quantized in Planck`s - cave Space quantity as*** , $(\frac{h}{\pi})$ = Spin = $2.[wr]^3$ in $r = 10^{-35}$ m ***i .e.***

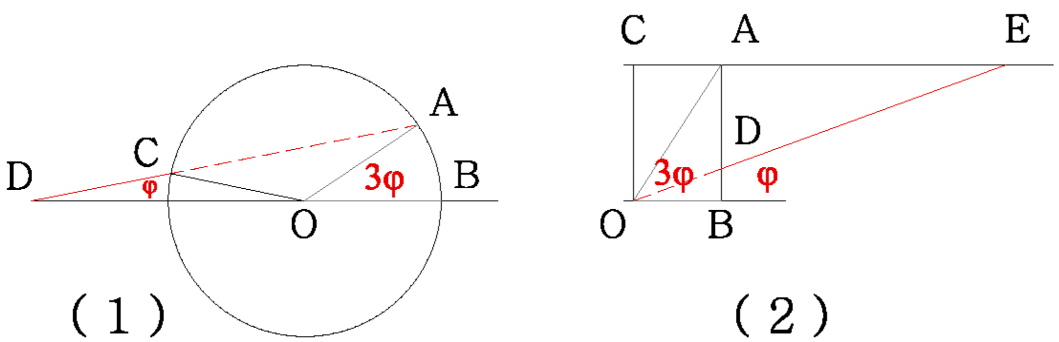
Energy Space quantity ,**wr** , is Quantized , and becomes the New Space quantity , $\mathbf{h/\pi = 2.[wr]^3}$, *doubled* , following the Euclidean Space-mould of *Duplication of the cube by changing frequency,* in tiny Sphere volume $V = (4\pi/3).[wr/2]^3$. *Also* , Since $w = E / [h/2\pi] = m.c^2/[h/2\pi] = 2\pi.mc^2/h = 2r.s^2 = 2.r^3.w^2$, then mass $\mathbf{m} = \frac{(wr)^3}{c^2} = \frac{2}{c^2}(wr)^3$, is Doubled as above with Space-mould and , *is what is called conversion factor mass* ,**m**, and it is an index of the energy changes .

All Energy magnitudes from , 0 → ∞ , deposit in the same Space , *resonance* , by changing frequency

3.. The Trisection of Any Angle .

Because of the three *master-meters* , where is holding the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (*a continuous analogy*) in all Spaces , the solution of this problem , as well as of those before , is linearly transformed .

The present method is a Plane method *, i.e. straight lines and circles ,* as the others and is not required the use of conics or some other equivalent . Archimedes and Pappus proposals are both instinctively right .



F.8. → (1) *Archimedes* , (2) *Pappus Method*

The Present method :

It is based on the Extrema geometrical analysis of the mechanical motion of shapes related to a system of poles of rotation .

The classical solutions by means of conics , or reduction to a , νεύσις , is a part of Extrema method . This method changes the ***Idle*** between the edge cases and *Rotates* it through constant points , *The Poles* , Fig.11 .

The basic triangle AOD_1 is such that angle $OD_1A=30$▫ and rotating through pole O .

The three edge positions are ,

a). Angle AOB = 90▫ when OD_1 ≡ - OE and then point D_1 is at point E on OB axis ,

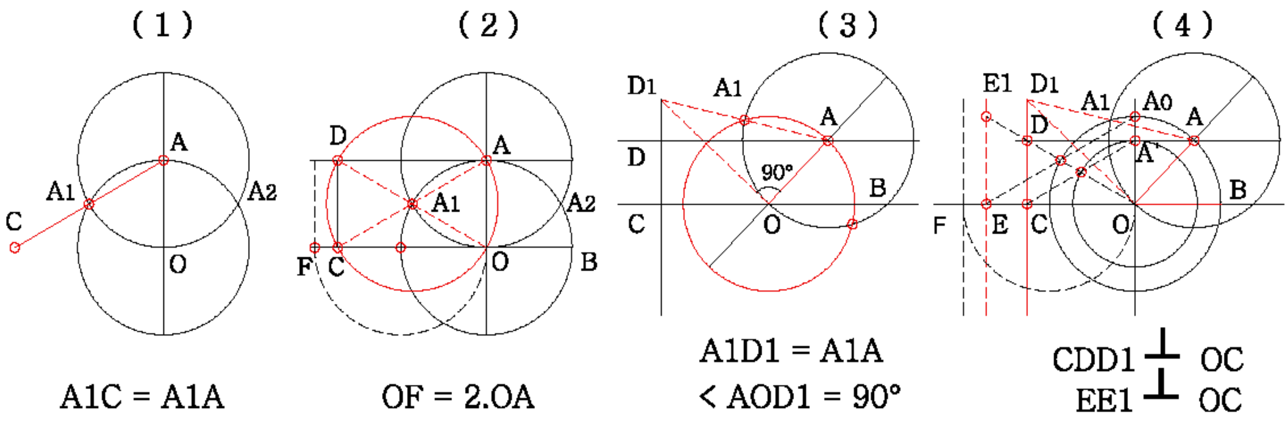
b). Angle AOB = 0 - 90▫ when OD_1= OE and then point D_1 is perpendicular to OB axis ,

c). Angle AOB = 0 when OA ≡ O and then point D_1 is perpendicular to OB axis.

This moving geometrical mechanism acquires common circles and constant common poles of rotation which are defined with initial ones .

This geometrical motion happens between the Extrema cases referred above ..

The steps of the basic Rotating Triangle AOD_1 between the extrema cases AOB=180▫ , AOB = 0



F.9. → ***The proposed Contemporary Trisection method .***

We extend Archimedes method as follows :

a . F9.-(2) **. Given an angle < AOB =AOC= 90°**

1.. Draw circle (A , AO = OA) with its center at the vertex A intersecting circle (O , OA = AO) at the points A_1 , A_2 respectively .
2.. Produce line AA_1 at C so that $A_1C = A_1A = AO$ and draw AD // OB .
3.. Draw CD perpendicular to AD and complete rectangle AOCD .
4.. Point F is such that OF = 2 . OA

b . F9.(3-4) . **Given an angle < AOB < 90°**

1.. Draw AD parallel to OB .
2.. Draw circle (A , AO = OA) with its center at the vertex A intersecting circle (O , OA = AO) at the points A_1 , A_2 .
3.. Produce line AA_1 at D_1 so that $A_1 D_1 = A_1A = OA$.
4.. Point F is such that $OF = 2 .OA = 2.O A_o$
5.. Draw CD perpendicular to AD and complete rectangle A′OCD .
6.. Draw A_o E Parallel to A′ C at point E (or sliding E on OC) .
7.. Draw A_o E′ parallel to OB and complete rectangle $A_oOE E_1$ **.**
8.. In F10 - (1-2-3) , Draw AF intersecting circle (O,OA) at point F_1 and insert after F_1 and on AF segment $F_1 F_2$ equal to OA → $F_1 F_2 = OA$.
9.. Draw AE intersecting circle (O , OA) at point E_1 and insert after E_1 on AE segment E_1E_2 equal to OA → $E_1E_2 = OA = F_1 F_2$.

To show that :

a). For all angles equal to 90° Points C and E are at a constant distance OC = OA . √ 3 and OE = OA_O . √ 3 , from vertices O , and also A′C // A_OE .
b). The geometrical locus of points C , E is the perpendicular CD , EE_1 line on OB .
c). All equal circles with their center at the vertices O , A and radius OA = AO have the same geometrical locus $EE_1 \perp OE$ for all points A on AD , or All radius of equal circles drawn at the points of intersection with its Centers at the vertices O , A and radius OA = AO lie on CD , E E_1 perpendicular lines .
d). Angle < D_1 OA is always equal to 90° and angle AOB is created by rotation of the right-angled triangle AOD_1 through vertex O .

e). Angle < AOB is created in two ways , by constructing circle (O , OA = OA_O) and by sliding , of point A_1 on line A_1 D Parallel to OB from point A_1 , to A .
f). Angle < AOB is created in two ways , either by constructing circle (O , OA = OA_O) and by sliding , of point A′ on line A′ D Parallel to OB from point A′ , to A , or on OA circle .
g). The rotation of lines AE , AF (*minimum and maximum edge positions*) on circle (O,OA = OA_O) from point E to point F which lines intersect circle (O , OA) at the edge points E_1 , F_1 respectively , **fixes a point** G on line EF and a point G_1 common to line AG and to the circle (O , OA) **such that** $G G_1$ = OA .

Proof :

a) .. F.9 .(1 - 2 - 4)

Let OA be one-dimensional Unit perpendicular to OB such that angle < AOB = AOC = 90▫ Draw the equal circles (O,OA) , (A , AO) and let points A_1 , A_2 be the points of intersection . Produce AA_1 to C on OB axis such that A_1C = A A_1 .
Since triangle AOA_1 has all sides equal to OA (AA_1 = AO = OA_1) then it is an equilateral triangle and angle < A_1AO = 60 ▫
Since Angle < CAO = 60 ▫ and AC = 2. OA then triangle ACO is right-angled and since angle < AOC = 90▫ , so the angle ACO = 30 ▫.
Complete rectangle AOCD , and angle < ADO = 180 – 90 – 60 = 30 ▫ = ACO = 90 ▫ / 3 = 30 ▫
From Pythagoras theorem $AC^2 = AO^2 + OC^2$ or $OC^2 = 4.OA^2 - OA^2 = 3.OA^2$ and

OC = OA . √ 3 .

For OA = OA_O then A_OE = 2. OA_O and **OE = OA_O . √ 3 .**

Since OC / OE = OA / OA_O → then line CA′ is parallel to EA_O .

b) .. F.9.(3 - 4)

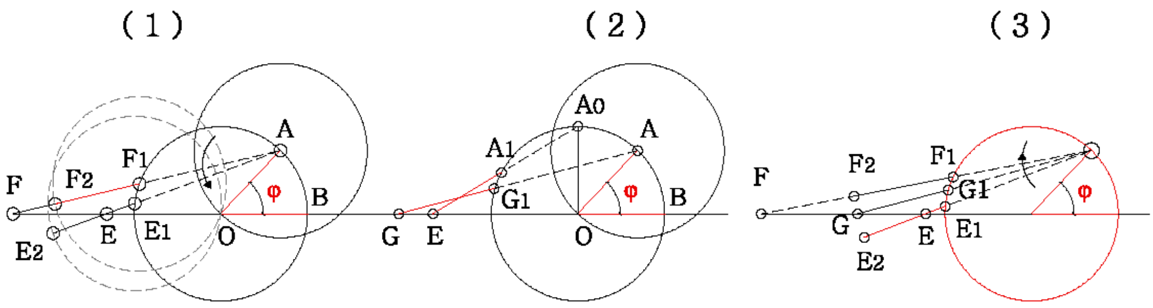
Triangle OAA_1 is isosceles , therefore angle < A_1 AO = 60 ▫ . Since A_1D_1 = A_1O , triangle D_1 A_1 O is isosceles and since angle < OA_1A = 60 ▫ , therefore angle < OD_1A = 30 ▫ or , Since A_1 A = A_1D_1 and angle < A_1 AO = 60 ▫ then triangle AOD_1 is also right-angle triangle and angles < D_1OA = 90 ▫ , < OD_1A = 30 ▫.
Since circle of diameter D_1 A passes through point O and also through the foot of the perpendicular from point D_1 to AD , and since also ODA = ODA′ = 30 ▫ , then this foot point coincides with point D , therefore the locus of point C is the perpendicular CD_1 on OC .
For A A_1 > A_1D_1 , then D_1 is on the perpendicular D_1 E on OC.

Since the Parallel from point A_1 to OA passes through the middle of OD_1 , *and in case where is* AOB = AOC = 90 ▫ *through the middle of* AD , then the circle with diameter D_1A passes through point D which is the base point of the perpendicular , **i.e.**

The geometrical locus of points C , or E , is CD and EE_1 , the perpendiculars on OB .
c) .. F.9.(3 - 4)

Since A_1A = A_1D_1 and angle < A_1AO = 60▫ then triangle AOD_1 is a right - angle triangle and ***angle < D_1OA = 90 ▫ . Since angle < A D_1O is always equal to 30 ▫ and angle < D_1 O A is always equal to 90 ▫ , therefore angle < AOB is created by the rotation of the right -angled triangle A-O-D_1 through vertex O .***

Since the tangent through A_O on to circle (O , OA′) lies on the circle of half radius OA , then this is perpendicular to OA and equal to A′A . (F.8)



F F1 > F1F2 = OA A1E = G1G = OA E E1 < E1E2 = OA

F.10. → ***The three cases of the Sliding segment OA = F_1F_2= E_1 E_2 between a line OB and a circle (O,OA) between the Maxima - Edge cases F_1F , E_1E or between F , E points .***

On AF, AE lines of F.10 exists :

$F F_1$ > OA GG_1 = OA , A_1E = OA_O $E E_1$ < OA
F_2 F_1 = OA A_1E = OA_O , EA_1= OA E_1 E_2 = OA

d) .. F.9-(4) - (F.10 - F.11)

Let point **G** be sliding on OB between points **E** and **F** where lines AE , AG , AF intersect circle (O, OA) at the points E_1, G_1 ,F_1 respectively where then exists $F F_1$ > OA , $G G_1$ = OA , $E E_1$ < OA . ***Points E , F are the limiting points of rotation*** of lines AE , AF (because then for angle < AOB = 90 ▫ → A_1 C = A_1 A = OA , A_1A_O = A_1E = OA_O and for angle < AOB = 0▫ → OF = 2 .OA) . Exists also E_1 E_2 = OA , F_2 F_1 = OA and point G1 common to circle (O , OA) and on line AG such that $G G_1$ = OA .
AE Oscillating to AF passes through AG so that $G G_1$ = OA and point G on sector EF.
When point G_1 of line AG is moving (rotated) ***on circle*** **(E_2 , E_2E_1 = OA**) and Point G_1 of ***G_1G is stretched on circle (O , OA)*** , then G_1G ≠ OA .

A position of point G_1 is such that , when G G_1 = OA point G lies on line EF.

When point G_1 of line AG is moving (rotated) ***on circle (*** **F_2 , F_2 F_1 = OA**) and point G_1 of ***G_1G is stretched on circle*** **(O, OA)** then length G_1G ≠ OA .

A position of point G_1 is such that , when G G_1= OA point G lies on line EF without stretching .

For both opposite motions there is only one position where point G lies on line OB and is not needed point G_1 of GA ***to be stretched*** on circle (O , OA) .

This position happens at the common point , P, of the two circles which is their point of intersection . At this point , P, exists only rotation and is not needed G_1 of GA to be stretched on circle (O , OA) so that point G to lie on line EF.
This means that point P lies on the circle (G , G G_1 = OA) , or GP = OA .
Point G_1 in angle < BOA is verged through two different and opposite motions , i.e.

1.. From point A′ to point A_O where *is done a parallel translation* of CA′ to the new position $E A_O$, *this is for all angles equal to 90* ▫ , and from this position to the new position EA by rotating $E A_O$ to the new position EA having always the distance $E_1 E_2$ = OA .
This motion is taking place on a circle of center E_1 and radius $E_1 E_2$.

2.. From point F , *where OF = 2. OA , is done a parallel translation of A′F to* FA_O , and from this position to the new position FA by rotating $F A_O$ to FA having always the distance $F_1 F_2$ = OA .

The two motions coexist , *limit* , again on a point **P** which is the point of intersection of the circles (E_2 , $E_2 E_1$ = OA) and (F_2, $F_2 F_1$ = OA) .

f) ..(F.9 .3 - 4) **-** (F.10 -3)

Remarks – Conclusions :

1.. Point E_1 is common of line AE and circle (O, OA) and point E_2 is on line AE such that E_1E_2 = OA and exists E E_1 < E_2E_1 **.** Length E_1E_2 = OA is stretched ,*moves* on EA so that point E_2 is on EF . Circle (E , E E_1 < E_2E_1 = OA) cuts circle (E_2 , E_2E_1 = OA) at point E_1 . There is a point G_1 on circle (O , OA) such that G_1 G = OA , *where point G is on EF* , ***and is not needed*** **G_1G** ***to be stretched*** on GA where then , circle (G , G G_1 = OA) cuts circle (E_2 , E_2E_1 = OA) at a point P .

2.. Point F_1 is common of line AF and circle (O,OA) and point F_2 is on line AF such that $F_1 F_2$ = OA and exists F F_1 > $F_2 F_1$ **.** Segment $F_1 F_2$ = OA is stretched , *moves* on FA so that point F_2 is on FE . Circle (F , F F_1 > $F_2 F_1$ = OA) cuts circle (F_2 , $F_2 F_1$ = OA) at point F_1 . There is a point G_1 on circle (O,OA) such that G_1G = OA ,*where point G is on FE* , ***and is not needed*** **G_1G** ***to be stretched*** on OB where then circle (G ,G G_1 = OA) cuts circle (F_2 , $F_2 F_1$ = OA) at a point P .

3.. ***When point G is at such position on*** **EF** ***that*** **G G_1 = OA ,** ***then point G must be at*** **A COMMON ,** ***to the three lines*** **E E_1 , G G_1 , F F_1 ,** ***and also to the three circles*** **(E_2 , $E_2 E_1$ = OA) , (G , G G_1 = OA) , (F_2 , $F_2 F_1$ = OA)**
This is possible at the common point **, P ,** ***of Intersection of circle*** **(** E_2 , $E_2 E_1$ = OA **)** ***and*** **(** F_2 , $F_2 F_1$ = OA**)** ***and since*** **G G_1** ***is equal to*** **OA** ***without*** **G G_1** ***be stretched on*** **GA ,** ***then also*** **GP = OA .**

4.. In additional , for point G_1 **:**

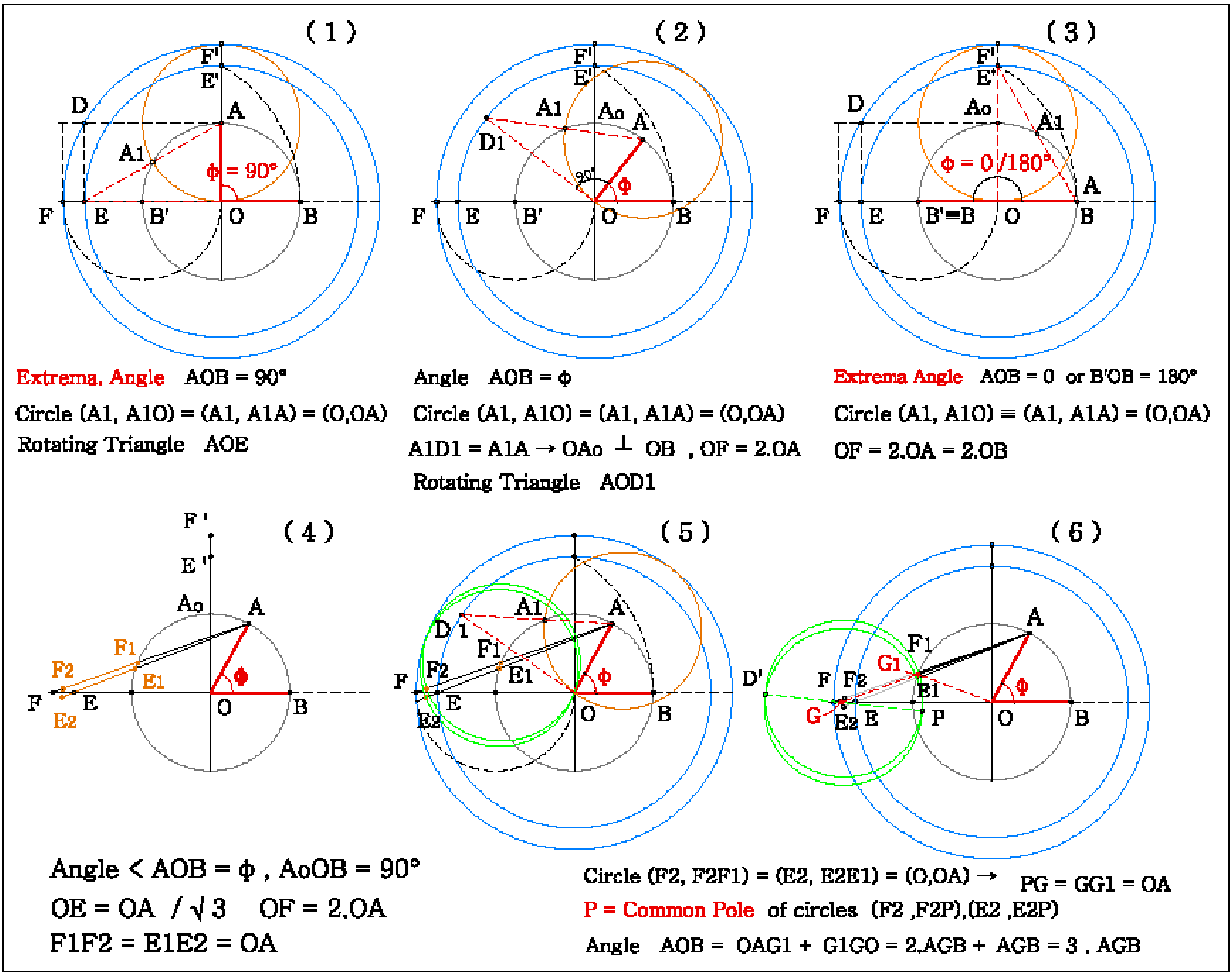
a.. Point G_1 , *from point* E_1 , moving on circle (E_2 , $E_2 E_1$ = OA) formulates Segment A E_1E such that E_1E = G_1 G < OA , for G moving on line GA .
There is a point on circle (E_2 , $E_2 E_1$ = OA) such that G G_1 = OA .

b.. Point G_1, *from point* F_1 , moving on circle (F_2 , $F_2 F_1$ = OA) formulates AF_1F such that F_1 F = G G_1 > OA , for G moving on line GA .
There is a point on circle (F_2 , $F_2 F_1$ = OA) such that G G_1 = OA .

c.. Since for both Opposite motions there is a point on the two circles that makes G G_1 = OA then point say P , is common to the two circles .

d.. Since for both motions at point P exists G G_1 = OA then circle (G , G G_1 = OA) passes through point P , and since point P is common to the three circles , then fixing point P as the common to the two circles (E_2 , $E_2 E_1$ = OA) , (F_2 , $F_2 F_1$ = OA) , then point G is found as the point of intersection of circle (P , PG = OA) and line EF . This means that the common point P of the three circles is constant to point P of the three circles and is constant to this motion.

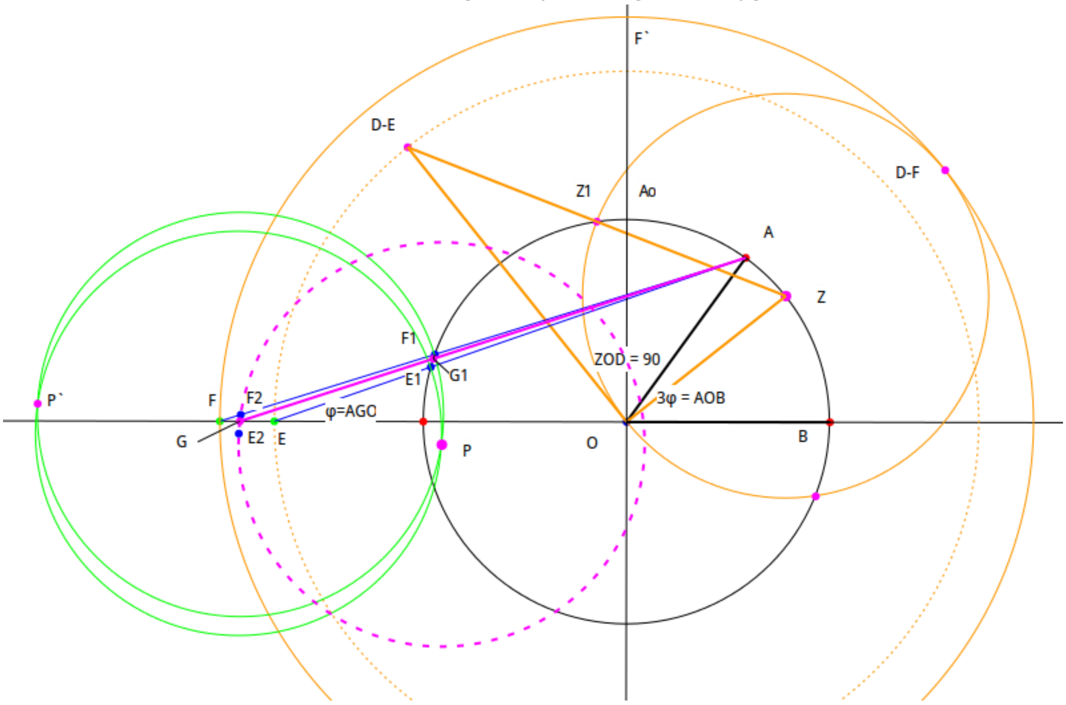
e.. Since , happens also the motion of a constant Segment on a line and a circle , then it is Extrema Method of the moving Segment as stated . The method may be used for part or Blocked figures either sliding or rotating . In our case , the Initial triangle forming 1/3 angle is formulating in all cases the common pole ,P, of the three circles .
From all above the geometrical trisection of any angle is as follows ,



F. 11 → ***The extrema Geometrical method of the Trisection of any angle < AOB***

In F.11- (1) Basic triangle AO D_1= OAE defines point E such that angle < AEO = 30° = AOB/3 .
In F.11- (2) Basic triangle AO D_1 defines D_1 point such that angle A D_1O = 30° = AOB/3 .
In F.11- (3) Basic triangle AO D_1 defines E` point such that angle AE`O = 30° , and it is the Extrema Case for angles AOB = 0° , B`OB = 180°

In F.11- (4) The two Edge cases (1),(3) issue for any angle AOB= φ° where F_1F_2= OA < F_1F , E_1E_2= OA< E_1E
In F.11- (5) The two circles with centers F_1 , E_1 correspond to Edge cases (1),(3) issuing for any angle AOB = φ°
In F.11- (6) The three circles [F_2, F_2F_1= OA] , [E_2, E_2E_1= OA] ,[G,GG_1= OA = GP] corresponding to Edge cases (1) , (3) define the common axis P P` of all movable poles and point , P , of this rotational system , such that GG_1= OA is stretched on (O,OA) circle and OB line , of any angle AOB = φ° .



F.11-A. → ***Presentation of the Trisection Method on Dr. Geo - Machine Macro –constructions .***

In F.11- A From Initial position of triangle AOB , *where angle* < AOB = 90▫ *and Segment* A_1C = OA , to the Final position of triangle , *where angle* < AOB = BOB = 0▫ *and* AOB = B`OB = 180 ▫, *through the Extrema position between edge - cases of triangle* ZOD *where* AOB = φ ▫ *and* GG_1 = GP = OA .

3.1. ***The steps of Trisection of any angle < AOB* = 90 ▫ → 0 ▫** F.11-[1-6]

1.. Draw circles (O , OA = OB) , (A , AO) , intersected at $A_1 \equiv Z_1$ point .
2.. Draw $OA_O \perp OB$ where point A_O is on the circle (O ,OA) and on a general circle (Z , D-E = 2. OA) . The circle (O , OD-E) intersects line OB at the Edge point E .
3.. Fix Edge point F on line OB such that → OF = 2. OA
4.. Draw lines AF , AE intersecting circle (O,OA) at points F_1 , E_1 respectively .
5.. On lines F_1 A , E_1 A fix points F_2 , E_2 such that $F_1 F_2$ = OA and $E_1 E_2$ = OA .
6.. Draw circles (F_2, $F_2 F_1$ = OA) , (E_2 , $E_2 E_1$ = OA) and fix point P as their common point of intersection .
7.. Draw circle (P , PG = OA) intersecting line OB at point G and draw line GA intersecting circle (O ,OA) at point G_1 , ***Then Segment*** **GG_1 = OA** , ***and angle*** **< AOB = 3. AGB** .
Proof :

1. Since point P is common to circles (F_2, $F_2 F_1$ = OA) , (E_2 , $E_2 E_1$ = OA) , then PG = PF_2 = PE_2 = OA and line AG between AE , AF intersects circle (O,OA) at the point G_1 such that GG_1 = OA . (F10.1 -2) - (F.11-5)
2. Since point G_1 is on the circle (O , OA) and since GG_1 = OA then triangle GG_1O is isosceles and angle < AGO = G_1OG .
3. The external angle of triangle Δ = GG_1O is < AG_1O = AGO + G_1OG = 2. AGO
4. The external angle of triangle GOA is angle < AOB = AGO + OAG = 3.AGO .
5.

Therefore angle < AGB = (1 / 3) . (AOB) (ο.ε.δ.)

A General Analysis :

Since angle < D_1OA is always equal to 90▫ then angle AOB is created by rotation of the right-angled triangle AOD_1 through vertex O . The circle (A , AO = A_1O) and triangle AO D_1 consists the geometrical Mechanism which creates the maxima at positions from , AOE , to A_OOE and to BOF` triangles , on (O , OE = √3.OA) , (O, OF = 2.OA) circles. F.11- (5)

In (1) Angle AOB = 90▫ , AE = 2.OA = OF , and point A_1 common to circles (O , OA) , (A, AO) define point E on OB line such that A_1E = OA . This happens for the extrema angle AOB = 90▫ .

In (2) Angle is , 0 < AOB < 90▫ , AD_1 = 2.OA and point A_1 common to circles (O,OA) , (A,AO) defines point D_1 on (O,OE =√3.OA) circle such that A_1D_1 = OA and on (O, OF = 2.OA) circle at any point D_f .

In (3) Angle < AOB = 0 or B`OB =180▫ , AE = 2.OA = BB` and point A_1 common to (O , OA) , (A, AO) circles define point E on OA_O line such that E ≡ E`, where then point D ≡ F` .

This happens for the extrema angle < AOB = 0 or 90▫ **.**

In (4-5) where angle is , 0 < AOB < 90▫ , and Segments $F_1F_2 = E_1E_2$ = OA the equal circles

(F_2, F_2F_1 = OA) , (E_2 , E_2E_1 = OA) define the common point P .

Since this geometrical formulation exists on Extrema edge angles , 0 *and* 90▫ ,then this point is constant to this formulation , and this point as center of a radius OA circle defines the extrema geometrical locus on it. All Poles are movable except the common Pole line PP` representing the Extrema case of this changeable system .

In (6) Since angle AOB is , 0 → 90▫ , and point P is constant , *and this because extrema circle* (P, PG = OA) *where* G *on* OB *line* , then is defining (G, GG_1) circle on GA segment such that point G_1 , *tobe the common point of segment* AG *and to circles* (O, OA) , (G,G G_1) .

The Physical notion of the Trisection :

This problem follows the two dimensional logic, *where* , ***the geometrical magnitudes and their unique circle*** *, have a linear relation* (continuous analogy) *in all Spaces as , in one in two in three dimensions , and as this happens to Compatible Coordinate Systems , happens also in Circle-arcs.*

The Compact-Logic-Space-Layer exists in Units , (***The case of 90*** ▫ ***angle***) ,where then we may find a new machine that produces the 1/3 of angles as in F.11. [11]

Since angles can be produced from any monad OB ,*and this because monad can formulate a circle of radius* OB *, and any point* A *on circle can then formulate angle* < AOB , therefore the logic of *continuous analogy of monads in all spaces* issues also and on OA radius equal to OB .

Application in Physics **:**

According to math theory of Elasticity , the total work on free edges where there is no shear becomes from Principal stresses only and work is $W = \frac{\sigma^2}{2E} + \frac{\tau^2}{2G}$ and the analogous Energy in monads is $W = \frac{1}{2}[\varepsilon E^2 + \mu H^2]$ and spread as the ***First Harmonic*** and equal to outer Spin $\overline{S}$ = E / w = 2πr.c .

Equation of Planck`s Energy E = h.f = (h/λ).c is equal to the Isochromatic pattern fringe-order in monad as→ σ1-σ2=(a/d).N=(a/d)nf1=(8πr²/3).n.f1. where n = the order of isochromatic , *a number* , f1= the frequency of Fundamental-Harmonic .

***Since total Energy in cave* (wr)² *is dependent on frequency only , and stored in the Fundamental and the first Six Harmonics , so the summations bands of these Seven Isochromatic Quantized interference fringe-order-patterns , is the total energy* E** in the same cave (wr)² as ,

$$E = Spin.w = \bar{S}.w = (h/2\pi).2\pi f = [\frac{8\pi r^2 f1}{3}].[\frac{n(n+1)}{2}] = [\frac{4\pi r^2 f1}{3}]\ n.(n+1) \quad ....(a)$$

When stress (σ1-σ2) go up then , **n** = ***order fringe defining Energy*** goes up also ,and the colors cycle through a more or less repeating pattern and the Intensity of the colors diminishes .
Since phase φ = k**x**-w**t** = *Spatial* and *Time Oscillation* dependence ,
For n = 1 , *Energy in the First Harmonic* is , E =2πr.c = $[\frac{4\pi r^2}{3}]$.f1.[**1**] , and
for n = 2 *Energy in the First and Second Isochromatic Harmonic* is , E = $[\frac{4\pi r^2}{3}]$.f1.[**3**] ***in threes , and φ is trisected*** with Energy-Bunched variation f2 , i.e.
Energy stored in a homogeneous ***resonance*** , is spread in the First of Seven-Harmonics beginning from the Fundamental and after the filling with frequency f1 , follows the Second-Harmonic .

In Second-Harmonic energy as frequency is doubled and this because of sufficient keeping homogeneously in Spatial dependence Quantity kx =(2π/λ).x which is in threes , meaning that , → ***Dipole – energy*** is Spatially-trisected in Space -Quantity Quanta the Spin = h/2π as the angle φ , of phase φ=kx-wt=(2π/λ).x , and Bisected by the Energy-Quantity Quanta as in an RLC circuit. [49] .

The Physical notion of the Regular Polygons :

According to Archimedes , *Geometric means* , speaking of numbers , *whether solid or square* , observes that , Between Plane One - mean suffices , but to connect two solids Two – means are necessary .
This denotes that between two square numbers there is one mean proportional number and between two cubes there are two means proportional numbers .

It was proved that Odd numbers become from any two consequent Even numbers , so the sum of two irrationals may be either rational or irrational .

The *Cattle – Problem* of Archimedes may be further analysed reaching to equations of any degree.
It was shown in pages 43 – 49 that , all n-Regular Polygons End to equations of n-degree Segment , by finding a suitable value of the Segment , x , That is we have in the general case to solve one or two equations of the form :

$$A . R^0 . x^n - B . R^2 . x^{n-2} + C . R^{n-6} . x^3 - D . R^{n-4} . x^2 + E . R^{n-2} . x^1 - F . R^n . x^0 = 0$$

for The Even Polygons , and

$$A .R^2 . x^{n-2} - B .R^{n-2} . x^{n-3} + C .R^{2(n-4)} . x^3 - D .R^{2(n-3)} . x^2 + E .R^{2(n-2)} . x^1 - F .R^{2(n-1)} . x^0 = 0$$

for The Odd Polygons , where A , B , C , D are constants .

The Presented Geometrical method is the solution of the above equation in the general case . Because , the n_{th} - degree - equations are the vertices of the n-polygon in circle so number , π , is their mould .
In Mechanics , by Scanning any Chord $K K_1$ to chord $K K_2$ of the circle ,then the *Work* (Energy as → Kinetic or Dynamic) *produced from any Removal , is Stored* . in the Inverted triangles OO_kK_2 ,$K_2P_kP_a$ as in page 60 .

4. The Parallel Postulate, *is not an Axiom* , is a Theorem.

The Parallel Postulate. **F.13**

General : Axiom or Postulate is a statement checked if it is true and is ascertained with logic (the experiences of nature as objective reality).

Theorem or Proposition is a non-main statement requiring a proof based on earlier determined logical properties.
Definition is an initial notion without any sensible definition given to other notions.
Definitions, Propositions or Postulates created Euclid geometry using the geometrical logic which is that of nature, the logic of the objective reality.
Using the same elements it is possible to create many other geometries but the true uniting element is the before refereed.

4.1. The First Definitions (**Dn**) = (D) , of Terms in Geometry but the true uniting ,

D1: A point is that which has no part (Position).
D2: A line is a breathless length (for straight line, the whole is equal to the parts) .
D3: The extremities of lines are points (equation).
D4: A straight line lies equally with respect to the points on itself (identity).
D : A midpoint C divides a segment AB (of a straight line) in two. CA = CB any point C divides all straight lines through this in two.
D : A straight line AB divides all planes through this in two.
D : A plane ABC divides all spaces through this in two .

4.2. Common Notions (**Cn**) = (CN)

Cn1: Things which equal the same thing also equal one another.
Cn2: If equals are added to equals, then the wholes are equal.
Cn3: If equals are subtracted from equals, then the remainders are equal.
Cn4: Things which coincide with one another, equal one another.

Cn5: The whole is greater than the part.

4.3. The Five Postulates (**Pn) =** (P) for Construction

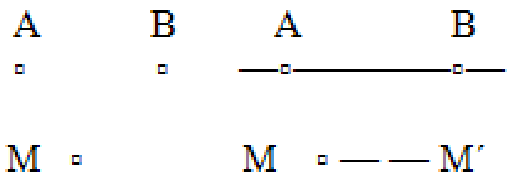
P1.. To draw a straight line from any point A to any other point B .
P2.. To produce a finite straight line AB continuously in a straight line.
P3.. To describe a circle with any center and distance. P1, P2 are unique.
P4.. That, all right angles are equal to each other.
P5.. That , if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles, or (for three points on a plane) . Three points consist a Plane .
P5a. The same is plane's postulate which states that, from any point M, not on a straight line AB, only one line MM´ can be drawn parallel to AB.

Since a straight line passes through two points only and because point M is the third , then the parallel postulate it is valid on a plane (three points only).
AB is a straight line through points A, B , AB is also the measurable line segment of line AB , and M any other point . When MA+MB > AB , then point M is not on line AB . (differently if MA+MB = AB , then this answers the question of why any line contains at least two points) ,

i.e. for any point M on line AB where is holding

MA+MB = AB , *meaning that line segments* MA,MB *coincide on* AB , is thus proved from the other axioms and so D2 is not an axiom . →

To prove that , one and only one line MM´ can be drawn parallel to AB.

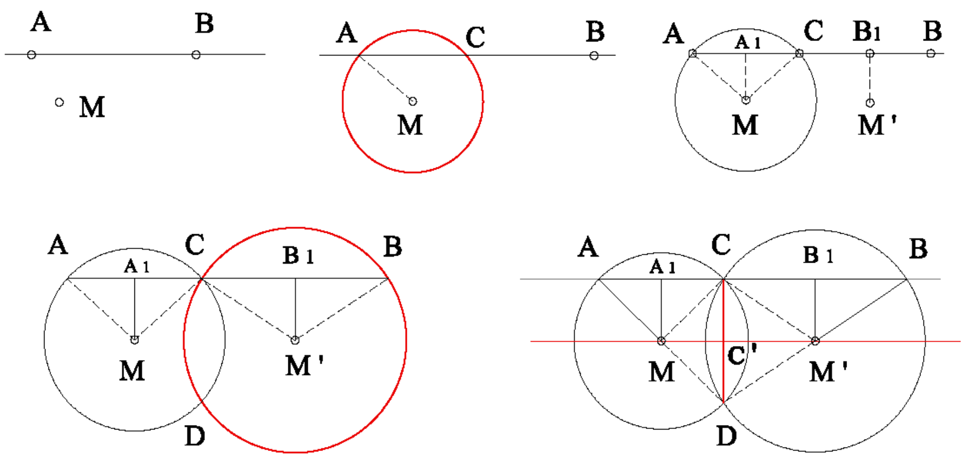


F.12. → *In three points (in a Plane).*

4.4. ***The Process in order to prove the above Axiom is necessary to show*** **: F.13 ,**

a..The parallel to AB is the locus of all points at a constant distance **h** from the line AB , and for point M is MA_1,

b..The locus of all these points is a straight line.



F.13. → *The Parallel Method*

Step 1

Draw the circle (M, MA) be joined meeting line AB in C. Since MA = MC, point M is on mid - perpendicular of AC. Let A_1 be the midpoint of AC, (it is A_1A+A_1C = AC because A_1 is on the straight line AC). Triangles MAA_1, MCA_1 are equal because the three sides are equal, therefore angle < $MA_1A = MA_1C$ (CN1) and since the sum of the two angles < MA_1A+MA_1C = 180° (CN2, 6D) then angle < $MA_1A = MA_1C$ = 90 °.(P4) so, MA_1 is the minimum fixed distance **h** of point M to AC.

Step 2

Let B_1 be the midpoint of CB , (it is $B_1C+ B_1B$ = CB because B_1 is on the straight line CB) and Draw B_1M' = h equal to A_1M on the mid-perpendicular from point B_1 to CB . Draw the circle (M′, M′B = M′C) intersecting the circle (M , MA = MC) at point D .(P3)

Since M′C = M′B, point M′ lies on mid- perpendicular of CB. (CN1)

Since M′C = M′D, point M′ lies on mid-perpendicular of CD. (CN1) Since MC = MD, point M lies on mid-perpendicular of CD . (CN1) Because points M and M′ lie on the same mid-perpendicular (This mid - perpendicular is drawn from point C′ to CD and it is the midpoint of CD) and because only one line MM′ passes through points M , M ′then line MM′ coincides with this mid-perpendicular (CN4) .

Step 3

Draw the perpendicular of CD at point C′. (P3, P1)

a..Because $MA_1 \perp AC$ and also $MC' \perp CD$ then angle < $A_1MC' = A_1CC'$. (Cn 2,Cn3,E.I.15)
Because $M'B_1 \perp CB$ and also $M'C' \perp CD$ then angle < $B_1M'C' = B_1CC'$. (Cn2, Cn3, E.I.15)

b..The sum of angles $A_1CC' + B_1CC' = 180° = A_1MC' + B_1M'C'$. (6.D), and since Point C′ lies on straight line MM′, therefore the sum of angles in shape $A_1B_1M'M$ are < $MA_1B_1 + A_1B_1M' +$ $[B_1M'M + M'MA_1] = 90° + 90° + 180° = 360°$ (Cn2) , i.e. The sum of angles in a Quadrilateral is 360° and in Rectangle all equal to 90 °. (m)

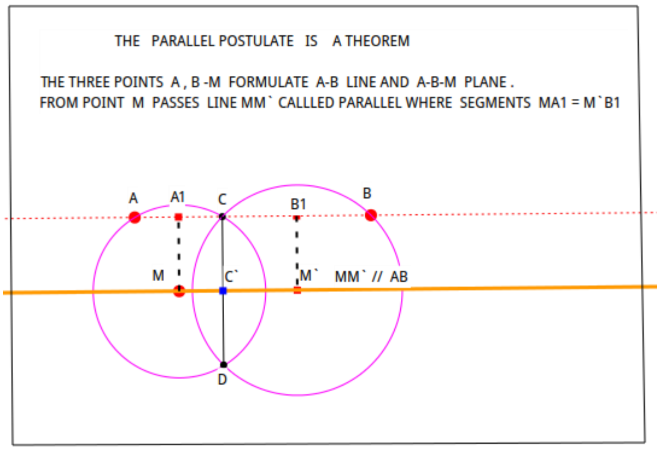
c..The right-angled triangles MA_1B_1, $M'B_1A_1$ are equal because $A_1M = B_1M'$ and A_1B_1 common, therefore side $A_1M' = B_1M$ (Cn1). Triangles A_1MM',$B_1M'M$ are equal because have the three sides equal each other, therefore angle < $A_1MM' = B_1M'M$, and since their sum is 180° as before (6D), so angle < $A_1MM' = B_1M'M = 90°$ (Cn2).

d.. Since angle < $A_1MM' = A_1CC'$ and also angle < $B_1M'M = B_1CC'$ (P4), therefore the three quadrilaterals $A_1CC'M$, $B_1CC'M'$, $A_1B_1M'M$ are Rectangles (CN3).
From the above three rectangles and because all points (M , M′ and C′) equidistant from AB, this means that C′C is also the minimum equal distance of point C′ to line AB or , $h = MA_1 = M'B_1 = CD / 2 = C'C$ (Cn1) Namely , line MM′ is perpendicular to segment CD at point C′ and this line coincides with the mid-perpendicular of CD at this point C′ and points M , M′ ,C′ are on line MM′. Point C′ equidistant ,h, from line AB , as it is for points M ,M′, so the locus of the three points is the straight line MM′, so the two demands are satisfied , ($h = C'C = MA_1 = M'B_1$ and also $C'C \perp AB$, $MA_1 \perp AB$, $M'B_1 \perp AB$) . (ο.ε.δ.) –(q.e.d)

e.. The right-angle triangles A_1CM , MCC' are equal because side $MA_1 = C'C$ and MC common so angle < $A_1CM = C'MC$, and the Sum of angles $C'MC + MCB_1 = A_1CM + MCB_1 = 180°$

F.13-A. → *Presentation of the Parallel Method on Dr. Geo - Machine Macro – Constructions .*

a.. The three Points A , B , M consist a Plane and so this Proved Theorem exist only in plane .
b.. Points A , B consist a Line and this because exists postulate P1 .
c.. Point M is not on A B line and this because when segment MA+MB > AB then point M is not on line AB according to *Markos* definition .
d.. When Point M is on AB line , and this because segment MA+ MB = AB then point M being on line AB is an Extrema case , and then formulates infinite Parallel lines coinciding with AB line in the Infinite (∞) Planes . All for the extrema Geometry cases in [44-46].



4.5 *The Succession of Proofs :*

1.. Draw the circle (M , MA) be joined meeting line AB in C and let A_1, B_1 be the midpoint of CA, CB.

2.. On mid-perpendicular B1M′ find point M′ such that M′B1 = MA_1, and draw the circle (M′, M′B = M′C) intersecting the circle (M , MA = MC) at point D.

3.. Draw mid-perpendicular of CD at point C′.

4..To show that line MM′ is a straight line passing through point C ′and it is such that $MA_1 = M'B_1 = C'C = h$, i.e. a constant distance , h , from line AB or , also The Sum of angles $C'MC + MCB_1 = A_1CM + MCB_1 = 180$ ▫

Proofed Succession

1.. The mid-perpendicular of CD passes through points M , M ′.

2.. Angle < $A_1MC' = A_1MM' = A_1CC'$, Angle < $B_1M'C' = B_1M'M = B_1CC'$ < $A_1MC' = A_1CC'$ because their sides are perpendicular among them i.e. $MA_1 \perp CA$, $MC' \perp CC'$.

a.. In case < $A_1MM' + A_1CC' = 180$▫ and $B_1M'M + B_1CC' = 180$▫ then < $A_1MM' = 180$▫ - A_1CC', $B_1M'M = 180$▫ - B_1CC', and by summation < $A_1MM' + B_1M'M = 360$▫ - A_1CC'- B_1CC' or Sum of angles < $A_1MM' + B_1M'M = 360 - (A_1CC' + B_1CC') = 360 -180$▫ = 180▫

3.. The sum of angles $A_1MM' + B_1M'M = 180$▫ because the equal sum of angles $A_1CC' + B_1CC' = 180$▫, so the sum of angles in quadrilateral MA_1B_1M' is equal to 360▫.

4.. The right-angled triangles MA_1B_1 , $M'B_1A_1$ are equal , so diagonal $MB_1 = M'A_1$ and since triangles A_1MM', $B_1M'M$ are equal, then angle $A_1MM' = B_1M'M$ and since their sum is 180 ▫, therefore angle < $A_1MM' = MM'B_1 = M'B_1A_1 = B_1A_1M = 90$ ▫

5.. Since angle $A_1CC' = B_1CC' = 90$▫, then quadrilaterals $A_1CC'M$, $B_1CC'M'$ are rectangles and for the three rectangles MA_1CC', $CB_1M'C'$, MA_1B_1M' exists $MA_1 = M'B_1 = C'C$

6.. The right-angled triangles MCA_1, MCC' are equal , so angle < $A_1CM = C'MC$ and since the sum of angles < $A_1CM + MCB_1 = 180$ ▫ then also $C'MC + MCB_1 = 180$ ▫ →

which is the second to show , as this problem has been set at first by Euclid.

All above is a Proof of the Parallel postulate due to the fact that the parallel postulate is dependent of the other four axioms (***now is proved as a theorem from the other four***).

Since line segment AB is common to ∞ Planes and only one Plane is passing through point M (Plane ABM from the three points A, B, M, then the Parallel Postulate is valid for all Spaces which have this common Plane , as Spherical, n-dimensional geometry Spaces. It was proved that it is a necessary logical consequence of the others axioms, agree also with the Properties of physical objects, d + 0 = d , d * 0 = 0 , now is possible to decide through mathematical reasoning , that the geometry of the physical universe is Euclidean . Since the essential difference between Euclidean geometry and the two non-Euclidean geometries , Spherical and hyperbolic geometry , is the nature of parallel line, i.e. the parallel postulate so ,

<< ***The consistent System of the – Non - Euclidean geometry - have to decide the direction of the existing mathematical logic*** >>.

The above consistency proof is applicable to any line Segment AB on line AB,(segment AB is the first dimensional unit, as AB = 0 → ∞), from any point M not on line AB, [MA + MB > AB for three points only which consist the Plane. For any point M between points A, B is holding MA+MB = AB i.e. from two points M , A or M , B passes the only one line AB. A line is also continuous (P1) with points and discontinuous with segment AB [14] ,which is the metric defined by non- Euclidean geometries ,and it is the answer to the cry about the < crisis in the foundations of Euclid geometry >

A Line Contains at Least Two Points , is Not an Axiom Because is Proved as Theorem

Definition D2 states that for any point M on line AB is holding MA+MB = AB which is equal to

< segment MA + segment MB is equal to segment AB > i.e. the two lines MA , MB coincide on line AB and thus this postulate is proved also from the other axioms, thus D2 is not an axiom, which form a system self consistent with its intrinsic real-world meaning. F.12-13.

4.6. Conclusions.

Parallel line.

A line (*two points only*) is not a great circle (*more than three points being in circle`s Plane*) so anything built on this logic is a mislead false .

The fact that the sum of angles on any triangle is 180▫ is springing for the first time, in article

(Rational Figured numbers or Figures) [9] .

This admission of two or more than two parallel lines, instead of one of Euclid's, does not proof the truth of the admission. The same to Euclid's also, until the present proved method . Euclidean geometry does not distinguish , Space from time because time exists only in its deviation - Plank's length level - ,neither Space from Energy - because Energy exists as quanta on any first dimensional Unit AB , which as above connects the only two fundamental elements of Universe , that of points or Sector = Segment = Monad = Quaternion , and that of Energy. [23]-[39].

The proposed Method in articles , based on the prior four axioms only , proofs , (not using any other admission but a pure geometric logic under the restrictions imposed to seek the solution) that , through point M on any Plane ABM (three points only that are not coinciding and which consist the Plane) , passes only one line of which all points equidistant from AB as point M ,

i.e. the right is to Euclid Geometry.

The what is needed for conceiving the alterations from Points which are nothing , to segments , i.e. quantization of points as , *the discreteting = monads = quaternion* , to lines , plane and volume , is the acquiring and having Extrema knowledge .

In Euclidean geometry the inner transformations exist as ***pure*** Points , segments , lines , Planes , Volumes, etc. as the Absolute geometry is (*The Continuity of Points*) , automatically transformed through the three basic Moulds *(the three Master moulds and Linear transformations exist as one Quantization*) to Relative external transformations , which exist as the , ***material*** , Physical world of matter and energy *(Discrete of Monads)* . [43-44]

The new Perception connecting the Relativistic Time and Einstein`s Energy - is Now Refining Time and Dark –matter Force - clearly proves That Big -Bang have Never been existed .

In [17-45-46] is shown the most important ***Extrema Geometrical Mechanism in this Cosmos*** which is that of STPL lines , that produces and composite , All the opposite space Points from Spaces to Anti-Spaces and to Sub-Spaces as this is in a Common Circle , *this is the Sub-Space* , to lines into a Cylinder .

This extrema mould is a Transformation , i.e. a Geometrical Quantization Mechanism , → for the Quantization of Euclidean geometry, *points* , to the Physical world ***, to Physics ,*** and is based on the following geometrical logic ,

Since Primary point ,A, is nothing and without direction and it is the only Space , and this point to exist , *to b*e , at any other point ,B, which is not coinciding with point ,A, then on this couple exists the Principle of Virtual Displacements $W = \int_A^B P.\,ds = 0$ or $[ds.(P_A+P_B) = 0]$, i.e. for any $ds > 0$ Impulse $P = (P_A+P_B) = 0$ and Work $[\,ds\,.\,(P_A+P_B) = 0\,]$, *Therefore* , Each Unit $AB = ds > 0$, exists by this Inner Impulse (P) where $P_A + P_B = 0$.

The Position and Dimension of all Points which are connected across the Universe and that of Spaces , exists , because of this equilibrium Static Inner Impulse and thus show the Energy-Space continuum . Applying the above logic on any monad = *quaternion* $(s + \bar{v}.\nabla i)$, *where* , s = the real part and $(\bar{v}.\nabla i)$ the imaginary part of quaternion so ,

Thrust of two equal and opposite quaternion is the , Action of these quaternions which is ,

$(s +\bar{v}.\nabla i) . (s + \bar{v}.\nabla i) = [s+\bar{v}.\nabla i]^2 = s^2+|\bar{v}|^2.\nabla i^2 + 2|s|x|\bar{v}|.\nabla i = s^2 - |\bar{v}|^2 +2|s|x|\bar{w}.\bar{r}|.\nabla i =$
$[s^2] – [|\bar{v}|^2] + [2\bar{w}.|s| |\bar{r}|.\nabla i]$ where,

$[+s^2] \rightarrow s^2 = (w.r)^2$, → is the real part

of the new quaternion which is , the positive Scalar product , of Space from the same scalar product ,s,s with ½ ,3/2,,, spin and this because of ,w, and which represents the massive , Space , part of quaternion → monad .

$[-s^2] \rightarrow - |\bar{v}|^2 = - |\bar{w}.\bar{r}|^2 = - [|\bar{w}|.|\bar{r}|]^2 = - (w.r)^2$ → is the always , the negative Scalar product , of Anti-space from the dot product of ,$\bar{w},\bar{r}$ vectors , with -½ ,-3/2, spin and this because of , - w , and which represents the massive , Anti-Space , part of quaternion → monad .

$[\nabla i] \rightarrow 2.|s|$ x $| \bar{w}.\bar{r} |.\nabla i =2|wr|.|(wr)|.\nabla i = 2.(w.r)^2$ → is a vector of , the velocity vector product , from the cross product of $\bar{w}$,$\bar{r}$ vectors with double angular velocity term giving 1,3,5, spin and this because of , ± w, in inner structure of monads , and represents the , Energy Quanta , of the Unification of the Space and Anti-Space through the Energy (*Work*) part of quaternion .

A wider analysis is given in articles [40-43] .

When a point ,A, is quantized to point ,B, then becomes the line segment AB = vector AB = quaternion [AB] → monad , and is the closed system ,A B, ***and since*** also from the law of conservation of energy , *it is the first law of thermodynamics* , which states that the energy of a closed system remains constant , therefore *neither increases nor decreases without interference from outside* , and so the total amount of energy in this closed system , AB , in existence has always been the same , ***Then*** the Forms that this energy takes are constantly changing , i.e.

The conservation of energy is realized when stored in monads and following the physical laws in E-geometry where then are Material → Points , monads , etc ← ***This is the unification of this Physical world of , what is callled matter and Energy , and that of Euclidean Geometry which are , Points , Segments , Planes and Volumes .*** For more in [48] .

The three Moulds (i.e. The three Geometrical Mechanism) of Euclidean Geometry which create the METERS of monads and which are , *Linear* for a perpendicular Segment , *Plane* for the Square equal to the circle on Segment , *Space* for the Double Volume of initial volume of the Segment , (*the volume of the sphere is related to Plane which is related to line and which is related to segment*) , ***Exist on Segments*** in Spaces , Anti-spaces and Sub-spaces .

This is the Euclidean Geometry Quantization to its constituents (i.e. Geometry in its moulds) .
The analogous happens when E-Geometry is Quantized to Space and Energy monads [48].

METER of Points A is the Point A , the
METER of line is the Segment ds = AB = monad = constant and equal to monad , or to the perpendicular distance of this segment to the set of two parallel lines between points A,B , the
METER of Plane is that of circle on Segment = monad and which is that Square equal to the circle , number ,*π*, the
METER of Volume ,$\sqrt[3]{2}$, is that of Cube , on Segment = monad which is equal to the Double Cube of the Segment and Measures all the Spaces , the Anti-spaces and the Subspaces in this cosmos .

Generally is more referred ,

a). There is ***not any Paradoxes of the infinite*** because is clearly defined what is a Point a cave and what is a Segment .

b). ***The Algebra of constructible numbers and number Fields is an Absurd theory*** , based on groundless Axioms as the fields are , and with direction the non-Euclid orientations purposes which must be properly revised .

c). ***The Algebra of Transcental numbers has been devised to postpone the Pure geometrical thought*** , which is the base of all sciences , by changing the base-field of solutions to Algebra as base .

Pythagorians discovered the existence of the incommensurable of the diagonal of a square in relation to its side without giving up the base , which is the geometrical logic.

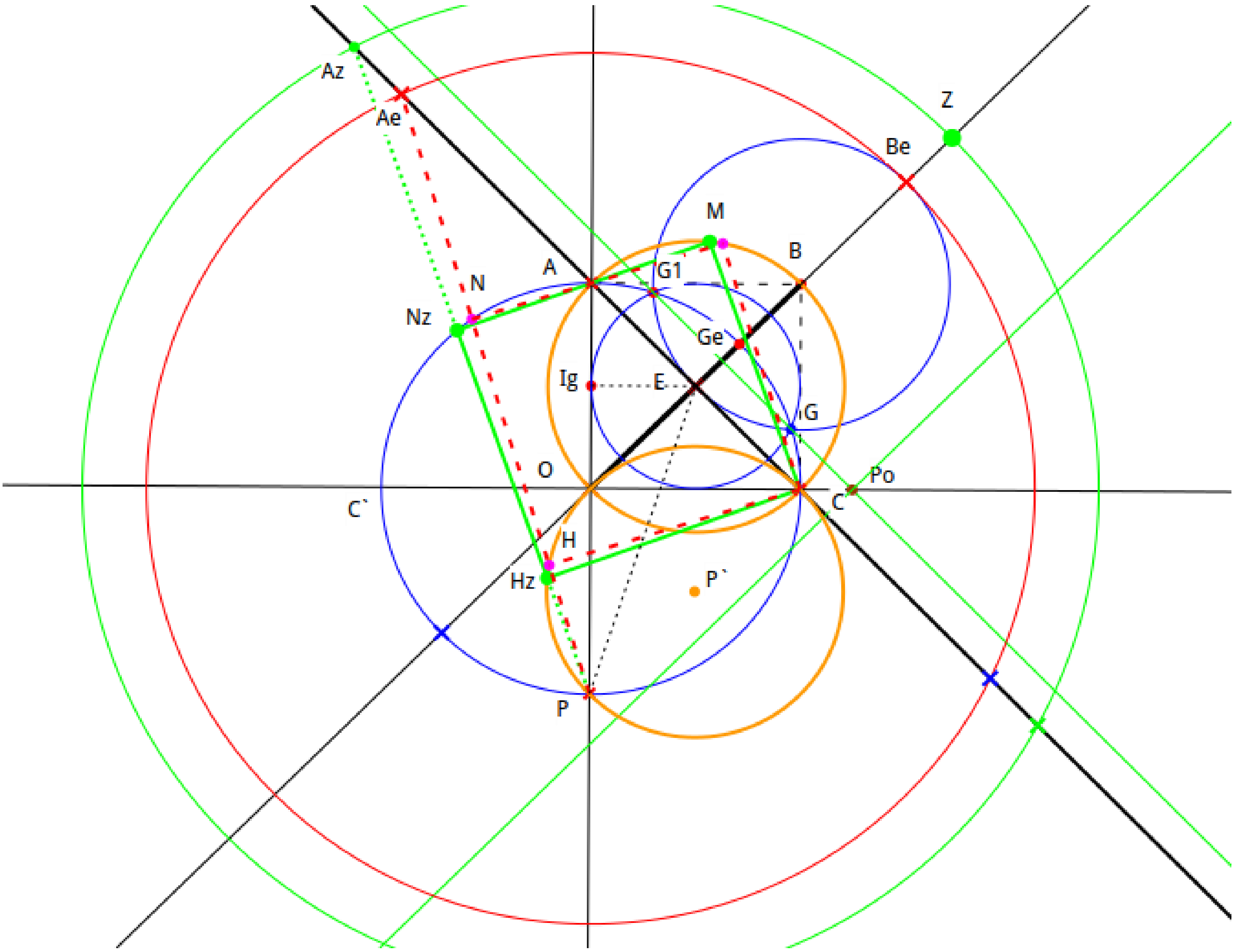
d). All theories concerning ***the Unsolvability of the Special Greek problems are based on Cantor`s shady proof*** , < *that the totality of All algebraic numbers is denumerable* > and not edifyed on the geometrical basic logic which is the foundations of all Algebra .

The problem of Doubling the cube F.4-A , as that of the Trisection of any angle F.11-A, is a Mechanical problem and could not be seen differently and the proposed Geometrical solutions is clearly exposed to the critic of all readers .

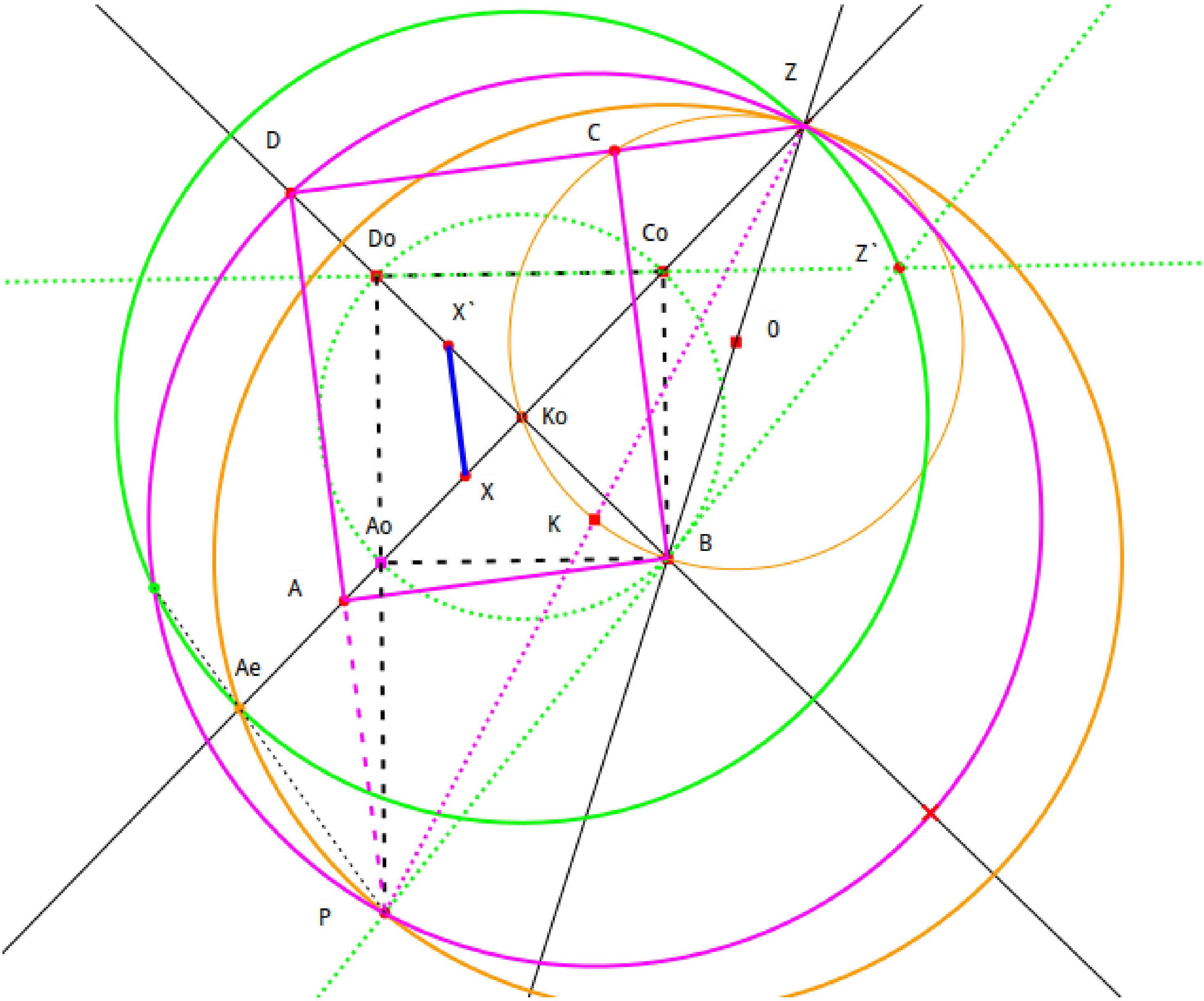
All trials for Squaring the circle are shown in [44] and the set questions will be answerd on the Changeable System of the two Expanding squares ,*Translation* [T] *and Rotation* [R] .

The solution of Squaring the circle using the Plane Procedure method is now presented in F-1,2 , and consists an , *Overthrow* , to all existing theories in Geometry , Physics and Philosophy .

e). Geometry is the base of all sciences and it is the reflective logic from the objective reality , which is nature , to our mind .



F.2-A → ***A Presentation of the Quadrature Method on Dr. Geo-Machine Macro - constructions .***
The Inscribed Square CBAO , *with Pole-line* AOP , *rotates through Pole* P , ***to the*** → *Circle-Square* CMNH *with Pole-line* NHP , ***and to the*** → *Circumscribed Square* CAC`P , *with Pole-line* C`PP = C`P , *of the circle* E , EO = EC *and at position* Be , A_eNHP Pole-line formulates square CMNH = π . EO ^{2} which is the Squaring of the circle . Number $\pi = \frac{CM^2}{EO^2}$ as in [Fig.2-A]



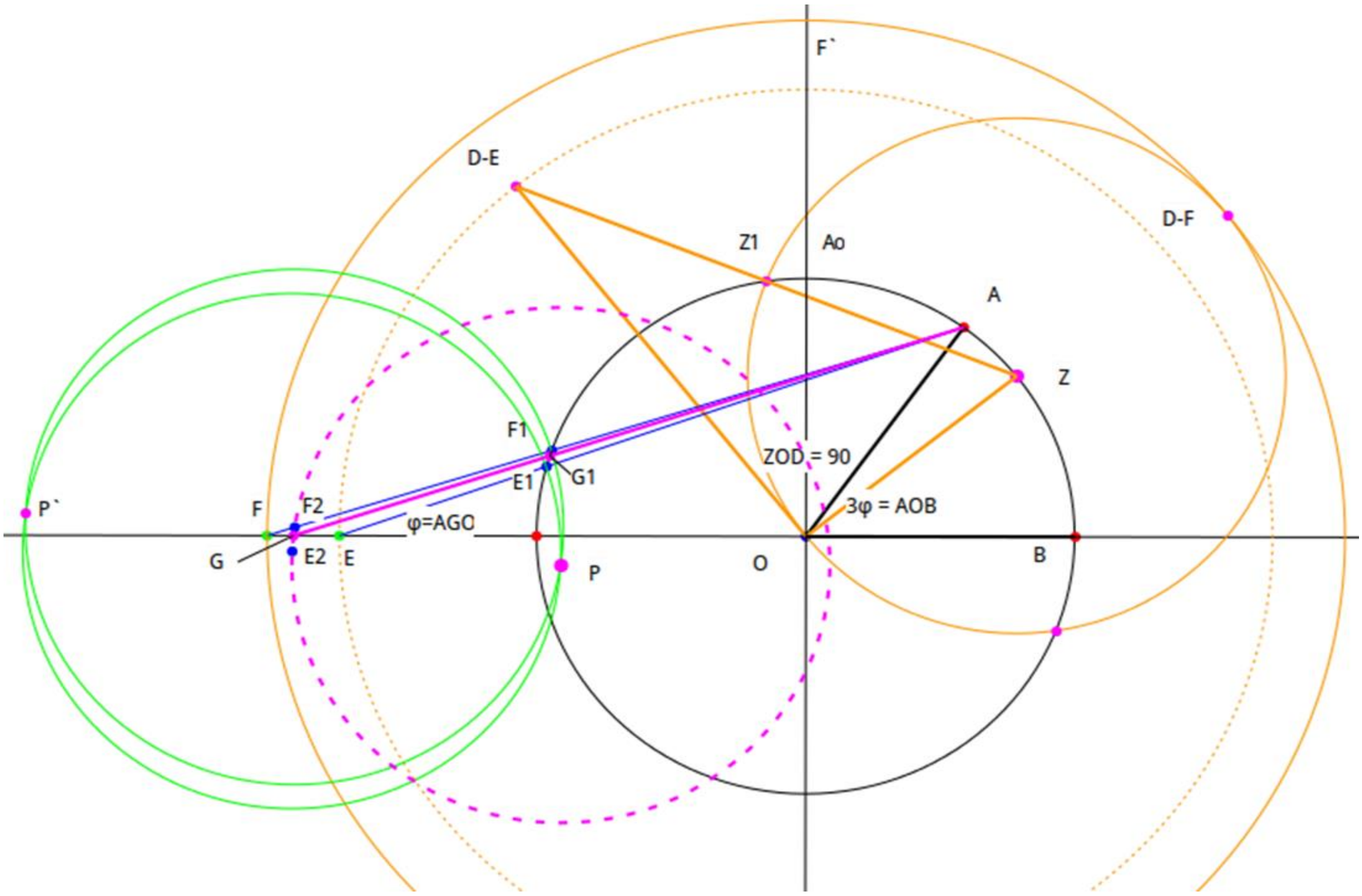
F.4-A. → ***A Presentation of the Dublication Method on Dr.Geo - Machine Macro - constructions***

BCDA Is the In-between Quadrilateral , on (K,KZ) Extrema-circle , and on K_OZ-K_OB Extrema –

lines of common poles Z , P , mechanism . *The Initial Quadrilateral* $BC_0D_0A_0$, *with Pole- lines* D_0A_0P , D_0C_0 Z ` , ***rotates*** *through Pole* P *and the moveable Pole* Z ` *on* Z `Z *arc* , ***to the*** → *Extreme Quadrilateral* BCDA *through Pole-lines* DAP - DCZ *with point* Do , *sliding on* B K_0D_0 *Pole-line*, and then at point D , $KD^3 = 2.KoA^3$ which is the Dublication of the Cube .

For any initial segment K_0X issues $(K_0X`)^3 = 2 . (K_0X)^3$ where $K_0X` = K_0D.(\frac{KoX}{KoA})$ and

$\sqrt[3]{2} = (\frac{KoD}{KoA}) . (\frac{KoX}{KoX`}) = [\frac{KoD}{KoA}]^2 = \frac{KoD^2}{KoA^2}$ → as in [Fig4-A] , and since $(\frac{KoD}{KoA}) = (\frac{KoX}{KoX`})$



F.11-A. → ***Presentation of the Trisection Method on Dr. Geo - Machine Macro – constructions .***

From Initial position of triangle AOC , *where angle* AOB = 90▫ *and Segment* A_1C = OA , to the Final position of triangle , *where angle* AOB = BOB = 0▫ *and* AOB = B`OB = 180 ▫ , *through the Extrema position between edge- cases of triangle* ZOD *where* AOB = φ ▫ *and at common point* P , PG = OA = GP = G $G_1 = G_1O$ and at point G , then $G_1G = G_1O$ = OA which is the Trisection of angle AOB , and Angle < AGB = ($\frac{1}{3}$). AOB .

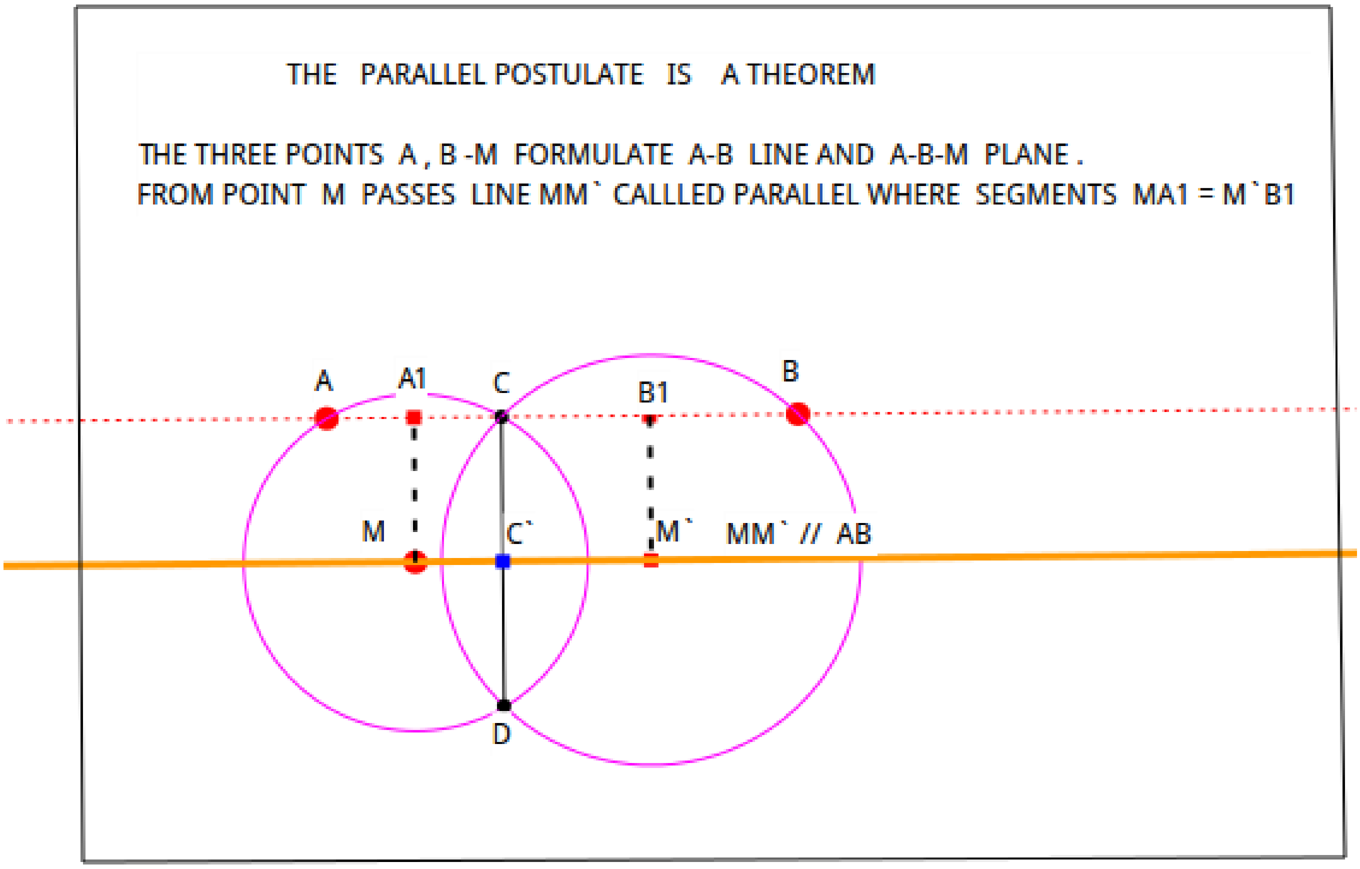
The Presentation of the Parallel Method .

a.. The three Points A , B , M consist a Plane and so this Theorem exist only in plane .

b.. Points A , B consist a Line and this because exists postulate [P1] .

c.. Point M is not on A B line and this because when segment MA+MB > AB then point M is not on line AB and $MA_1 = M`B_1$.

d.. When Point M is on A B line , and this because segment MA+ MB = AB then point M being on line AB is an Extrema case , and then formulates infinite Parallel lines coinciding with AB line in the Infinite (∞) Planes through AB.



F.13-A. → ***Presentation of the Parallel Method on Dr. Geo - Machine Macro – Constructions***

5.. THE REGULAR POLYGONS :

5.1. *THE ALGEBRAIC SOLUTION* :

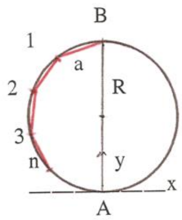
It has been proved by De Moivre's , that the n-th roots on the unit circle AB are represented by the vertices of the Regular n-sided Polygon inscribed in the circle .

It has been proved that the Resemblance Ratio of Areas , of the circumscribed to the inscribed squares (Regular quadrilateral) which is equal to 2 , leads to the squaring of the circle.
It has been also proved that , Projecting the vertices of the Regular n-Polygon on any tangent of the circle , then the Sum of the heights y_n is equal to $n * R$.
This is a linear relation between Heights , h , and the radius of the circle , *the monad* .
This property on the circle yields to the Geometrical construction (As Resemblance Ratio of Areas is now controlled) , and the Algebraic measuring of the Regular Polygons as follows :

when : R = The radius of the circle , with a random diameter AB .
a = The side of the Regular **n** -Polygon inscribed in the circle
n = Number of sides , a , of the n -Polygon , then exists :

$$\mathbf{n . R = 2 . R + 2 . y_1 + 2 . y_2 + 2 . y_3 + \dots\dots\dots 2 . y_n} \qquad \dots\dots\dots\dots \mathbf{(n)}$$

the heights y_n are as follows :



$y_B = [2 . R]$
$y_1 = [4.R^2 - a^2] / (2 . R)$

$y_2 = [4.R^4 - 4. R^2. a^2 + a^4] / (2. R^3)$

$$y_3 = \frac{[8. R^6 - 10. R^4. a^2 + 6 . R^2. a^4 - a^6] - a^2 .\sqrt{64. R^8 - 96. R^6.a^2 + 52. R^4. a^4 - 12.R^2. a^6 + a^8}}{2. R^5}$$

$y_n = [\dots\dots\dots\dots\dots] / 2. R^n$

THE ALGEBRAIC EQUATIONS OF THE REGULAR n - POLYGONS

(a) REGULAR TRIANGLE ۞ :

The Equation of the vertices of the Regular Triangle is :

$3.R = 2.R + [\frac{4.R^2 - a^2}{R}]$ >>> $R^2 = 4 . R^2 - a^2$ >>> $a^2 = 3. R^2$

The side $\mathbf{a_3 = R . \sqrt{3}}$ ……….(1).

(b) REGULAR QUADRILATERAL ۞ (SQUARE) :

The Equation of the vertices of the Regular Square gives :

$4.R = 2.R + [\frac{4.R^2 - a^2}{R}]$ >>> $a^2 = 2 . R^2$

The side $\mathbf{a_4 = R . \sqrt{2}}$ ……….(2)

(c) REGULAR PENTAGON ۞ :

The Equation of the vertices of the Regular Pentagon is :

$5.R = 2.R + [\frac{4.R^2 - a^2}{R}] + [\frac{4. R^4 - 4.R^2. a^2 + a^4}{R^3}]$ >>> $a^4 - 5. R^2. a^2 + 5. R^4 = 0$

Solving the equation gives :

$R^2 = \frac{5. R^2 - \sqrt{25. R^4 - 20. R^4}}{2} = \frac{5.R^2 - R^2 . \sqrt{5}}{2} = [\{ 5.R^2 - R^2.\sqrt{5} \} / 2] = \frac{R^2}{2} (5 - \sqrt{5})$

$a^2 = \{ \frac{R^2}{4} \} . [10 - 2 \sqrt{5}]$ >>> The side $\mathbf{a_5} = |\frac{R}{2}| \sqrt{10 - 2.\sqrt{5}}$

…..(3)

(d) REGULAR HEXAGON ۞ :

The Equation of the vertices of the Regular Hexagon is :

$6.R = 2.R + [\frac{4.R^2 - a^2}{R}] + [\frac{4. R^4 - 4.R.^2.a^2 + a^4}{R^3}]$ >> $a^4 - 5. R^2.a^2 + 4. R^4 = 0$

Solving the equation gives :

$a^2 = \frac{5. R^2 - \sqrt{25. R^4 - 16. R^4}}{2} = [\frac{5 - 3}{2}] . R^2 = R^2$ The side $\mathbf{a_6 = R}$

…..(4)

(e) REGULAR HEPTAGON ۞ :

The Equation of the vertices of the Regular Xeptagon is :

$$7. R = 2. R + [\frac{4. R^2 - a^2}{R}] + [\frac{4. R^4 - 4. R^2 . a^2 + a^4}{R^3}] + [\frac{8. R^6 - 10. R^4. a^2 + 6. R^2. a^4 a - a^6}{2. R^5}] -$$

$$- [\frac{a^2}{2. R^5}] . \sqrt{64 . R^8 - 96. R^6. a^2 + 52 . R^4. a^4 - 12. R^2 . a^6 + a^8}$$

Rearranging the terms and solving the equation in the quantity **a** , obtaining :

$R^2 . a^{10} - 13 . R^4 . a^8 + 63 . R^6 . a^6 - 140 . R^8 . R^4 + 140 , R^{10} . a^2 - 49 . R^{12} = 0$ for $a^2 = x$

$$x^5 - 13 . R^2 . x^4 + 63 . R^4 . x^3 - 140 . R^6 . x^2 + 140 . R^8 . x^1 - 49 . R^{10} = 0 \quad \ldots\ldots\ldots\ldots(7)$$

Solving the 5 nth degree equation the Real roots are the following two :

$x_1 = R^2 . [3 - \sqrt{2}]$, $x_2 = R^2 . [3 + \sqrt{2}]$ which satisfy equation (7)

Having the two roots , the Sum of roots be equal to 13 , their combination taken 2 ,3, 4 at time be equal to 63 , - 140 , 140 , the product of roots be equal to – 49 , then equation (7) is reduced to the third degree equation as :

$$z^3 - 7. z^2 + 14. z - 7 = 0 \quad \ldots..(7a)$$

by setting $\psi = z - (-7/3)$ into (7a) , then gives $\boldsymbol{\psi^3 + \rho. \psi + q = 0}$ **.... (7b)** where ,

$\rho = 14 - (-7)^2 / 3 = 14 - 49/3 = - 7 / 3$ > $\rho^2 = 49 / 9$ > $\rho^3 = - 343 / 27$
$q = 2. (-7)^3 / 27 + 14 .(-7) / 3 - 7 = 7 / 27$ > $q^2 = 49 / 729$

Substituting ρ , q then $\boldsymbol{\psi^3 - (7 / 3) . \psi + (7 / 27) = 0}$ **... (7b)**

The solution of this third degree equation (7b) is as follows : $\rho = - 7 / 3$
$q = 7 / 27$

Discriminant $D = q^2 / 4 + \rho^3 / 27 = (49 / 729 .4) - (343 / 27.27) = - [49 / 108] < 0$

$D = - 49 /108 = i^2 (3.21^2 / 4 .27^2) = i^2 (21 . \sqrt{3} / 2 .27)^2 = i^2 (21. \sqrt{3} / 54)^2$

$D = [7 . \sqrt{3} / 18]^2 . i^2$ also $\sqrt[2]{D} = \left| \frac{7 . \sqrt{3}}{18} \right| . i$

Therefore the equation has three real roots :

Substituting $\psi = w - \rho / 3.w = w + 7 / 9.w$ > $\psi^2 = w^2 + 49 / 81 .w^2 + 14 / 9$
> $\psi^3 = w^3 + 343/729w^3 + 49 /27w + 7w / 3$
to (7b) then becomes $w^3 + 343 /729 w^3 + 7 / 27 = 0$
and for $z = w^3$ $z + 343 /729 z + 7 / 27 = 0$

$$\boldsymbol{z^2 + 7. z / 27 + 343 / 729 = 0} \quad \ldots(7c)$$

The Determinant D < 0 therefore the two quadratic complex roots are as follows :

Z_1 = $[-7/27 - \sqrt{49/27.27 - 4.\ 343/729}\ ] / 2 = [-7/27 - \sqrt{49/27.27.4 - 49.7.\ 4/27.27.4}\ ] / 2$

$= [-7/27 - \sqrt{(49 - 49.28)/ 27.27.4}\ ] / 2 = [-7 - 7 .\sqrt{-27}\ ] / 27 . 2$

$= [-7 - 21 . \sqrt{-3}\ ] / 3^3 . 2 = [\frac{-7}{2}] . (1 - 3 . i . \sqrt{3}) / 27 = (-7 / 54).[1 - 3.i. \sqrt{3}\ ]$

Z_2 = $[-7/2 . (1 - 3 . i\ \sqrt{3}\ ] / 27 = (- 7 / 54) . [1 + 3.i. \sqrt{3}\ ]$

The Process is beginning from the last denoting quantities to the first ones :

Root $W_{1,2} = \sqrt[3]{3} = \frac{1}{3} \sqrt[3]{\frac{-7 \pm 21 . i . \sqrt{3}}{2}} = \frac{1}{3} \sqrt[3]{-(\frac{7}{2}) . [1 \pm 3.i . \sqrt{3}\ ]}$ ……(1)

Root $\psi = W + 7 / 9.W = \frac{1}{3 .} \sqrt[3]{\frac{-7 \pm 21. i . \sqrt{3}}{2}} + \frac{7}{3 . \sqrt[3]{\frac{-7 \pm 21 i. \sqrt{3}}{2}}}$ …….(2)

Root $X = \psi - \rho / 3 = \psi + 7 / 3 = \frac{7}{3} + \frac{1}{3} . \sqrt[3]{\frac{-7 \pm 21 .i . \sqrt{3}}{2}} + \frac{7}{3 . \sqrt[3]{\frac{-7 \pm 21 .i . \sqrt{3}}{2}}}$

$$X = \frac{1}{3} \left| \frac{7. \sqrt{\frac{-7 \pm 21 . i . \sqrt{3}}{2}} + \sqrt{\frac{-7 \pm 21 . i . \sqrt{3}}{2}}^{\,3/2} + 7}{\sqrt[3]{-7 \pm 21 . i . \sqrt{3}}} \right| . R^2 \quad \text{……… (3)}$$

The root a_7 of equation (7) equal to the side of the regular Heptagon is $a_7 = \sqrt{X}$

$$\mathbf{a_7 = \sqrt[2]{\frac{1}{3} \left| \frac{7. \sqrt[3]{\frac{-7 \pm 21. i . \sqrt{3}}{2}} + \sqrt{\frac{-7 \pm 21 . i . \sqrt{3}}{2}}^{\,3/2} + 7}{\sqrt[3]{-7 \pm 21 . i . \sqrt{3}}} \right| . R}} \quad \textbf{…….. (4)}$$

Instead of substituting $\psi = w - \rho / 3.w$ into (7.b), is substituted $\boldsymbol{\psi = u + v}$ and then gives the equation of second degree as $z^2 + 7.z / 27 + 343 / 729 = 0$ which has the two complex roots as follows :

$$Z_{1,2} = \frac{7}{54} . [-1 \pm 3 . i . \sqrt{3}] = \frac{1}{27} . [(-7 \pm 21 . i . \sqrt{3}) / 2]$$ and the side a_7 is as :

$$a_7 = \sqrt[2]{\sqrt[3]{\frac{7}{Z_1}} + \sqrt[3]{\frac{7}{Z_2}} + \frac{7}{3}}$$

and by substituting Z_1, Z_2 into (7b) becomes the same formula as in (4) .

It is easy to see that $\sqrt[3]{- (7/2) . [1 - 3 . i . \sqrt{3}]} * \sqrt[3]{- (7/2) . [1 + 3 . i . \sqrt{3}]} = 7$

Analytically is :

$x = R^2 . [0,753\,020\,375\,967\,025\,701\,777]$ >> $x^2 = 0,56704$

$a_7 = \sqrt{x} = R . [0,867\,767\,453\,193\,664\,601 \ldots]$

By using the formula of the **real** root of equation (7a) then :

$a.x^3 + b.x^2 + c.x + d = 0$ >>> for $a = 1$, $b = -7$, $c = 14$, $d = -7$ then $x^3 - 7. x^2 + 14. x - 7 = 0$

$$x = -\frac{b}{3} - \frac{2^{1/3} .(- b^2 + 3.c)}{3 [-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}]^{1/3}} + \frac{[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}]^{1/3}}{3\, 2^{1/3}} .$$

Substituting the coefficients to the upper equation becomes :

$-b^2 +3.c = - (-7)^2 + 3.14 = - 49 + 42 = -7$ $-2.b^3 + 9.b.c -27.d = -2.(-7)^3 + 9.(-7).14 - 27.(-7) = 686 -882 + 189 = -7$

$4.(-b^2+3.c)^3 = 4 (-7)^3 = - 1372$ $(-2.b^3 +9.b.c -27.d)^2 = (-7)^2 = 49$ $32^{1/3} = \sqrt[3]{8.4} = 2 . \sqrt[3]{4}$

$$X = \frac{7}{3} - \frac{\sqrt[3]{2} .(-7)}{3 . \sqrt[3]{-7 + 21. i .\sqrt{3}}} + \frac{\sqrt[3]{-7 + 21. i .\sqrt{3}}}{2 .\sqrt[3]{4}}$$ and

$$a_7 = \sqrt{X} = \sqrt{\frac{7}{3} + \frac{7 \cdot \sqrt[3]{2}}{3 \cdot \sqrt[3]{-7 + 21 \cdot i \cdot \sqrt{3}}} + \frac{\sqrt[3]{-7 + 21 \cdot i \cdot \sqrt{3}}}{2 \cdot \sqrt[3]{4}}}$$

The Side of the

Regular Heptagon

(4.a)

Further Analysis to the Reader

(f) REGULAR OCTAGON :

The equation of vertices of the Regular Octagon is

$$8.R = 2.R + \frac{(a^2)}{R} + \frac{(4.R^2.a^2 - a^4)}{R^3} + \frac{10.R^4.a^2 - 6.R^2.a^4 + a^6 + a^2.\sqrt{64.R^8 - 96.R^6 a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}}{2.R^5}$$

Rearranging the terms and solving the equation in the quantity **a ,** is a 10th degree equation , and by reduction ($x = a^2$) is find the 5th degree equation as follows :

$$a^{10} - 13.R^2.a^8 + 62.R^4.a^6 - 132.R^6.a^4 + 120.R^8.a^2 - 36.R^{10} = 0$$

$$x^5 - 13.R^2.x^4 + 62.R^4.x^3 - 132.R^6.x^2 + 120.R^8.x^1 - 36.R^{10} = 0 \quad \ldots. (a)$$

Solving the 5th degree equation is find the known algebraic root of Octagon of side **a** as :

The roots are >>>>>>>>>> $x_1 = R^2 . [\, 2 - \sqrt{2}\,]$, $x_2 = R^2 . [\, 3 - \sqrt{3}\,]$

$$a_8 = \sqrt{x} = R . \sqrt{2 - \sqrt{2}} \quad \ldots\ldots\ldots (b)$$

Verification :

$x = a^2 = R^2 (2 - \sqrt{2})$ $\quad x^2 = R^4 . (6 - 4\sqrt{2})$ $\quad x^3 = R^6 . (20 - 14.\sqrt{2})$

$x^4 = R^8 . (68 - 48\sqrt{2})$ $\quad x^5 = R^{10} . (232 - 164\sqrt{2})$ $\quad \ldots\ldots\ldots\ldots(c)$

by substitution (c) in (a) becomes :

$$R^{10}. [\, 232 - 164 . \sqrt{2}\,] = R^{10}. [\, 232 - 164 .\sqrt{2}\ 2\,]$$

$$- R^{10} . [\, 884 - 624 . \sqrt{2}\,] = R^{10}. [\, -884 + 624 . \sqrt{2}\,]$$

$$R^{10} . [\, 1240 - 868 . \sqrt{2}\,] = R^{10}. [\, 1240 - 868 . \sqrt{2}\,]$$

$- R^{10} . [792 - 528 . \sqrt{2}] = R^{10}. [-792 + 528 . \sqrt{2}]$

$R^{10}. [240 - 120 . \sqrt{2}] = R^{10} . [240 - 120 . \sqrt{2}]$

$- R^{10} . [36] = R^{10} . [- 36]$

$R^{10} . [1712 - 1712 + (1152 - 1152) . \sqrt{2}] = 0$

$R^{10} . [0+0] = 0$ therefore Side $a_8 = R . \sqrt{2 - \sqrt{2}}$ …… (b)

(g) CONCLUTION :

By summation the heights **y** on any tangent in a circle ,which hold for every ***Regular n-sided Polygon*** inscribed in the circle as the next is :

$$\mathbf{n . R = 2.R + 2. y_1 + 2. y_2 + 2. y_3 + ... 2 . y_n} \qquad \textbf{………………(n)}$$

the sides a_n of all these Regular n-sided Polygons are Algebraically expressed .

The Geometrical Construction of all Regular Polygons has been proved to be based on the solution of the moving Segment ZD of the figure of page 8 and it is the Master Key of Geometry , because so , the n_{th} degree equations are the vertices of the n-polygon .

In this way , all Regular p - gon are constructible and measureable .

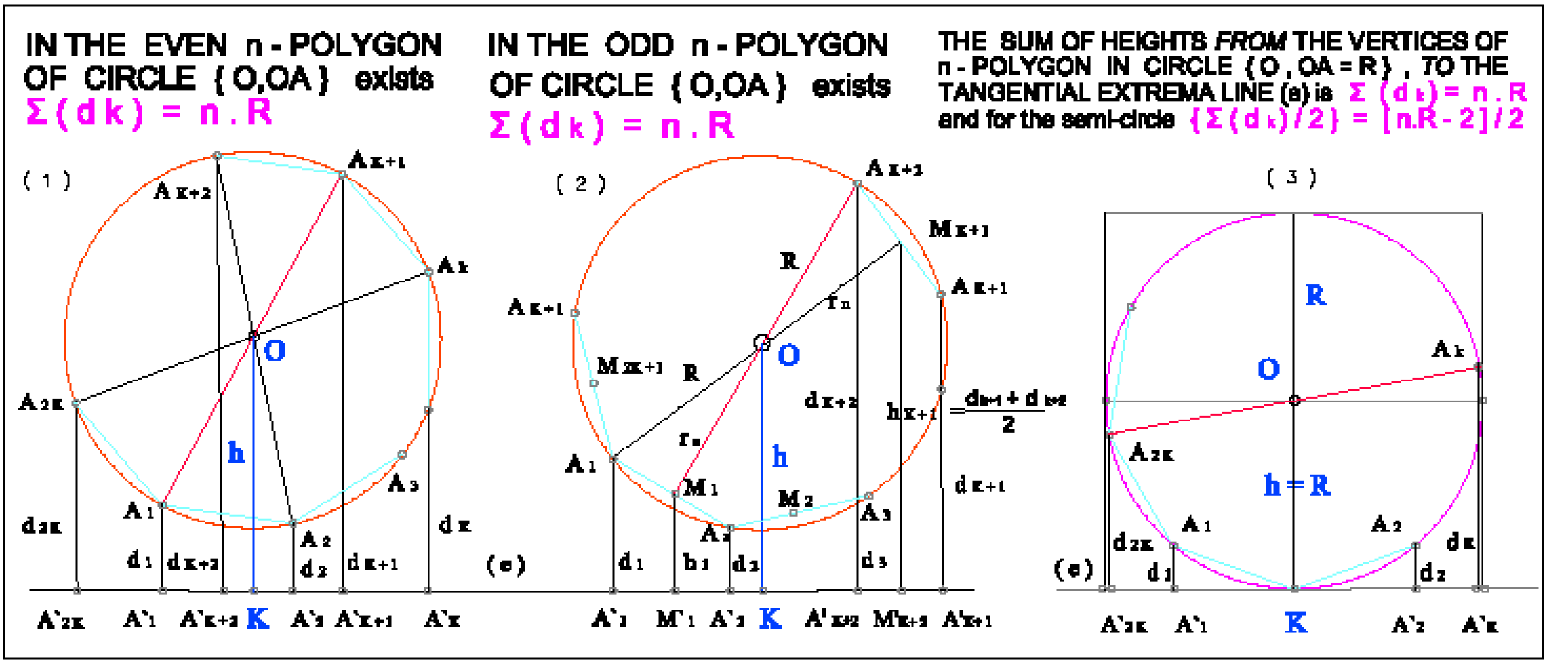
The mathematical reasoning is based on Geometrical logic exclusively alone .
As the Resemblance Ratio of Areas on the 4 - gone is equal to 2 , the problem of squaring the circle has been approached and solved by extending Euclid logic of Units (*under the restrictions imposed to seek the solution* , *with a ruler and a compass* ,) on the unit circle AB , to unknown and now the Geometrical elements . (*the settled age-old question for all these problems is not valid*) .

The Regular Heptagon :

According to Heron , the regular Heptagon is equal to six times the equilateral triangle with the same side and is the approximate value of $\sqrt{3} . R / 2$
According to Archimedes , given a straight line AB we mark upon it two points C , D such that $AD.CD = DB^2$ and $CB.DB = AC^2$, without giving the way of marking the two points . According to the Contemporary Method , the side of the Regular Heptagon is the root of a third degree equation with three real roots , one of which is that of the regular Heptagon as analytically presented.

5.2. *THE GEOMETRICAL SOLUTION OF THE POLYGONS* :

a.. The Even and Odd n-Polygons :



F.14 → An Even and an Odd n-Polygon in circle O ,OA with diameters , $A_k A_{2k}$, passing from A_{2k} , as vertex (apex) of the Polygone , and diameters , $A_{k+2} M_1$ perpendicular to side $A_1 A_2$.

Let be the n-Polygon A_1 , A_2 , A_3 , A_k , A_{k+1} , A_{k+2} , A_{2k} , in circle (O , OA_1) ,

(e) a straight line not intersecting the circle
d_1 , d_2 , d_{2k} , The heights of the vertices to (e) line ,
h_1 , h_2 , h_{2k+1} , The heights of the midpoints M_k M_{k+1} of the sides to (e) line and
OK = h , The height from the center O to (e) line .

To proof :

In any n - Polygon , The Sum , Σ = Σ (h) , of the Heights , d_1 , d_2 , d_{2k} , of the Vertices A_1 , A_2 , A_3 , A_k , A_{k+1} , A_{k+2} , A_{2k} , *where n = 2k* , from any straight line (e) is equal to

Σ = Σ (h) = n . OK = n . h

Proof F.14 :

From any vertex A_k , of the n-Polygon draw the diameter (A_k O A_{2k})

a.. When n = 2.k → then Vertex A_{2k} belongs to the Polygon
b.. When n = 2.k + 1 → then line $A_k O$, is mid-perpendicular to one of the sides .

Case a.. n = 2.k F.14 –(1)

Exists $\frac{n}{2} = \frac{2\,k}{2} = k$, and are the pairs of vertices in opposite diameters as in A_1 , A_{k+1} , and the , k , Trapezium which has bases the heights of the vertices in opposite diameters from (e) line , and which have height OK = h , as Common Height from their Diameter , i.e.

From trapezium A_1 , $A`_1$ , $A_{k+1}$ , $A`_{k+1}$ exists $d_1 + d_{k+1} = 2.h$ and analogically ,
$d_2 + d_{k+2} = 2.h$
$d_3 + d_{k+3} = 2.h$ ………….. $d_k + d_{k+1} = 2.h$

And by Summation ,

$$d_1 + d_2 + \dots\ d_k + d_{2k} = 2.h \quad \text{or} \quad \Sigma = (2k) . h = n . h = n . OK \qquad \text{……..….(1)}$$

Case b.. n = 2.k +1 F.14 –(2)

$A_1 A_2$, $A_2 A_3$, …. $A_{2k+1} A_1$, the sides of the Polygon .
M_1 , M_2 , M_{2k+1} , are the midpoints of sides from line (e)
h_1 , h_2 , …. h_{2k+1} the corresponding heights of midpoints from (e) .
The diameter from vertex A_1 is perpendicular to side $A_{k+1} A_{k+2}$ which has the midpoint M_{k+1} ,
while $A_1 M_{k+1} = A_1O + OM_{k+1} = R + r_n$
In trapezium $A_1\ A`_1\ M`_{k+1}\ M_{k+1}$ with Bases $A_1\ A`_1$ and $M`_{k+1}\ M_{k+1}$, both perpendicular to (e) line is parallel to height OK = h and bisects $A_1O = R$ and $OM_{k+1} = r_n$ and from figure , exists

$$OK = h = \frac{R\,h_{k+1} + r_n . d_1}{R + r_n} \qquad \text{……..….(2)}$$

i.e. Height OK is common to all 2k+1 trapezium which are formed as $A_1\ A`_1\ M`_{k+1}\ M_{k+1}$ and OK Height divides also the corresponding to $A_1\ M_{k+1}$ side with the same analogy as $\frac{R}{r_n}$.
By summation of 2k+1 parts of (2) which are all equal to OK = h , then from the 2k+1 different Between them trapezium referred exists ,

$$(2k+1).\ h = \frac{R\{h_{k+1} + h_{k+2} + h_{k+1} + h_1 + \dots h_k\} + r_n . \{d_1 + d_2 + \dots . d_{k+1} + d_{2k+1}\}}{R + r_n} = n . h = \frac{R.S + r_n \Sigma}{R + r_n} \text{ .....(3)}$$

where $S = h_1 + h_2 + \dots\ h_k + d_{2k+1}$. Since h_1 , h_2 , …. h_k , d_{2k+1} are the diameters of trapezium with bases d_1 , d_2 to h_1 , d_2 , d_3 to h_2 and so on and also d_{2k+1} , d_1 to h_{2k+1} then $S = \frac{d_1 + d_2}{2} + \frac{d_2 + d_3}{2} + \dots\ \frac{d_{2k} + d_1}{2} = \frac{2\{d_1 + d_2 + \dots\ d_{2k+1}\}}{2} = d_1 + d_2 + \dots\ d_k + d_{2k} = \Sigma$ and (3) is $n . h = \frac{R.S + r_n \Sigma}{R + r_n} = [\frac{R + r_n}{R + r_n}] . \Sigma = \Sigma$ ***i.e.*** $\boldsymbol{\Sigma = n . h}$ ***for all Even and Odd n-Polygons*** .

A relation between Heights and the Number of the Regular Polygons .

Case c.. Line (e) is Extrema as Tangential to circle F.14 –(3)

In this case height ,h , is equal to radius R and OK = h = R .

Since the Sum of Heights of the vertices in any n-Polygon is Σ = n . h = n .OK then **Σ = n . R**

This remark helps to construct Geometrically , *i.e. with a Ruler and a Compass* , all the Regular n-Polygons because gives the relation of the Apothem , the radius r_n of the inscribed circle which is related to the Interior angle w = { $\frac{n-2}{n}$ }. 180^o .

i.e. Angles , w , in a circle of radius , R , define the n-Sides , $A_1 A_2$ ***, of the Regular Polygon which in turn define the Sum , Σ , of their heights equal to Σ = n . R***

Since also the relation of radius ,R, between the Circle and ,r, of the Inscribed circle is extended to Heights , this helps Extrema - Method to be applicable on the solution which follows .

b.. The Theory of Means :

It was known from Pappus the how to exhibit in a semicircle all three means , namely , The Arithmetic , The Geometric , and The Harmonic mean .

In Fig.15 –(1a) → On the diameter AC of circle (O , OA = OC) , C is any Pont on OC .
Draw BD at right angles to AC meeting the semi - circle in D .
Join OD and draw BE perpendicular to OD .
Show that DE is the Harmonic - Mean between AB , BC

Proof :

For , since ODB is a right – angled triangle , and BE is perpendicular to OD then ,
DE : BD = BD : DO or DE . DO = BD ² = AB . BC
But DO = $\frac{1}{2}$ (AB + BC) therefore DE .(AB+BC) = 2 . AB.BC . By rearranging
is AB . (DE – BC) = BC . (AB – DE) or AB : BC = (AB –DE) : (DE –BE) ,

that is , DE is the Harmonic Mean between AB and BC .

In Fig.15 –(1b) → Is given only Segment AB and is defined Harmonic mean AM between AB ,MB
Draw BC at right angles to AB meeting center C of circle (C , CB = AB / 2) .
Join AC intersecting circle (C ,CB) at points D , E where DE = 2.DC = AB .
Draw circle (A , AD) intersecting AB at point M .
Show that AM is the Harmonic - Mean between AB , MB .

The Proof :

For , since ABC is a right – angled triangle , and DE = AB then ,
AB ² = AD . AE = AD . (AD + DE) = AD . (AD + AB) = AD ² + AD . AB therefore ,
AD ² = AB ² - AD . AB = AB . (AB – AD) or AD ² = AB . MB

That is , AM is the Harmonic Mean in AB Segment , or between AB and MB .

6.. Markos Theory ***, on Segments and Angles Relation*** **:**

The Three Circles Method :

In Fig.15 –(2) → Two Even ,n, and ,n+2, Regular Polygons on the same circle (O , OA) where ,

n , n +2 are the number of sides differing by an Even number

λ_a = The length of a side of a – [n - Polygon].

λ_b = The length of a side of b – [n+2 Polygon].

r_a = The Apothem (the radius of the inscribed circle of a – Polygon) .

r_b = The Apothem (the radius of the inscribed circle of b – Polygon) .

h_A = The Height of KA_1 side of a – Polygon .

h_B = The Height of KB_1 side of b – Polygon .

$\Delta h = h_A - h_B$, the difference of heights .

$\Delta r = r_a - r_b$, the difference of apothems .

S = The sum of interior angles equal to (n-2).180^o = (n-2).π

$\frac{h_A}{\lambda_a} = \sin\varphi_a$, $\frac{h_B}{\lambda_b} = \sin\varphi_b$, $\frac{h}{\lambda} = \varphi$,

$w_a = [\frac{2}{n}].180 = [\frac{2}{n}]\pi$, The Interior angle of the [n - Polygon].

$w_b = [\frac{2}{n+2}].180 = [\frac{2}{n+2}].\pi$, The Interior angle of the [n+2 Polygon].

w_o = An Extrema-angle between w_a , w_b which is related to Heights .

$\varphi_a = [\frac{n-2}{2.n}]\pi$, The angle of side λ_a to (e) line for Even , n-Polygon.

$\varphi_b = [\frac{n}{2(n+2)}]\pi$, The angle of side λ_b to (e) line for Even , n+2 Polygon.

$\varphi_o = [\frac{n-1}{2(n+1)}]\pi$, The angle of side λ_o to (e) line for Odd – Polygon .

Show that , the Extrema-angle , w_o , and the complementary angle , φ_o , define the In-between Odd-Regular n-Polygons on the same circle* (O , OA) *, by Scanning the* , Δh , *difference Height , on Circles - Heights - System , and following the Harmonic – Mean of Heights .

Proof : Fig.15 – (2 , 3)

a.. Draw on OK circle , the Tangent at point K , and from K any two Chords KA and KB .
From Points A , B draw the Perpendiculars AA`,BB` and the Parallels AA_1,BB_1 , to Tangent (e).

b.. Draw the circle of Heights (A_1 , A_1B_1)

In right angles triangles KAA` , KBB` , ratios $\frac{AA`}{KA} = \frac{h_A}{\lambda_a} = \sin\varphi_a$ and $\frac{BB`}{KB} = \frac{h_B}{\lambda_b} = \sin\varphi_b$,

where $h_A = \lambda_a . \sin\varphi_a$ and $h_B = \lambda_b . \sin\varphi_b$ and the difference $\Delta h = h_A - h_B$, or

$$\Delta h = h_A - h_B = \lambda_a . \sin\varphi_a - \lambda_b . \sin\varphi_b \quad \text{……….} \ (1)$$

Since between the two sequent , n , n+2 , Even – Regular – Polygons exists the Geometric logic of AB Monads , i.e. *In a Segment the whole is equal to the parts , and to the two halves* , and for angle φ_a to become φ_b is needed to pass through another one angle φ_o , which is between the two , *therefore* ,

a.. Between the two sequence Even -Regular-Polygons exists another one Regular-Polygon .

b.. According to Pappus theory of Proportion and Means , between the three terms h , λ , φ exists one of the three means .

c.. For since the Sum { it is algebraically n + (n+2) = 2n + 2 = 2.(n+1) } must be an Integer which can be divided by 2 .

d.. Between the two Even -Regular-Polygons exists the only one (n+1) Odd-Regular-Polygon .

For the commonly divergence angle , φ , equation (1) becomes h_φ ,

$$\Delta h = h_A - h_B = (\lambda_a - \lambda_b) . \sin\varphi = \Delta\lambda . \sin\varphi \qquad \text{..........} \quad (2)$$

or , $h_A - h_B = (2. r_a . \sin\varphi - 2. r_b . \sin\varphi) . \sin\varphi = 2(r_a - r_b) . \sin^2\varphi$ i.e.

$$h_A - h_B = 2(r_a - r_b) . \sin^2\varphi \quad \text{or} \quad \frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{1/2(r_a - r_b)} \qquad \text{..........} \quad (3)$$

***That is* , $\sin\varphi = \left(\frac{h_\varphi}{\lambda_\varphi}\right)$, *is the Harmonic - Mean between* [$h_A - h_B$] , [$\frac{1}{2(r_a - r_b)}$]**

From (1) $\Delta h = \lambda_a . \sin\varphi_a - \lambda_b . \sin\varphi_b = \frac{\lambda_a^2}{4R^2} - \frac{\lambda_b^2}{4R^2} = \frac{1}{4R^2}(\lambda_a^2 - \lambda_a^2)$ or

$$2.R . \Delta h = (\lambda_a^2 - \lambda_b^2) = [\lambda_a - \lambda_b] . [\lambda_a + \lambda_b] . \qquad \text{..........} \ (4)$$

Show that , the Extrema-angle* , w_o , *formulates the complementary angle* , φ , *defining the In-between Odd - Regular n-Polygons on the same circle* (O , OK) *, using the Extreme cases of this System* { $\Delta h = h_A - h_B = A_1B_1$ } *, on the Circles of difference of Height .

Analysis :

1.. From above relation of Heights and circle radius for two Sequent – Even - Polygons then ,

$\Sigma\ h_n = n . R = n . OK$ (a) and $\Sigma\ h_{n+2} = (n+2) . R = (n+2) . OK$ (b)

By Subtraction (a) , (b)

$$\Sigma\ h_{n+2} - \Sigma\ h_n = (n+2)R - nR = 2.R \qquad \rightarrow \text{constant}$$

By Summation (a) , (b)

$$\Sigma\ h_{n+2} + \Sigma\ h_n = (n+2)R + nR = (n+1).2.R \qquad \rightarrow \text{constant}$$

i.e. in the System of Regular - Polygons the , ***Interior angles* (w) *and Gradient* (φ) , *Heights* (h) *and their differences*** , **Δh** ***, – Summation and Subtraction of Heights are Interconnected and Intertwined at the Common Circle* [A , $\Delta h = h_A - h_B$] *producing the Common* (n+1) , Odd – Regular - Polygon** .

2.. In Fig.15 - (2-3) → For , KA , KB , chords exists $\lambda_a = 2R.\sin\varphi_a$, $\lambda_b = 2R.\sin\varphi_b$,

and their product [POP] = ($\lambda_a . \lambda_a$) = $4R^2.[\sin\varphi_a . \sin\varphi_b]$ …………… (5)

The sum of heights for the n and n+2 Even Regular Polygon is $\Sigma h_A = n.R$ and $\Sigma h_B = (n+2).R$ and the In-between Odd Regular Polygon $\Sigma h_o = (n+1).R$. The corresponding Interior angles

$$w_a = [\frac{2}{n}]\pi \quad \text{and} \quad \varphi_a = [\frac{n-2}{2.n}]\pi$$

$$w_b = [\frac{2}{n+2}]\pi \quad \text{and} \quad \varphi_b = [\frac{n}{2.(n+2)}]\pi$$

$$w_o = [\frac{2}{n+1}]\pi \quad \text{and} \quad \varphi_o = [\frac{n-1}{2.(n+1)}]\pi$$

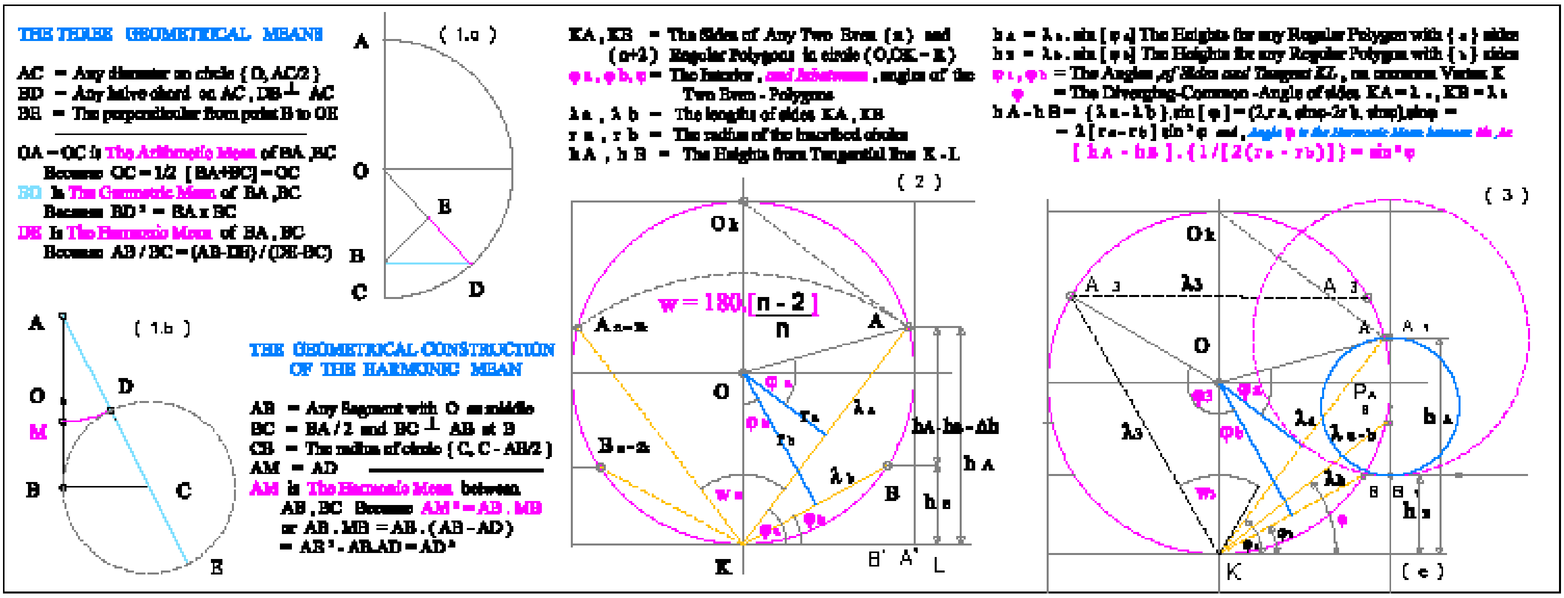
The Power of point O to circle of diameter Δh is for $\lambda_o = 2R.\sin\varphi_o$, $\lambda`_o = 2R.\sin\varphi_o$,
[POP] = [$\lambda_o . \lambda`_o$] = $4R^2.\sin^2\varphi_o$ ……… (6) and equal to (5) therefore

$$\sin\varphi_a . \sin\varphi_b = \sin^2\varphi_o \quad \text{or} \quad \frac{\sin\varphi_a}{\sin\varphi_o} = \frac{\sin\varphi_o}{\sin\varphi_b} \qquad \text{………….(7)}$$

i.e. Angle φ_o follows the Harmonic-Mean between angles φ_a , φ_b on Δh Difference of Heights.

3.. Since Product of magnitudes $\lambda_a . \lambda_b$ = constant and also ($\lambda_a - \lambda_b$) . ($\lambda_a + \lambda_b$) = constant , *therefore , the Power of any point IN and OUT of the circle of Heights is Constant* , meaning that exists another one Regular – Polygon , between the two Even - Sequence i.e.

The Outer are the two Even-Regular* N *and* N+2 *Polygons ,
and The Inner is the* N+1 *Odd – Regular Polygon .



F.15 → In (1) are shown the two ways for constructing the three Means on One or Two Segments .
In (2) is shown the Divergency of Sides to Heights of Two n , and (n+2) Even Polygons .
In (3) is shown the locus of the Two - Circles of Heights (A_1, A_1B_1) and the parallels to (e) .
to be Extrema case for the two segments KA , and KB .

6.1. *Analysis of the Geometrical Construction .* Fig.16 - (3)

The construction of all the *Even - Regular - Polygons* is possible by dividing the circle (O , OK) in 2 , 4 , 6 , 8 , 10 , 12 , 14 …2n parts as $w_a = [\frac{2}{n}]\pi$ and $\varphi_a = [\frac{n-2}{2.n}]\pi$, n = 1 , 2 , 3 ....

The construction of all the *Odd - Regular -Polygons* is possible by Applying the Circles on Heights between the chords of the Even-Sequence of Polygons on [2 , 4] – [4 , 6] – [6 , 8] – [8 ,10] … [(2n) – (2n+2)] as formulas $w_o = [\frac{2}{n+1}]\pi$ and $\varphi_o = [\frac{n-1}{2.(n+1)}]\pi$ founded from point K .

Case A → Digone .

Step 1 :

Draw from point K , *of any circle* (O , OK) , Tangent (e) at K and Chord KA which is the diameter (because diameter of the circle is the Side of the Regular - Digone) and any KB , corresponding to the Even (n) and (n+2) Regular Polygon .

Step 2 :

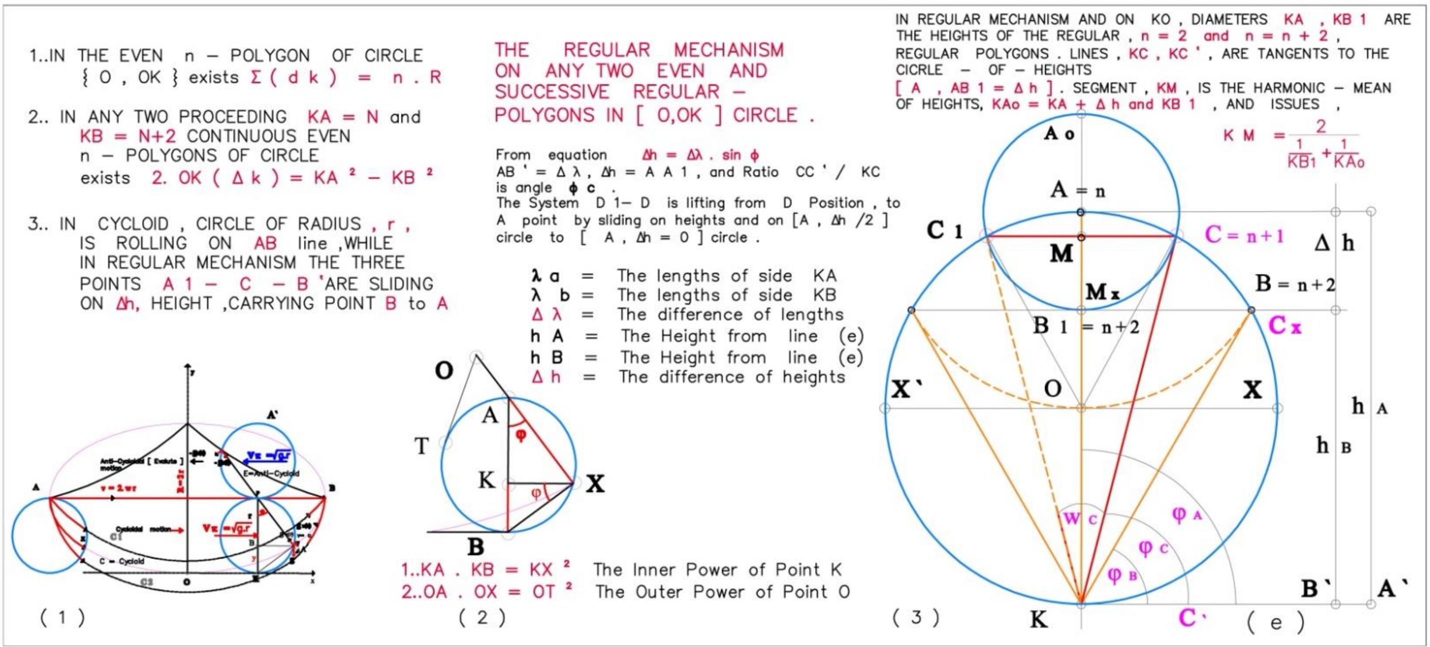
Draw from points A , B , the perpendiculars to (e) and define the difference $\Delta h = h_A - h_B = AB_1$ on diameter KA and Draw circle (A , AB_1) with radius Δh , and line KA intersecting circle at point A_o .

Step 3 :

Draw tangents KC , KC_1 and chord CC_1 intersecting circle (O , OA) at point C .

Step 4 :

Draw Chord KC which is the Side of the Regular Odd – (n +1) - Regular - Polygon on angle φ_c



F.16 → In (1) is shown the Rolling of a circle on a straight line and forming the Cycloid .
In (2) is shown the Inner - Outer Power of Points , K , O , on circle of AB diameter .
In (3) is shown the How and Why KM Segment is the Harmonic-Mean between KA , KB_1 .

Proof :

1.. Because triangle A C K is rightangled then AC is perpendicular to KC therefore Segment KC is perpendicular to AC and it is Tangential to circle (A , AB_1) .
The same also for KC_1 , which is also tangent to circle (A , AB_1) .

2.. From relations $KA_o = KA + AA_o = KA + AB_1$

$KB_1 = KA - AB_1 = KA - (KA_o - KA) = 2.KA - KA_o$ or ,

$2.KA = KA_o + KB_1 = (h_A + \Delta h) + h_B$ ……….. (1) therefore

$KA = \frac{h_A + \Delta h + h_B}{2}$ ……….. (2) *The Arithmetic – Mean* .

3.. From the Power of point K to circle (A , AB_1) exists $[KC]^2 = [KB_1].[KA_o]$ therefore

$KC = \sqrt{KB_1 . KA_o} = \sqrt{[h_A + \Delta h] . h_B}$ ….(3) *The Geometric – Mean*

4.. From the right angled triangle A C M exists $KM . KA = KC^2 = (KB_1).(KA_o)$ or

$$KM = \frac{KA_o.KB_1}{KA} = \left[\frac{KA_o.KB_1}{KA_o + KB_1}\right].2 = \left[\frac{2}{\frac{1}{KA_o} + \frac{1}{KB_1}}\right] \quad …….. (4) \text{ i.e.}$$

KM is the *Harmonic - Mean between* KA_o *and* KB_1 or $(h_A + \Delta h)$, h_B .

For n = 2 , then KA is the Side of the Regular - Digone and equal to the diameter of the circle .

For n = n+2 = 4 , then KB is the Side of the Regular - Pentagon sided on the perpendicular to KA side . Exist $h_A = KA$, $h_B = KO = KB_1$, $\Delta h = AB_1$, and A_3 point coincides with A_2 , and consequence with C point . Parallel line DA_4 coincides with the parallel C C ` line and KC is the Side of the n+1 = 3 , Regular – Trigon on $KM = KO + \frac{\Delta h}{2} = 1,5. OK$.

Point A *is the Vertex and* KA *is the Side of the Regular Digone* .

Point* C *is the Vertex and* KC *is the Side of the Regular Trigon (*Triangle*).

Point B *is the Vertex and* KB *is the Side of the Regular Tetragon* .

In addition , from formula Σ = n . R = 3R = 3.OK , and since every half is $\frac{3}{2}$. OK = 1,5. OK then Point C is on half Δh , or height $h = KO + \frac{OA}{2}$.

For n = 4 , then KA is the Side of the Regular - Tetragon and equal $KX = OK.\sqrt{2}$ chord .

For n = n+2 = 6 , then KB is the Side of the Regular -Hexagon sided on circle (O , OA) .

For n = n+1 = 5 then it is the side of the Regular-Pentagon .

The How this is Geometrically achieved follows by the following three methods .

a.. The [*Antiphon – Archimedes*] Ancient Greek – Polygons method .

b.. The [*Euler - Savary*] Coupler-Curves curvature - centers method .

c.. The [*Markos*] Geometrical , Three – Circles - Method , in Polygons .

6.2. *The Geometrical Construction of ALL Regular Polygons* .

Preliminaries : The Coupler Curves .

Geometry :

Let A be a point on a Plane System ,S, rolling on the fixed system ,So, as in Fig-17.1
K_A is the center of curvature , the Instaneous center on the fix system .
P is the Instaneous center of curvature on the fix curve So (the pole P),
(p) , (π) are the coupler curves on , S , So
u = The translational velocity of pole P equal to ds/dt = AA`/dt
w = Angular velocity of pole P equal to dr/dt = d(APA`)/dt and for d = u / w then ,

Euler-Savary equation is $Ex = [\, 1/ r_D - 1/ R_D \,] \sin\varphi = 1/ d$ ………………… (a)

When point P lies on the radius of curvature of Polar path (φ = 90) then sinφ =1 and from Fig - 17.2 holds → $[\, 1/ r_D - 1/ R_D \,] = 1/ d$ and issues $r = r_D . \sin\varphi$ and $R = R_D . \sin\varphi$

i.e. The trajectories of points A on the circumference of circle radius $\boldsymbol{r_D}$, have their center of curvature on circumference of circle of radius $\boldsymbol{R_D}$.

Motion :

The motion of curves (p) , (π) is in Fig -17.3
Let $\overline{v_A}$, $\overline{v_P}$, $\overline{v_{KA}}$, be the velocities of points A , P , K_A to their systems .
For system S the curvature center K_A , the Instaneous center , is found from the intersection of A`P` and AP . For system ,So, the curvature center K_{AA} , the Instaneous center of K_A on fixed system (π) is found from the intersection of $P`K_{AA`}$ and PK_A .
From the above similar triangle $K_AAA`$ , $K_APP`$ exists ,

$(K_AA/PA) = (K_AA`/P`A`) = (K_{AA`}A`/PK_{AA`}) = K_AK_{AA}/PK_{AA}$ or $\{K_AA/PA\} = \{K_AK_{AA}/K_{AA}P\}$… (b)

i.e. ***The Points*** A , K_{AA} ***are harmonically divided by the points*** P , K_A and exists ,

$1/ PA + 1/ PK_{AA} = 2 / PK_A$

Inversing the two Systems by considering fixed system ,So, rolling on ,S, as in Fig-17.4 then ,

$Ex = [\, 1/r_A - 1/R_A \,] \sin\varphi_A = 1/ d$ and $[\, 1/r_{A`} - 1/R_{A`} \,] \sin\varphi`_A = 1/ d$ where in both cases issues ,

$(PK_A - PA)/(PK_A.PA) = -(PK`_A - PA`)/(PK_{A`}.PA`)$ or $Ex = (1/PA - 1/PK_A) = (1/PK_{A`} - PA`) = 1 / d$ … (c)

The Path of the Instaneous-center of curvature , $\mathbf{O_A}$, on (k) , (π) coupler envelope curves is proved that , During the rolling of curve (k) of system , S , and the fixed to it envelope (π) , then the Instaneous-center of curvature and those of the constant envelope (π) , ***coincides*** to the Instaneous-center of curvature K_A of (k) as in Fig-17.1

The center D , of a Rolling circle (p) on another circle (π) , executes a circular motion with K_D as center which coincides with the center of curvature of the second circle . Because angle φ =90˚ , then for every point A on (p) exists a center of curvature K_A on AP and C K_p as in Fig-17.2

During the rolling of a circle (p) on (π) line , then the corresponding Instaneous-center of curvature K_A of any point A is the common point of intersection of AP produced and the parallel to DP from point C and the Instaneous-center of curvature K_D for point D is in infinite and KD = ∞ .
The Euler-Savary equation involves the four points A , P , K_A , K_{AA} lying on the path normal.
Equation (b) may be written in the form $PA / AK_{AA} = AK_{AA}/ AK_A$ and is recognized that AK_{AA} is the mean proportional between PA and K_A A .

The Cubic of Stationary curvature :

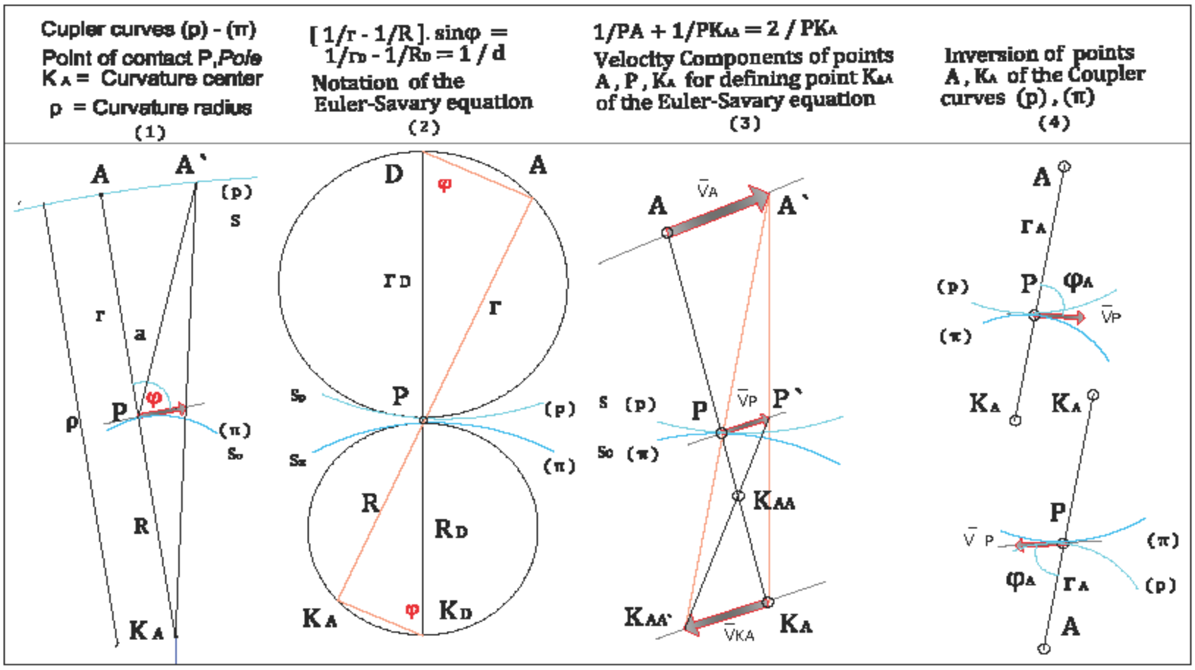
Euler-Savary formula apply to the analysis of a mechanism in a given position and vicinity .
It gives also the radius of curvature and the center of curvature of a couple-curve. Because couple-curve (Path ↔ Evolute) is the equilibrium of any moving system , then Complex-plane is involved and the E-S geometrical equations ,

$Ex = (1/ PA - 1/ PK_A)\, i.e^{i\varphi} = h\, [1/PA - 1/PK_A] = h . (\frac{d\varphi}{ds})$ and for the homothetic motion

(h = 1) then , $$Ex = \frac{1}{PA} - \frac{1}{PK_A} = \frac{1}{P K_{AA}} (\frac{d\varphi}{ds})$$ ………… (d)

Equation (d) is that of ***Rhodonea Hypocycloid curves*** .

The Inflation circle , Κύκλος Καμπής και Αντίστροφων Κέντρων , ***extrema case*** **,** shows the location of coupler points whose curves have an infinite radius of curvature , i.e. on inflection circle lie all centers of curvature of System curves and which , these are rolling on inflection point on the envelope .(Envelope here are the two or more surfaces in direct contact).The Cubic of Stationary curvature [COSC] indicates the location of coupler points that will trace segments of approximate circular arcs . In Geometry , the rolling of a circle , on a circle and or on a line is likewise to Mechanism as , Space Rolling on Anti=space , a Negative particle , Electron , on a Positive particle , Proton , or on many Protons , so the Wheel-Rims represent the , COSC in Mechanics .



F.17 → **In (1)** A point A on Coupler-curves (p) , (π) define the point of curvature KA , the Instaneous point **P ,** the pole on (π) .

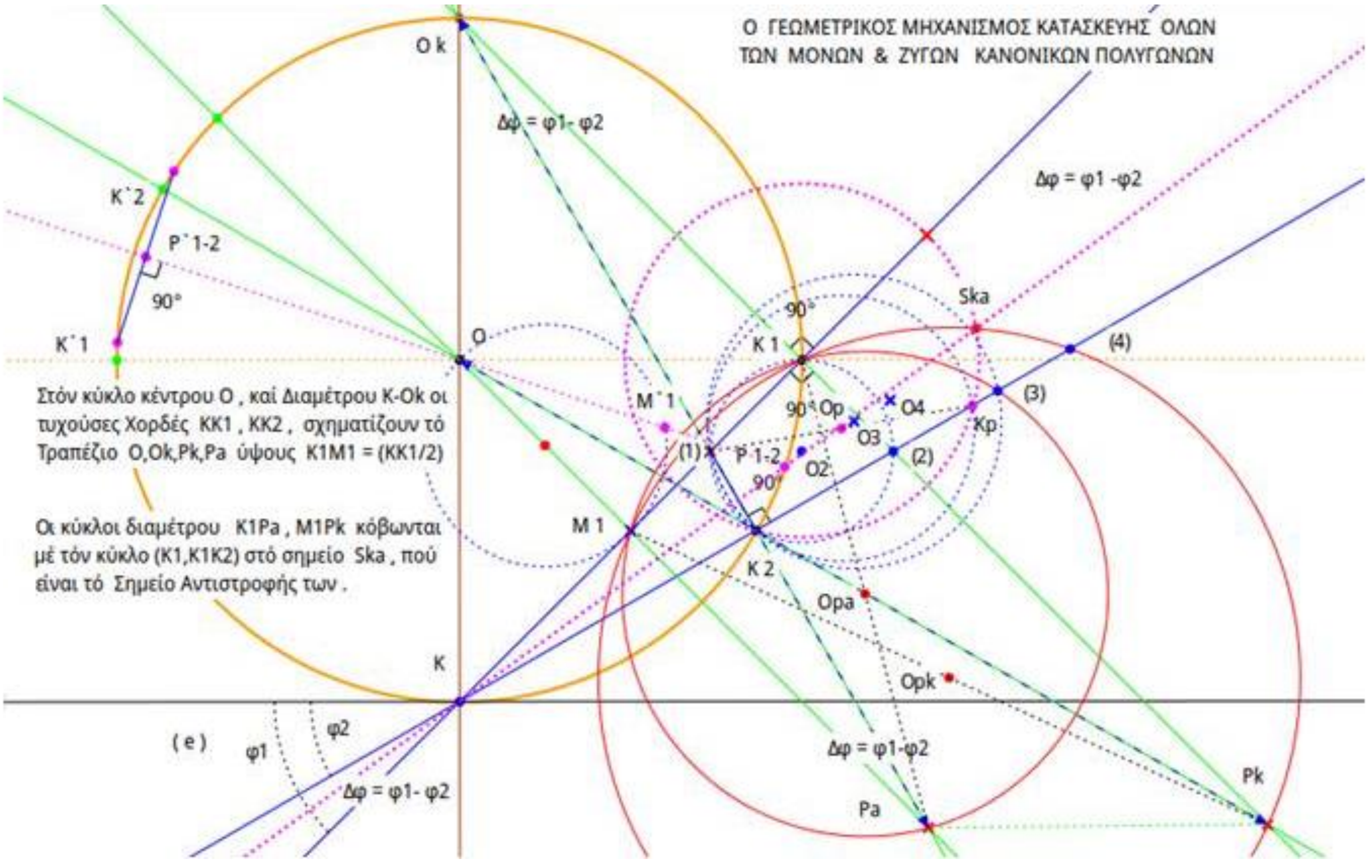
In (2) is the case of point A lying on radius of curvature of polar path (point D) where then the paths of points A in , S , system have the Instaneous center of curvature KA on the fixed system So .

In (3) The Velocity Instaneous center , for curvature point K_A , in S_o system is point K_{AA} .

In (4) The two points A , K_A , of Coupler-curves (p) , (π) , follow the inversed motion where Poles of rotation , A and K_A , are inverted .

Above F.17 is the Master-key for the solution to inscribe in a circle a regular polygon with any given number of sides either Mechanical or Geometrical - Solutions [63].

Η Μέθοδος αφιερώνεται στην Σύγχρονη - Ελλάδα , για να Μη Ξεχνά τους Προγόνους της .



F.17 - A ***Στον κύκλο*** (O ,OK) μέ την ευθεία (e) εφαπτομένη στο σημείο K , και με διάμετρο KO_K , ***Φέρομεν*** τις τυχουσες Χορδές $K K_1$, $K K_2$ και τις αντίστοιχες χορδές $O_K K_1$, $O_K K_2$, μέ τις γωνίες $< K_1K(\varepsilon) = \varphi_1$, $< K_2K(\varepsilon) = \varphi_2$ και $\Delta\varphi = \varphi 1 - \varphi 2$.
Από δε του σημείου O , ***Φέρομεν*** την OM_1 μεσοκάθετο τής χορδής $K K_1$.

1.. ***Προεκτείνομεν*** την $O_k K_1$, ώστε *να Κόβει* την προέκταση της OK_2 στό σημείο P_k , και ***Φέρομεν*** τον κύκλο (O_{pk} , $O_{pk}P_k = O_{pk}M_1$) κέντρου O_{pk} και διαμέτρου [P_kM_1] .
2.. ***Προεκτείνομεν*** την $O_k K_2$, ώστε *να Κόβει* την προέκταση της OM_1 στό σημείο P_a , και ***Φέρομεν*** τον κύκλο (O_{pa} , $O_{pa}P_a = O_{pa}K_1$) κέντρου O_{pa} καί διαμέτρου [P_aK_1] .
3.. Η ευθεία $K K_2$ ***Προεκτεινομένη Κόβει*** , Την προέκταση της $O_K K_1$ *Στο σημείο* (2) , Τον κύκλο (K_1 , K_1K_2) *Στο σημείο* K_p , Τον κύκλο διαμέτρου [P_aK_1] *Στο σημείο* (3) , και Τον κύκλο διαμέτρου [P_kM_1] *Στο σημείο* (4) . Ο κύκλος (K_1 , K_1K_2) κόβει τον κύκλο διαμέτρου [P_kM_1] στο σημείο S_{ka} , η δε Χορδή $K S_{ka}$ κόβει τον κύκλο (O , OK) στο σημείο P_{1-2} .
4.. ***Να δειχθεί*** ,
   α) Οι κύκλοι (O_{pa} , $O_{pa}K_1$) , (O_{pk} , $O_{pk}M_1$) είναι αι ***Ορθαί Προβολαί*** του Γεωμετρικού Μηχανισμού { [$O_K K_1$ // OM_1] και η γωνία < OP_aO_k= $P_aO_kP_k$ ***σε Θέση Αντιστροφής Μεγίστου –Ελαχίστου*** }του Συστήματος των Απείρων – Αντιθέτων – Κύκλων στά Ακρότατα σημεία Καμπής .
   β) Η ευθεία [$P_{1-2}O$] είναι ο Κοινός Ακραίος - Μηχανισμός [$M_1M`_1 \perp OM`_1$] Συστήματος Ορθών ***και Αντίθετων Προβολών πέριξ ευθείας*** διερχομένης από τού κέντρου O .
   γ) Στήν περίπτωση όπου οι χορδές $K K_1$, $K K_2$ ανήκουν σε δύο συνεχόμενα ***Ζυγά*** Κανονικά Πολυγωνα ,Τότε η χορδή [$K P_{1-2}$] ανήκει στο ενδιάμεσο ***Μονό*** Κανονικό Πολυγωνο.

ΑΠΟΔΕΙΞΗ :

1.. Τά τρίγωνα $K K_1 0_k$, $K K_2 0_k$, είναι ορθογώνια ***διότι*** η υποτείνουσα $K0_k$, είναι διάμετρος του κύκλου (O ,OK) . Επειδή η γωνία < $KK_1 0_k$ = 90°, ***άρα*** και η συμπληρωματική της < $KK_1 P_k$ = 90°. Το ίδιο και γιά την γωνία < $KK_2 0_k$ πού αντιστοιχεί η γωνία < (1) K_2(2) = 90 ,

2.. Επειδή στο τετράπλευρο [(1) K_1(2) K_2] , οι έναντι γωνιαι < (1) K_1(2) = (1) K_2(2) = 90° , ***άρα*** τούτο είναι εγγράψιμο σε κύκλο .

3.. Επειδή η γωνία < (1)$K_2 K_P$ = 90 , ***άρα*** τά σημεία (1) , K_2 , K_P , είναι εγγράψιμα σε κύκλο . Το ίδιο ισχύει και δια τά σημεία (1) K_2 (3) καί τά (1) K_2(4) .

4.. Η δύναμη των σημείων P_k , P_a στον κύκλο (O ,OK) είναι οι εφαπτομένες T_{pk} , T_{pa} τού κύκλου και ίσαι μέ $T^2{}_{pk} = (P_k 0)^2 - (OK)^2$ καί $T^2{}_{pa} = (P_a 0)^2 - (OK)^2$, ***άρα*** ισχύει ,

$$(OK)^2 = (P_k 0)^2 - T^2{}_{pk} = (P_a 0)^2 - T^2{}_{pa} \qquad \ldots\ldots..(1)$$

5.. Η δύναμη των σημείων P_k , P_a στον κύκλο (K_1 ,$K_1 K_2$) είναι οι εφαπτομένες T_{pk1} , T_{pa1} τού κύκλου και ίσαι μέ $T^2{}_{pk1} = [P_k K_1]^2 - [K_1 K_2]^2$, καί $T^2{}_{pa1} = [P_a K_1]^2 - [K_1 K_2]^2$, ***άρα*** ισχύει ,

$[K_1 K_2]^2 = [P_k K_1]^2 - T^2{}_{pk1}$ καί $[K_1 K_2]^2 = [P_a K_1]^2 - T^2{}_{pa1}$, οπότε $[P_k K_1]^2 - T^2{}_{pk1} = [P_a K_1]^2 - T^2{}_{pa1}$ ή

$$T^2{}_{pa1} - T^2{}_{pk1} = [P_a K_1]^2 - [P_k K_1]^2 \qquad \ldots\ldots..(2)$$

Επειδή η Χορδή $[P_k K_1]$ του κύκλου διαμέτρου $[P_k M_1]$, είναι ίση με $[P_k K_1]^2 = [P_k M_1]^2 - [M_1 K_1]^2$ η (2) γινεται $T^2{}_{pa1} - T^2{}_{pk1} = [P_a K_1]^2 - \{[P_k M_1]^2 - [M_1 K_1]^2\} = [P_a K_1]^2 - [P_k M_1]^2 + [M_1 K_1]^2$ .... (3)

Δηλαδή η Δύναμη του Συστήματος των ***Δύο Κύκλων*** σχετίζεται με τις Εντός - Εναλλάξ Διαμέτρους των $[P_a K_1]$, $[P_k M_1]$ ***και μόνο*** , επί του Ορθογωνίου Τραπεζίου $[P_k K_1 M_1 P_a]$ ύψους $K_1 M_1$, πού αμφότεραι προβάλλωνται στο αυτό ύψος $K_1 M_1$ όπου και ο κύκλος (K_1 ,$K_1 K_2$) .

6.. Επειδή το σημείο P_a ευρίσκεται επί της $0M_1$ // $0_K K_1$, ***άρα*** όλοι οι κύκλοι διαμέτρου $[P_a M_1]$ προβάλλωνται στο σημείο M_1 , και όταν το σημείο $P_a \rightarrow \infty$, *είναι στό άπειρο* , τότε ο κύκλος (P_a , $P_a \infty$) ταυτίζεται με την χορδή $K K_1$. Επίσης το σημείο P_k ευρίσκεται επί της $0_K K_1$ // $0M_1$, ***άρα*** όλοι οι κύκλοι διαμέτρου $[P_k K_1]$ προβάλλωνται στο σημείο K_1 , και όταν το σημείο $P_k \rightarrow \infty$ *είναι στό άπειρο* , τότε ο κύκλος (P_k , $P_k \infty$) ταυτίζεται επίσης μέ την χορδή $K K_1$. ***Δηλαδή*** ,

Η χορδή $K K_1$ είναι ο ***Γεωμετρικός τόπος των άπειρων κύκλων*** επί των παραλλήλων $0M_1$, $0_K K_1$, του Τραπεζίου $[O0_k P_k P_a]$ με τις χορδές του να κόβωνται επί του κύκλου (K_1 ,$K_1 K_2$) .

7.. Η χορδή $K K_1$ περιστρεφομένη πέριξ του σημείου K , στήν ευθεία KS_{ka} , ***καθορίζει*** το κοινό σημείο S_{ka} τών κύκλων (K_1, $K_1 K_2$) καί τού κύκλου της μεγαλυτέρας διαμέτρου $[P_a K_1]$ ή $[P_k M_1]$, πού είναι ο κοινός ***Γεωμετρικός -Τόπος διάβασης σε Θέση Μεγίστου – Ελαχίστου*** (*το κρίσιμο σημείο αλλαγής*) τού Συστήματος από το Απειρο , ∞ , στην θέση $[K K_2]$, ήτοι η Θέση Μεγίστου – Ελαχίστου μεταξύ των σημείων $K_1 K_2$ οπου και γίνεται η Αντιστροφή .

Η ευθεία , $P_{1-2}O$, πού περνά από το σημείο O , είναι η Ακραία Κοινή ευθεία Ορθής Προβολής του Συστήματος του Τραπεζίου $[O0_k P_k P_a]$ μεταξύ των Χορδών $[K K_1]$, $[K K_2]$.

8.. Στήν περίπτωση όπου οι χορδές $K K_1$, $K K_2$ ανήκουν σε δύο συνεχόμενα ***Ζυγά*** Κανονικά Πολύγωνα η χορδή , $K P_{1-2}$, ανήκει στο ενδιάμεσο ***Μονό*** Κανονικό Πολυγωνο , διότι στο σημείο Αναστροφής των κύκλων , ***είναι η Θέση Μεγίστου – Ελαχίστου { Ακροτάτου σημείου Καμπής }*** , καί η Διάμετρος $P_{1-2}0$ γίνεται κάθετος της πλευράς του ,ή, η *Αντιστροφή των γωνιών πέριξ του άξονος* , P_{1-2} - $P`_{1-2}$.

ο.ε.δ.

Ακολουθούν οι διάφορες σκέψεις Προσεγγιστικές και Μη πού έγιναν .

6.3. Αι Μέθοδοι :

Προκαταρκτικά : Το Θέμα , F.16(3).

Ο τυχόν κύκλος (Ο , ΟΚ) είναι δυνατόν να χωριστεί **σε** ,

α.. Δύο ίσα μέρη από την διάμετρο ΚΑ [Είναι το Δίπολο ΑΚ] με γωνία < ΑΟΚ = 180 ° .

β.. ***Τέσσερα*** ίσα μέρη από την Διχοτόμο των 180 ° πού είναι η Κάθετη δεύτερη Διάμετρος Χ ` Χ

γ.. ***Οκτώ*** ίσα μέρη από την Διχοτόμο των τεσσάρων γωνιών πού είναι 90 ° .

δ.. ***Δεκαέξει*** ίσα μέρη από την Διχοτόμο των Οκτώ γωνιών πού είναι 45 ° και ούτω καθ` εξής .

ε.. Ο κύκλος έχων 360 ° = 2π ακτίνια δύναται να χωριστεί σε ,

Τρία ίσα μέρη 360° / 3 = 120° πού είναι δυνατό [Το Ισόπλευρο τρίγωνο] ,

Έξη ίσα μέρη 360° / 6 = 60° πού είναι δυνατό με τις διχοτόμους του τριγώνου [Το Κανονικό Εξάγωνο] ,

Δώδεκα ίσα μέρη 360° / 12 = 30° πού είναι δυνατό με τις διχοτόμους του Εξαγώνου [Το Κανονικό Δωδεκάγωνο] , και ούτω καθ` εξής σε 15° , 7,5° ……...

Παρατήρηση .

α… Η σειρά των Ζυγών αριθμών είναι 2 , 4 ,6 , 8 ,10 ,12 ,14 ,16 ,18 ,20 ,…………….

Η σειρά των Μονών αριθμών είναι 1 ,3 ,5 ,7,9 ,11 ,13 ,15 ,17,19 ,21 , ……………

προερχομένη από το ημι-άθροισμα του Προηγούμενου και του Επόμενου Ζυγού αριθμού π.χ.

Ο αριθμός $5 = \frac{4+6}{2} = \frac{10}{2} = 5$. Η Λογική της Πρόσθεσης ισχύει και στην Γεωμετρία αλλά στα δικά της πλαίσια πού είναι η Λογική του Υλικού – Σημείου , δηλαδή το Μηδέν (0 = Τίποτα) Υπάρχει ως άθροισμα του Θετικού + Αρνητικού [ίδε , Υλική Γεωμετρία 58-60-61]

β… Στην άνω παράγραφο 5.5(Case c) απεδείχθη η σχέση (1) Σ(h) = (2k) . h = n .h = n .OK , *όπου*

Σ = Το άθροισμα των Υψών , των Κορυφών του Κανονικού (n) – Πολυγώνου ,

από των Κορυφών K_n , μέχρι της εφαπτομένης (e) στο σημείο Κ ,

h = ΟΚ , Το ύψος του κέντρου Ο από την (e) ,

n = Ο αριθμός των Πλευρών του Κανονικού – Πολυγώνου , …… *και πού*

Μετατρέπει το Άθροισμα των Υψών από της Εφαπτομένης (e) σε πολλαπλάσιο αριθμό της ακτίνας του κύκλου , πού ***σχετίζεται άμεσα με τις γωνίες*** $\boldsymbol{\varphi_n}$ **,** και τις κορυφές των πλευρών , KK_n .

γ... Εις τυχούσα Χορδή KK_1 του κύκλου (Ο , ΟΚ) , η Κεντρική γωνία < KOK_1 , είναι διπλάσια της Εγγεγραμμένης της και η γωνία < $K O_K K_1 = KOM_1$. Η Μεσοκάθετος OM_1 είναι παράλληλος της Καθέτου $O_K K_1$, άρα τέμνονται στο άπειρο (∞) . Επειδή δε αι δύο Κάθετοι περνούν από τα σημεία Ο και O_K , αυτά αποτελούν τους Πόλους περιστροφής των .

Εις το Σχήμα F.18 – A **, το τυχόν Σημείο K_2** , επί του κύκλου , σχηματίζει την δεύτερη Χορδή **KK_2**

η δε Κάθετος $O_K K_2$ προεκτεινόμενη κόβει την OM_1 , *παράλληλο της* $O_K K_1$, σε ένα σημείο P_1 πού είναι ο Πόλος -Σχηματισμού των δύο Χορδών , ή , γωνιών .
Το γιατί είναι διότι το σημείο P_2 κινείται επί της OM_1 από το άπειρο μέχρι της διαμέτρου KP_1 .
Επί της διαμέτρου KP_2 του κύκλου (O_2 , $O_2P_2 = O_2K$) , και με κέντρο το O_2 , *Σχηματίζονται* οι ίδιες γωνίες φ_1 , φ_2 από τις Χορδές P_1M_1 , P_2K_2 , ώστε η γωνία < $M_1P_1K_2 = K_1KK_2 = OP_1O_k$

Δηλαδή , ***Σε δύο Χορδές , KK_1 , KK_2 , κύκλου (O , OK) , κοινής κορυφής K , η Μεσοκάθετος OM_1 της πρώτης , και η Κάθετος $O_K K_2$ της δεύτερης , κόβονται σε ένα σημείο P_1 πού σχηματίζει τον κύκλο (O`$_1$ , O`$_1P_1$) πού είναι ο Συζυγής του Κύκλος ,*** { είναι ο κύκλος των Ίσων-Γωνιών με τον κύκλο (O ,OK) } . ***Το ίδιο και με τον κύκλο (O_2,O_2P_2= O_2K).***

δ... Από την σχέση Σ = (2k) . h = n .h = n .OK , διά n = 2 τότε Σ = 2.h = 2 .OK δηλαδή η διάμετρος KO_K . Διά n = 3 τότε Σ = 3.h = 3 .OK και n = 4 τότε Σ = 4.h = 4 .OK . Επειδή οι Μονοί αριθμοί είναι ο Αριθμητικός - Μέσος των δύο γειτονικών Ζυγών άρα και το 3.OK είναι (2.OK +4.OK)/2 .
Η διαφορά των υψών είναι $\Delta h = h_{K1} - h_{K2} = K_1K`_1$ και μεταξύ των παραλλήλων των σημείων $K_1$ , $K_2$ , και της (e) . Ο κύκλος ( $K_1$ , $K_1K`_1$) είναι ο Κύκλος των *Υψομετρικών-Διαφορών* των Χορδών K K_1 , K K_2 , και μεταβάλλεται ανάλογα με το σημείο $K`_1$ ή το ίδιο με το K_2 . Δηλαδή ,

Ο Κύκλος των Υψομετρικών - Διαφορών (K_1 , $K_1K`_1$ ) αλληλοσχετίζεται με τις Χορδές , [ $KK_1$ ,$KK_2$ ] , [ $O_KK_1$ ,$O_KK_2$ ] τού κύκλου ( O , OK ) μέσω των αντίστοιχων κορυφών K , $O_K$ και με τον Κύκλο – Ίσων Γωνιών ( O`$_1$,O`$_1P_1$) μέσω της Μεσοκαθέτου OM_1 της πρώτης Χορδής K K_1 , και της Καθέτου O_KK_2 της δεύτερης Χορδής KK_2 .

Αυτός ο Αλληλοσχηματισμός των Τεσσάρων κύκλων ,

{ (O , OK) - (K_1 , $K_1K`_1$ ) - ( O`$_1$, O`$_1P_1$) - (O_2 , O_2P_2) }

καθέτων προς την εφαπτομένη (e) , επιτρέπει , ***Στον οποιονδήποτε κύκλο*** (O , OK) , να καθορίσει μέσω των Δύο Χορδών K K_1 , K K_2 , και γωνιών φ_1 , φ_2 , ***την μεταξύ των κίνηση*** , *ήτοι* ***Από την σχέση*** αθροίσματος των Υψών Σ = (2k) . h = n .h = n .OK , προκύπτει ότι το Άθροισμα των Υψών δύο συνεχομένων Κανονικών - Πολυγώνων n , n+2 είναι → $\frac{\Sigma\ 2(h1)}{2} + \frac{\Sigma\ 2(h2)}{2} =$
$[\frac{n_1}{2} + \frac{n_2}{2}]$.OK = $[\frac{n_1 + n_2}{2}]$.OK = n_3 .OK , όπου $n_3 = [\frac{n_1 + n_2}{2}]$ είναι ο Αριθμός των Κορυφών του μεταξύ των δύο Ζυγών n_1 , n_2 , ***Μονού – Αριθμού - Κορυφών*** του Κανονικού-Πολυγώνου .
Επί της Υψομετρικής –Διαφοράς $\Delta h = O_1K`_1$ καθέτου της (e) διατηρούνται οι ιδιότητες Άθροισης .
Από την ταυτόχρονο θέση των γωνιών φ_1 , φ_2 , **στους** δύο κύκλους ορίζονται και οι χορδές .

ε... Επειδή αι K K_1 , K K_2 , είναι κάθετοι των OP_1 ,O_KP_1 , ***άρα το σημείο K είναι το Ορθόκεντρο*** όλων των καθέτων των τριγώνων από τούτου , καθώς και της κοινής χορδής των δύο κύκλων (O_2 , O_2P_2) , (O , OK) . Επειδή δε ο ***Γεωμετρικός -Τόπος*** των Χορδών K K_1 , K K_2 , ***του Κοινού Ορθοκέντρου K είναι*** → για τον κύκλο (O ,OK) το τόξο K_1K_2 , για τον κύκλο (O_2 ,$O_2K=O_2P_2$) το τόξο M_1 K_2 , και για τον κύκλο (O_1 , $O_1P`_1$) το τόξο (1)-(2) με τα σημεία τομής των χορδών , **ΑΡΑ** τα σημεία (1) , M_1 είναι τα Ακραία σημεία των κύκλων τούτων ώστε να είναι K $M_1 \perp P_1M_1$.

Αι ανωτέρω δύο λογικές καταλήγουν στη ***Μηχανική και Γεωμετρική λύση*** πού ακολουθεί .

Η κατά προσέγγιση Μηχανική Απόδειξη :

Εις το σχήμα F. 18 - A. , έστω κύκλος (O , OK) με την ευθεία (e) εφαπτομένη στο σημείο, K , και την $K\,O_K$ διάμετρο του κύκλου .

Ορίζουμε επί του κύκλου και από της αρχής , K , τις Κορυφές K_1 , K_2 να αντιστοιχούν σε άκρα πλευρών ***Ζυγών - Κανονικών – Πολυγώνων*** και τις αντίστοιχες γωνίες των , φ_1 , φ_2 , μεταξύ των πλευρών $K\,K_1$, $K\,K_2$, και της εφαπτομένης (e) .

Φέρομεν από των σημείων K_1 , K_2 , τας παραλλήλους προς την (e) από δε της Κορυφής K_1 κάθετο προς την (e) πού να τέμνει την παράλληλο από του σημείου K_2 , στο σημείο $K`_1$ , και εν συνεχεία φέρομεν την κάθετο $K_1K`_1$ ως ακτίνα τού Κύκλου (K_1 , $K_1K`_1$) .

Φέρομεν την $O_K\,K_1$ πού προεκτεινόμενη τέμνει την OK_2 προεκτεινόμενη (από το σημείο O) στο σημείο P_2 από δε του O_2 (μέσου της διαμέτρου $K\,P_2$), φέρομεν τον κύκλο (O_2 ,$O_2\,K = O_2P_2$).

Προεκτείνομεν τις πλευρές O_kK_1 , O_kK_2 , ώστε να κόβουν τον κύκλο (O_1 , $O_1K`_1$ ) στα σημεία 1 , 1`, και 2 , 2`, αντίστοιχα και εν συνεχεία φέρομεν τις εναλλάξ χορδές 1 - 2` και 2 -1` .

Ορίζουμε το κοινό σημείο , T , των χορδών 1 - 2` και 2 - 1` και προεκτείνουμε την , O_kT, ώστε να κόβει τον κύκλο (O , OK) στο σημείο K_5 . Ή , με τον Αρμονικό - Μέσο

Φέρομεν από τού σημείου $K`_1$ κάθετο , $K`_1A =(K`_1K_1)/2$ και τον κύκλο ( A , $AK`_1$) ώστε να κόβει την χορδή O_1A στο σημείο B . Φέρομεν από το K_1 τον κύκλο (K_1 ,K_1B) ώστε να κόβει την κάθετο $K_1K`_1$ στο σημείο , C , από δε του σημείου C παράλληλο της (e) ώστε να κόβει τον κύκλο (O, OK) στο σημείο K_5 . ***Η χορδή* K K_5 *είναι η πλευρά του Μονού – Κανονικού - Πολυγώνου*** , διότι ,

O κύκλος (O_4 ,$O_4\,K = O_4$ O) είναι ο κύκλος των μέσον των χορδών KK_1 , KK_2 ***Άρα*** και της KK_5.
Οι γωνίες $< KM_1O_2 = KM_2O`_1 = 90^o$, $< KM_1P_1 = KM_1O = 90^o$, $< K\,K_2P_1 = KK_2O_\kappa = 90^o$,
Άρα το σημείο K είναι το Ορθόκεντρο των τριγώνων KOM_2 , KOP_1 , KO_kP_2 , $K\,O_kO_1$.
Οι γωνίες $<K_1KK_2$, $K_1O_k\,K_2$, OP_1O_k , OP_2O_k , P_2OP_1 είναι ίσες μεταξύ των ,

Διότι Είναι α) Εγγεγραμμένες στο ίδιο τόξο , K_1K_2 , τού κύκλου (O, OK) ,
β) Οι πλευρές των P_1M_1 , P_1K_2 , κάθετες των KK_1 , KK_2 ευρίσκονται εντός του κύκλου ($O`_1$ , $O`_1\,K = O`_1\,P_1$) ,
γ) Εντός εναλλάξ μεταξύ των δύο παραλλήλων , OP_1 , και O_kP_2 των κύκλων (O_4 , $O_4\,K = O_4$ O) , (O_2 , $O_2\,K = O_2\,P_2$) .

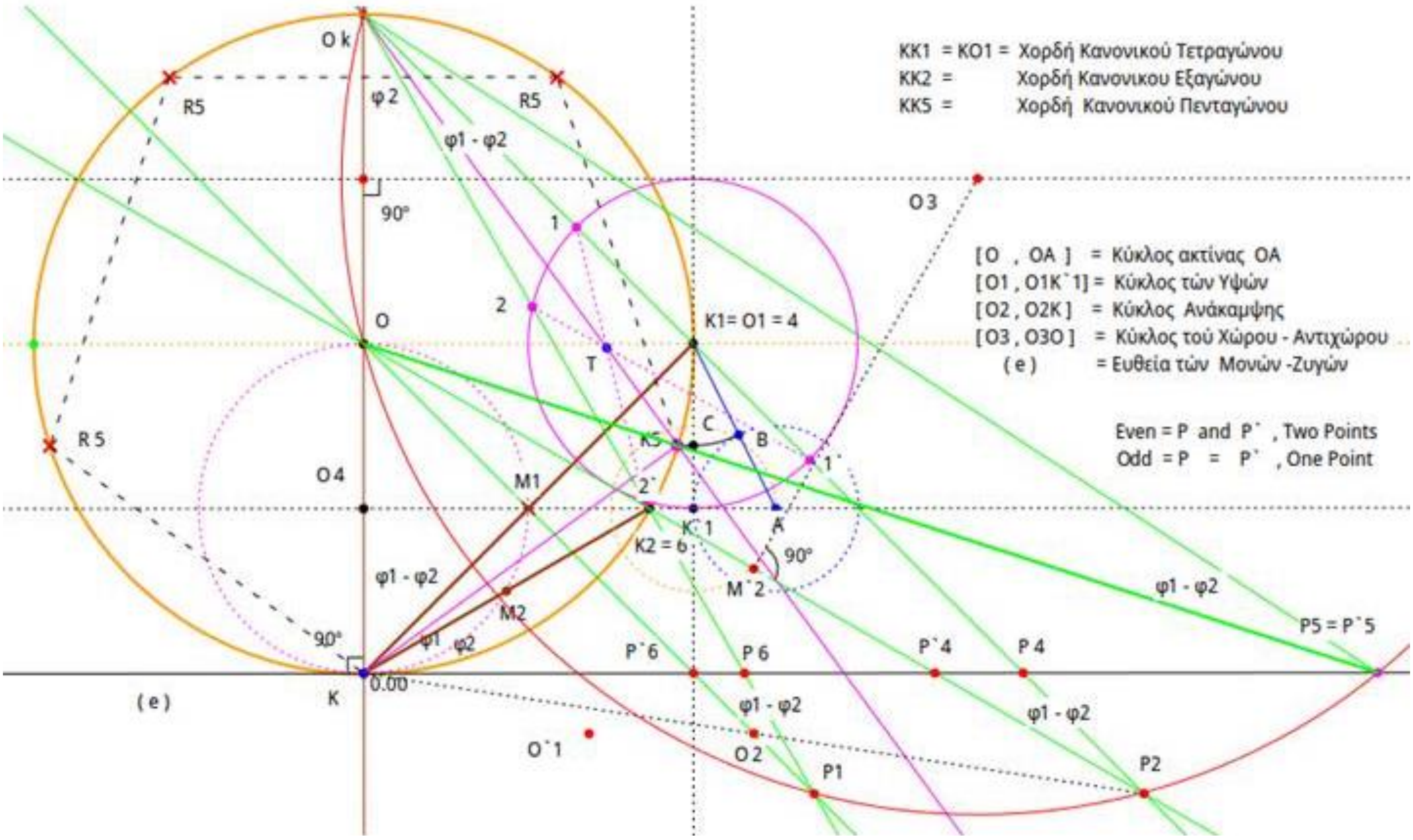
Οι Χορδές O_kK_1 , OM_1 είναι κάθετοι της χορδής KK_1 , ***Άρα*** είναι παράλληλοι ,
Οι Χορδές O_kK_2 , OM_2 είναι κάθετοι της χορδής KK_2 , ***Άρα*** είναι παράλληλοι ,
Ο Γεωμετρικός Τόπος του σημείου K_1 , ***από του Σημείου K_1 προς K_2*** , στο κύκλο (O , OK) είναι το τόξο K_1K_2 του κύκλου , ***ενώ*** επί του κύκλου (O_1 ,$O_1K`_1$ ) το τόξο 1 , 2` του κύκλου.
Ο Γεωμετρικός Τόπος του σημείου K_2 , ***από του Σημείου K_2 προς K_1*** , στο κύκλο (O , OK) είναι το τόξο K_2K_1 του κύκλου , ***ενώ*** επί του κύκλου (O_1 ,$O_1K`_1$ ) το τόξο 2 , 1` του κύκλου.
Ο Γεωμετρικός Τόπος από του σημείου , O, των παραλλήλων της Χορδής O_kO_1 , είναι οι Χορδές OP_1 , $O_4O`_1$ , ***από δε τού πόλου*** , $O_k$ , η τομή , T , των χορδών 1 , 2` και 2 , 1` αντίστοιχα .

Επειδή δε η γωνία $< O_kO_1K = O_kK_2K = 90^o$, ***Άρα*** η τομή , T , κινείται παράλληλα της O_1K , και είναι το κοινό σημείο των δύο ***Γεωμετρικών Τόπων*** .

Επειδή τα σημεία $\mathrm{K_1}$, $\mathrm{K_2}$ ***είναι οι Διαδοχικές Κορυφές των Χορδών - Ζυγών – Κανονικών - Πολυγώνων του κύκλου*** (O , OK) ***, και συνάμα τα σημεία*** $\mathbf{O_1}$ **,** $\mathbf{P_2}$ ***, οι αντίστοιχοι Ακραίοι πόλοι επί των κύκλων*** (O_1 , $O_1K`_1$) , (O_2 ,O_2 K) , πού ακολουθούν την ΚΟΙΝΗ δέσμευση του σημείου K , να είναι **Ορθόκεντρο και Αρχή των Πολυγώνων** και το σημείο , T , **ο σταθερός κοινός πόλος του συστήματος , ΑΡΑ** η ευθεία O_kT , είναι σταθερά και κόβει τον (O , OK) , ***στο σημείο*** $\boldsymbol{K_5}$ ***πού είναι η Κορυφή του Ενδιάμεσου Μονού –Κανονικού –Πολυγώνου ?? ,*** Ή

Επειδή , από την Αρμονική σχέση* (1) *και* (4)** ($K_1K`_1$) ² = ( $K_1C$ ). ( $K_1C + K_1K`_1$) ***ορίζεται το Αρμονικό ύψος $\mathbf{K_1C}$ ***και με την παράλληλο χορδή*** CK_5 , **το σημείο** $\mathbf{K_5}$ επί **τού κύκλου ,** (O , OK) ώστε να ***αντιστοιχεί η ανωτέρω Αρμονική σχέση , ΑΡΑ και η χορδή*** KK_5 *είναι επίσης του* ***Ενδιάμεσου Μονού –Κανονικού –Πολυγώνου .*** ***ο.ε.δ.***

Μάρκος , 5/5/2017



F.18 – A → Στον κύκλο (O , OK) , για **n = 4** , η Χορδή KK_1 είναι η πλευρά του Ζυγού-Κανονικού Τετραγώνου ενώ για , **n = n + 2 = 6** , η KK_2 είναι η πλευρά του Ζυγού - Κανονικού Εξαγώνου η δε Χορδή $\mathbf{KK_5}$ του ***Κανονικού – Μονού - Πενταγώνου*** .

Ο κύκλος (O_1 , $O_1K`_1$ ) είναι ο , ***κύκλος Καμπής*** , των Υψομετρικών Διαφορών με $\Delta h = h_{K1} - h_{K2} = K_1K`_1$,ο δε κύκλος (O_2, O_2P_2) είναι ο , ***κύκλος Ανακάμψεως ,*** [Euler-Savary].

Ο κύκλος (O_4 , $O_4K = O_4O$) είναι ο , ***κύκλος των Μέσων των Χορδών*** ,από του σημείου K .

Οι χορδές 1 , 2` και 2 , 1` κόβονται στο σημείο C , πού είναι το Σταθερό σημείο στις Περιβάλλουσες επί της παραλλήλου της $K O_1$ από του σημείου , C , και με κέντρο Καμπυλότητας το άπειρο , ∞ . Επειδή δε ο κύκλος των Υψομετρικών Διαφορών $[K_1 , K_1K`_1]$ είναι και Προβολή τού Κύκλου Ταχυτήτων $[K_1 , K_1K_2]$ πού είναι και κύκλος Καμπής , με κοινό το σημείο K_1 κέντρου Καμπυλότητας στο άπειρο ,∞, ***Άρα όλες οι Γεωμετρικές - Ιδιότητες των δύο Κύκλων είναι Κοινές .***

Πρώτη Προσεγγιστική Γεωμετρική Απόδειξη :

Επειδή οι πλευρές P_1O_k , P_1O είναι κάθετοι των , $K K_2$, $K K_1$ αντίστοιχα , ***Άρα*** η γωνία < $OP_1O_k = K_1KK_2$, και επειδή η P_2O , είναι χορδή μεταξύ των παραλλήλων P_1O , P_2O_k , ***Άρα*** και οι γωνίες < OP_1O_k , OP_2O_k , είναι ίσες , τόσον επί των Σταθερών πόλων , *κορυφών* , O , O_k , όσον και των κινουμένων πόλων , *των κορυφών* , P_1 , P_2 .

Επειδή οι γωνίες OP_1O_k , OP_2O_k , είναι ίσες ***Άρα*** βαίνουν επί κύκλου χορδής $O O_k$. Επειδή δε επί του ιδίου κύκλου βαίνουν οι πόλοι , O_k , O , P_1 , P_2 , ***Άρα*** το κέντρο του κύκλου τούτου ευρίσκεται ως τομή της Μεσοκαθέτου των χορδών αυτών , $O O_k$ και $O P_2$, και πού είναι το σημείο O_3 .

Το σημείο K , της ευθείας (e) είναι κοινό των Άπειρων (∞) Κανονικών - Πολυγώνων των κύκλων κέντρου O και με ακτίνα KO = 0 → ∞ , ***Άρα*** το Άπειρο - Κανονικό - Πολύγωνο είναι η ευθεία (e) το Κανονικό - Πολύγωνο του κύκλου (O , OK) είναι το ζητούμενο , το δε Μηδενικό – Κανονικό – Πολύγωνο το σημείο K .

Επειδή δε οι κινούμενοι πόλοι P_1 , P_2 , *των δύο Ζυγών Κανονικών Πολυγώνων* , ευρίσκονται επί του κύκλου [O_3 , O_3 O] , ***κύκλος του Αντιχώρου*** , [12] , ***Άρα*** ο ενδιάμεσος Κινούμενος πόλος του Μονού - Κανονικού – Πολυγώνου , *περνά από το* , ∞ , *πού είναι η τομή της ευθείας* (e) *και του κύκλου τούτου* , πού είναι το κοινό σημείο P_5 . Το ίδιο παρουσιάζεται και με την γωνία των 90^o πού συμβαίνει με δύο κάθετες ευθείες οι οποίες περνούν από το άπειρο .

Η χορδή OP_5 αντιστοιχεί στην Ανακαμπτομένη χορδή των κύκλων Ανακάμψεως [O_2 ,O_2P_2] στο άπειρο πού είναι το σημείο P_5 . Τα Δύο - ζεύγη των τομών P_4 , $P`_4$ και $P_6$ , $P`_6$, συγκλίνουν στο Ένα- Ζεύγος με ένα σημείο $P_5 = P`_5$, όπου τα δύο σημεία συμπίπτουν . ***ο.ε.δ.***

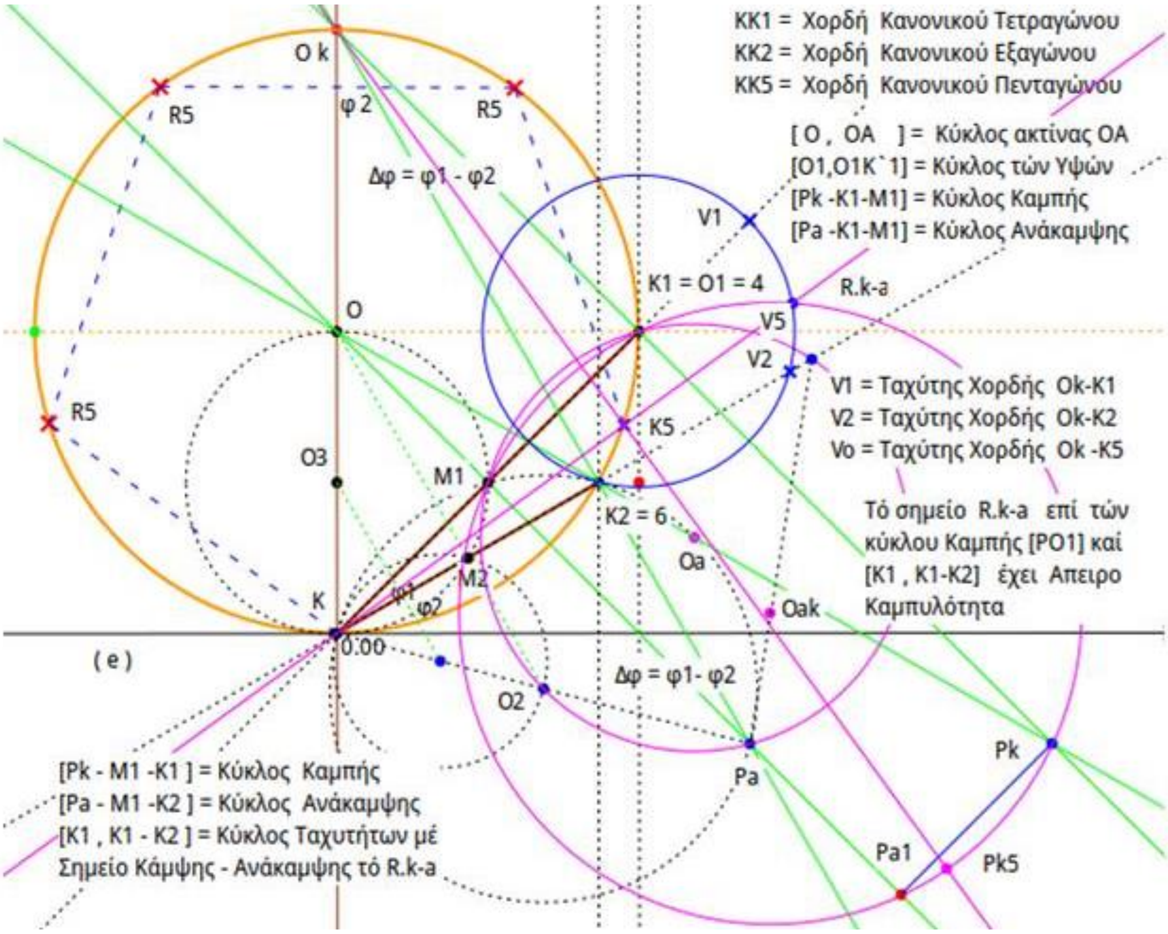
Παρατήρηση .

Η ανωτέρω Γεωμετρική Απόδειξη επιλύει μερικώς το πρόβλημα των Κανονικών – Πολυγώνων παρακάμπτοντας τούς μέχρι σήμερα περιορισμούς στην Αλγεβρική - θεωρία των Πρώτων προς αλλήλους αριθμούς . Στο σχήμα F16.(3) είναι OX ⊥ OA δηλαδή η γωνία < XOK = 90^o . Τυχούσα γωνία XOC < XOA < 90^o ισούται με την συμμετρική της $X`OC_1$ , εφόσον περάσει από την θέση OA όπου η γωνία < XOA = X `OA = 90^o και η πλευρά OC περνά από το άπειρο .

Στο σχήμα 18-B , λόγω του ότι οι χορδές O_kK_1 , O_kK_2 , είναι κάθετες των KK_1 , KK_2 , άρα και η γωνία < $K_1O_kK_2 = K_1K K_2$. Η αλλαγή της θέσης των καθέτων από του νέου κέντρου O , σχηματίζει την Αντισυμμετρική γωνία OP_aO_k ίση με τις άλλες εφόσον περάσει μία κάθετος παράλληλος της KK_2 από το άπειρο . Επειδή η Αντισυμμετρική γωνία βαίνει στη χορδή OO_k των δύο σταθερών κορυφών σχηματισμού των γωνιών , οι κύκλοι πού περνούν από τα σημεία K , K_2, P_a , είναι οι ***Κύκλοι Ανάκαμψης*** , λόγω του ότι οι σταθερές περιβάλλουσες KK_1, KK_2, KK_i όλων των πλευρών αυτού του Συστήματος των γωνιών Ανακάμπτονται στα σημεία συνάντησης των με κοινό το K_1 τού κύκλου , οι δε κύκλοι από τα σημεία K , K_1, P_k , είναι οι , ***Κύκλοι Καμπής ,*** πού αντιστρέφουν τις γωνίες των κύκλων Ανάκαμψης σε , *Εντός-Εναλλάξ ίσες γωνίες όπως είναι* < $OP_aO_k = OP_kO_k$ *επί των παραλλήλων* O_k ,OP_a .

Έτσι προκύπτει η Ακριβής Γεωμετρική Επίλυση τών Κανονικών - Μονών - Πολυγώνων .

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΠΕΝΤΑΓΩΝΟΥ



F.18 –B → Στον κύκλο (O , OK) , για **n = 4** , η Χορδή $K K_1$ είναι η πλευρά του Ζυγού -Κανονικού Τετραγώνου ενώ για , **n = n + 2 = 6** , η $K K_2$ είναι η πλευρά του Ζυγού - Κανονικού Εξαγώνου η δε Χορδή $\mathbf{K K_5}$ του ***Κανονικού – Μονού - Πενταγώνου*** .

Ο κύκλος (O_1 , $O_1K`_1$ ) είναι ένας , ***κύκλος Καμπής*** , των Υψομετρικών Διαφορών με Δh = $h_{K1}$- $h_{K2}$ = $K_1K`_1$,δε κύκλος (O_a, O_aP_a) είναι ο , ***κύκλος Ανακάμψεως ,*** [Euler-Savary] , και ο κύκλος [PO_1, PO_1P_k=PO_1M_1] είναι ο , ***Κύκλος Καμπής*** , από του σημείου O_k.

Η γωνία < $P_kO_kP_a$ = OP_aO_k είναι μία Περιβάλλουσα επί των κύκλων - Καμπής , η δε γωνία < OP_aO_k η αντίστοιχη Περιβάλλουσα επί των κύκλων Ανάκαμψης .

Το εγγεγραμμένο σχήμα $P_kK_1M_1$ P_{a1} , *εντός του Κύκλου Καμπής* , είναι ορθογώνιο διότι η γωνία < $P_kK_1M_1$ = $K_1M_1P_{a1}$ = 90 ° , Άρα και η χορδή P_kP_{a1}// K_1M_1 . Επειδή δε η γωνία < OP_kO_k έχει την πλευρά OP_k , μεταξύ των παραλλήλων πλευρών O_kP_k , OP_a , άρα είναι ίση με την Εντός - Εναλλάξ < $P_kO_kP_a$. Η γωνία < $P_kO_kP_{k5}$ = $OP_\infty O_k$ στην θέση O_kP_∞ όπου το σημείο P_∞ ευρίσκεται επί της παραλλήλου OP_a . ***Δηλαδή*** , F.18 - B ,

Στο σημείο $\mathbf{P_\infty}$ γίνεται η Αντιστροφή των γωνιών σε Εντός - Εναλλάξ μεταξύ των σημείων $\mathbf{P_k}$ του Κύκλου - Καμπής , και $\mathbf{P_a}$ του Κύκλου – Ανάκαμψης , αλλά Πού ??

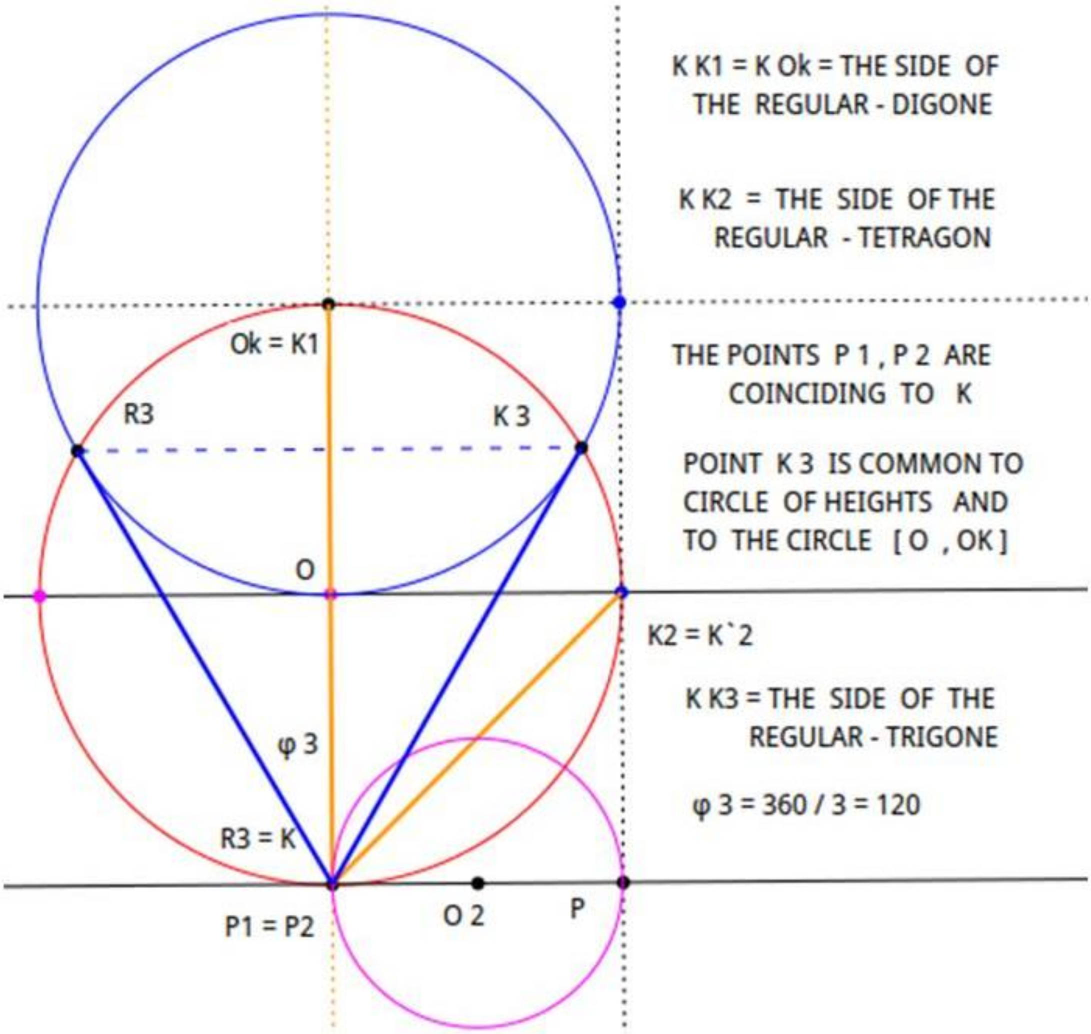
Να δειχθεί ότι ο κύκλος [$\mathbf{K_1}$, $\mathbf{K_1K_2}$] *Ταχυτήτων* ***, διέρχεται διά του Κύκλου-Καμπής .***

Φέρομεν τον κύκλο [K_1, K_1K_2] πού τον ονομάζουμε , *Κύκλο - Ταχυτήτων* , του σημείου K_1 , και τούτο διότι το σημείο K_1 κινούμενο επί του κύκλου [O ,OK] κατευθύνεται ακαριαία στο σημείο K_2 με ταχύτητα το μέγεθος , K_1K_2 . Από την θεωρία του Κέντρου Καμπυλότητας (Euler–Savary) η ταχύτης V_1 του σημείου K_1 , στρεφομένου πέριξ του σημείου O_k είναι ίση με $\overline{V_1} = K_1K_2$ και κάθετος της O_kK_1 , του δε σημείου K_2 στρεφομένου πέριξ του ιδίου πόλου O_k είναι $\overline{V_2} = K_1K_2$ και κάθετος της O_kK_2 , *δηλαδή* , Οι τροχιές των σημείων του κύκλου [K_1, K_1K_2] έχουν τα κέντρα καμπυλότητας των επί του κύκλου διαμέτρου $K\,O_k$, η δε κατεύθυνση των ταχυτήτων των σημείων K_1 , K_2 του κύκλου [K_1, K_1K_2] ευρίσκονται επί των καθέτων χορδών KK_1 , KK_2 αντίστοιχα .

Όταν όμως το σημείο K_1 κινείται επί της χορδής K_1K , τότε το Κέντρο καμπυλότητας αρχίζει από το σημείο P_k , κινείται επί της O_kP_k και κατευθύνεται προς το άπειρο ∞ σχηματίζοντας έτσι την Περιβάλλουσα των Κύκλων - Καμπής , όπου και Αντιστρέφεται η κίνηση προς τα πίσω όπως τούτο συμβαίνει σε γωνίες 90^o μεταξύ δύο καθέτων . Για να φτάσει το σημείο K_1 στη θέσητου σημείου M_1 από το άπειρο της ευθείας OP_a στο σημείο P_a , σχηματίζοντας έτσι την Περιβάλλουσα των Κύκλων – Ανάκαμψης περνά και από ένα Κοινό σημείο των δύο κύκλων το , R_{k-a} , πού είναι τέτοιο ώστε οι Εντός-Εναλλάξ γωνίες πού είναι ίσες ,να είναι και επί των πόλων K , O_k , και πού είναι στην θέση K_5 . Επειδή η Διάμετρος από τες Κορυφές K_1 , K_2 ***περνά από Κορυφές των , n , και , n+2 , Ζυγών Κανονικών Πολυγώνων , η δε Διάμετρος από την Κορυφή τού*** $K_{5=n+1}$ ***περνά από το μέσο M_5 τής έναντι Χορδής Άρα είναι και Μεσοκάθετος της , Δηλαδή περνά από Σημεία Καμπής σε Σημείο Ανάκαμψης όπως τούτο συμβαίνει και στους τρείς ανάλογους Κύκλους .***

Ο κύκλος (PO_1 , $PO_1\,K_1 = PO_1\,P_k = PO_1\,M_1$) είναι ο Οριακός – Κύκλος - Καμπής πού περνά από τα σημεία K_1, M_1 , P_k , ο δε κύκλος (O_a , $O_aK_1 = O_aP_a = O_a\,M_1$) είναι ο Οριακός -κύκλος -Ανάκαμψης πού περνά από τα σημεία K_1 , M_1 , P_a . Το σημείο K_1 με ταχύτητα V_1 επί τού κύκλου ταχυτήτων κινείται επί του κύκλου ταχυτήτων μέχρι του σημείου K_2 και με ταχύτητα $V_1 \rightarrow V_2$.
Επειδή η καμπύλη Κίνησης , *η Τροχιά* , του σημείου K_1 είναι η ευθεία , KK_1 μέχρι το Άπειρο , πού είναι και η Σταθερά περιβάλλουσα , Άρα το σημείο K_1 είναι και το αντίστοιχο κέντρο - καμπυλότητας της KK_1 , και οι τροχιές των , καθώς επίσης και ο κύκλος των ταχυτήτων των , έχουν το αντίστοιχο κέντρο καμπυλότητας στο άπειρο .
Το άκρο του Βέλους V_1 (*η αιχμή του* V_1) , διαγράφει κατά την στιγμήν αυτήν τροχιά παρουσιάζουσα Καμπή , ***Άρα η Αιχμή του V_1 διέρχεται διά του Κύκλου - Καμπής . ο.ε.δ.***
Επειδή επί της O_kK_1 άπειροι κύκλοι περνούν από το σημείο K_1 , ***Άρα είναι οι Οριακοί Κύκλοι – Ανάκαμψης Διαμέτρου τά τμήματα*** K_1P_k από τού σημείου $K_1 \rightarrow P_k \rightarrow \infty$ **.**
Το ίδιο συμβαίνει και με την Αιχμή του V_2 του σημείου K_2 , και τους Κύκλους Ανάκαμψης από τό K_2.
Επειδή δε ισχύει η σχέση των Υψών , **Σ = n .OK ,** και στα Μονά , **n+1** , Κανονικά Πολύγωνα η Διάμετρος από την Κορυφή , K , είναι κάθετος της έναντι πλευράς , Άρα πρέπει να υπάρχει ένα τέτοιο *Κοινό σημείο και στις Περιβάλλουσες* , πού είναι πράγματι το σημείο R_{k-a} .
Εις την περίπτωση πού , ο Οριακός - Κύκλος - Καμπής (PO_1 , $PO_{1-}K_1 = PO_{1-}P_k = PO_{1-}M_1$) τέμνει τον άξονα OO_k τότε το σημείο R_{k-a} , Αντιστρέφεται και κινείται επί του άξονος OP_k .

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΤΡΙΓΩΝΟΥ



F.19 → Στον κύκλο (O , OK) , για **n = 2** , η Χορδή KK_1 , είναι η πλευρά του Ζυγού -Κανονικού Διγώνου ενώ για , **n = n+2 = 4** , η KK_2 είναι η πλευρά του Ζυγού - Κανονικού – Τετραγώνου , η δε Χορδή $\mathbf{KK_3}$ του ***Κανονικού – Μονού – Τριγώνου.***

For n = 2 , then KK_1 is the Side of the Regular - Digone and equal to 2.OK..

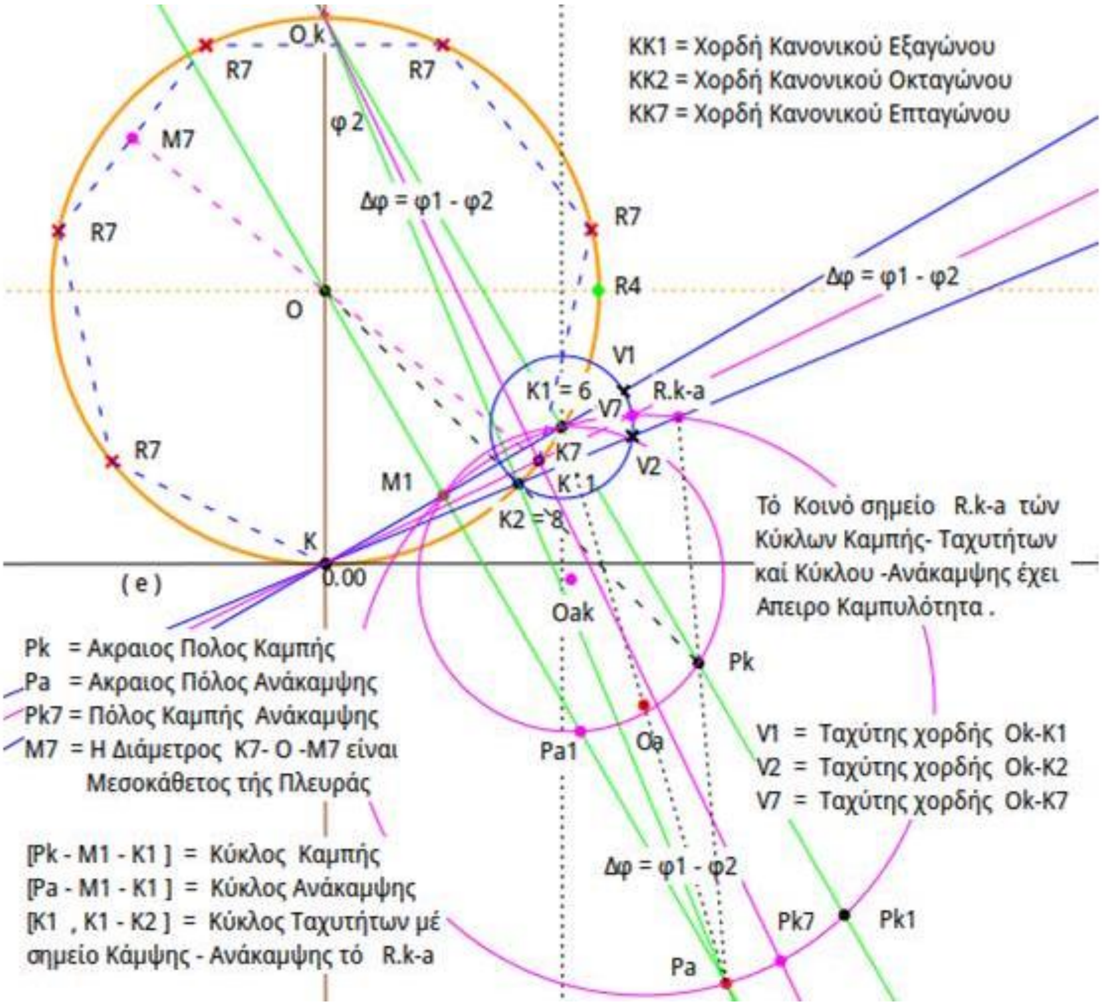
For n = n+2 = 4 , then KK_2 is the Side of the Regular –Tetragon and equal to $OK.\sqrt{2}$, the point K_2 on (O , OK) circle . Exist $\Delta h = h_{K1} - h_{K2} = O_k O$.

The Circle of Heights is (K_1 , $K_1 O$) . The Coupler - Circle is (O_2 , $O_2 P$) ,

Points P_1 , P_2 are the intersections of Sides KK_1 , KK_2 produced .

Point K_3 is the intersection of $P_2 O_k$ Segment , and the circle (O , OK) .

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΕΠΤΑΓΩΝΟΥ

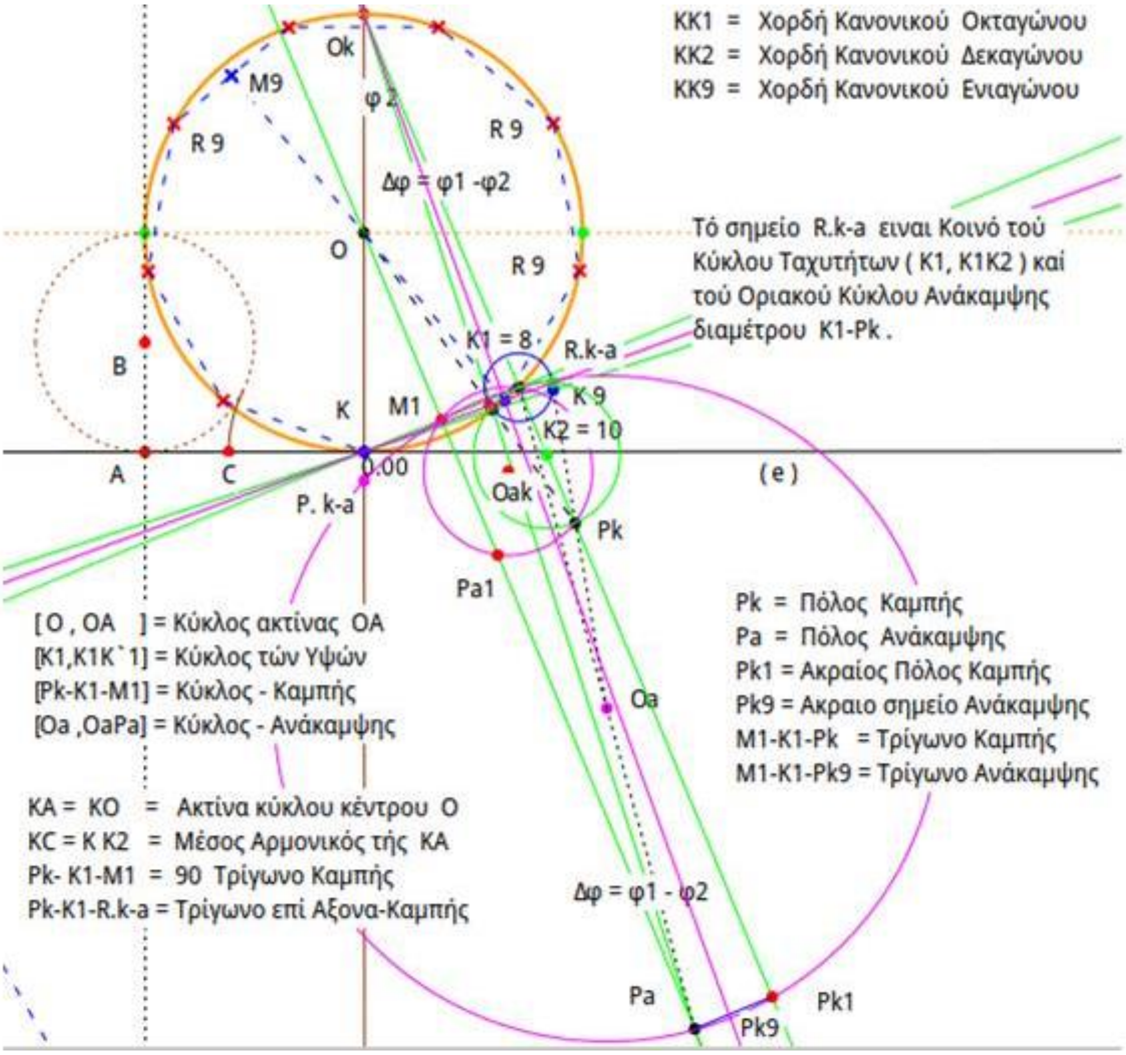


F.20 → Στον κύκλο (Ο , ΟΚ) , για **n = 6** , η Χορδή KK_1 είναι η πλευρά του Ζυγού -Κανονικού Εξαγώνου ενώ για , **n = n + 2 = 8** , η χορδή KK_2 είναι η πλευρά του Ζυγού -Κανονικού Οκταγώνου , η δε Χορδή **KK_7** του ***Κανονικού – Μονού – Επταγώνου .*** →

Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1P_a$, *εντός του Κύκλου Ανάκαμψης* , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία $< P_{k1}K_1M_1 = K_1M_1P_a = 90^{o}$, Άρα και η χορδή $P_{k1}P_a // K_1M_1$ η δε γωνία $P_{k1}P_aP_{k7} = K_1KK_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία $< P_{k1}P_aP_{k7} = P_aP_{k1}P_{\infty} = K_1KK_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής . ***Ο κύκλος Ταχυτήτων*** $[K_1, K_1K_2]$ απεδείχθη ότι είναι ένας κύκλος Καμπής πού ***κόβει τον Οριακό κύκλο Ανάκαμψης*** $[O_a, O_aP_a]$ στο σημείο R_{k-a} , η δε ευθεία KR_{k-a} κόβει τον κύκλο [O ,OK] στο σημείο K_7 πού η χορδή KK_7 είναι η πλευρά του Κανονικού Επταγώνου Στό Τραπέζιο $\mathbf{OO_kP_kP_a}$ η Εντός Εναλλάξ γωνία $< OP_aO_k = P_aO_kP_k = K_1KK_2 = \Delta\varphi_{(\varphi1-\varphi2)}$.

Οι Διάμετροι $K_1OK`_1$ , $K_2OK`_2$, των Κανονικών , Εξαγώνων – Οκταγώνων διέρχονται από τις έναντι κορυφές των $K`_1$ ,$K`_2$, ***ΕΝΩ η Διάμετρος*** K_7OM_7 διέρχεται του μέσου της έναντι Πλευράς και είναι Μεσοκάθετος της . Στο σημείο K_7 γίνεται η ***Αναστροφή της Διαμέτρου*** κατά γωνία 90^{o}.

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΕΝΙΑΓΩΝΟΥ

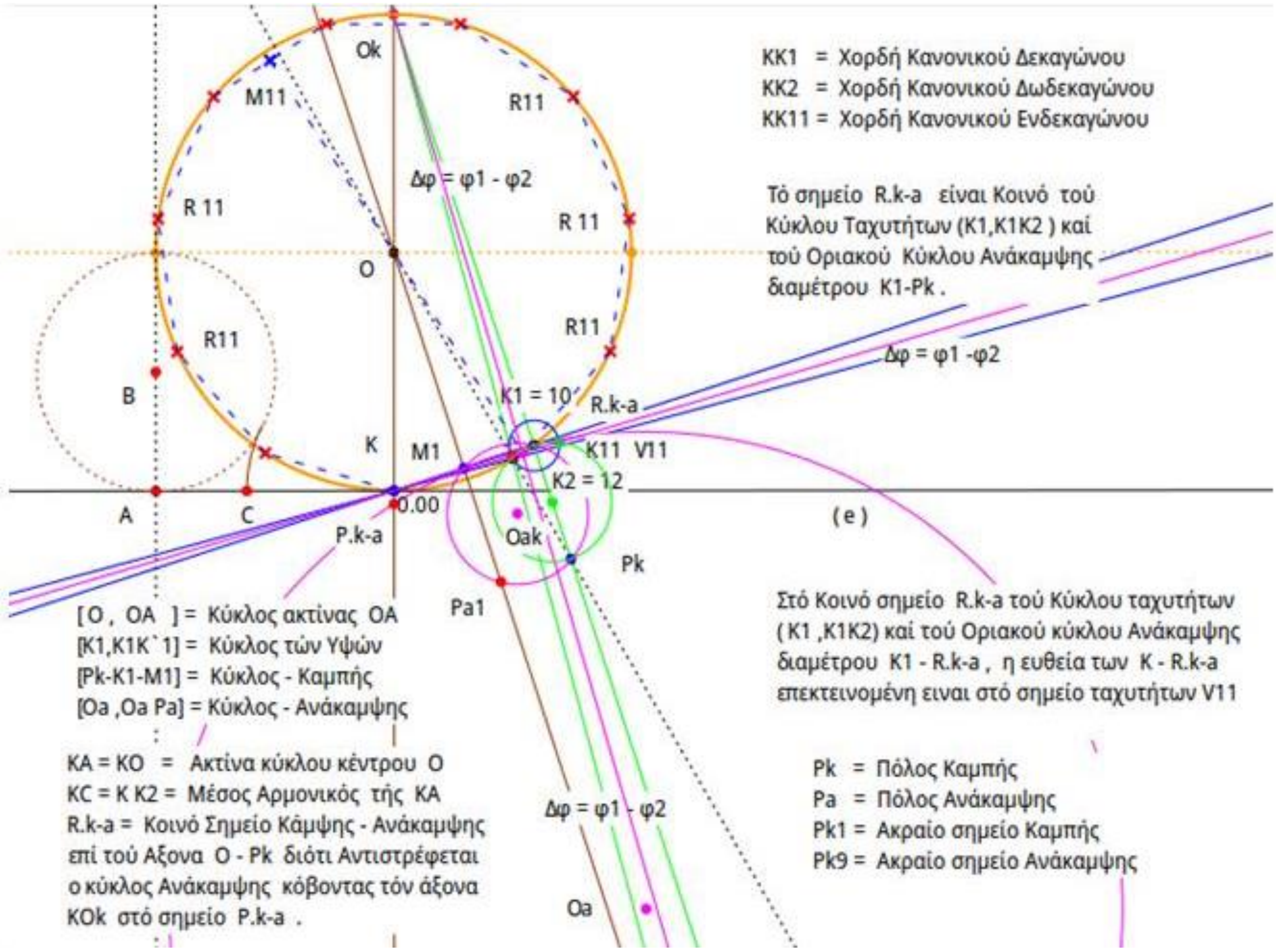


F.21 → Στον κύκλο (O , OK) , για **n = 8** , η Χορδή KK_1 είναι η πλευρά του Ζυγού - Κανονικού Οκταγώνου ενώ για , **n = n + 2 = 10** , η χορδή KK_2 είναι η πλευρά του Ζυγού - Κανονικού Δεκαγώνου , η δε Χορδή $\mathbf{KK_9}$ του ***Κανονικού – Μονού - Ενιαγώνου.*** →

Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1P_a$, *εντός του Κύκλου Ανάκαμψης* , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία $< P_{k1}K_1M_1 = K_1M_1P_a = 90^{o}$, Άρα και η χορδή $P_{k1}P_a \,//\, K_1M_1$ η δε γωνία $< P_{k1}P_aP_{k9} = K_1KK_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία $< P_{k1}P_aP_{k9} = P_aP_{k1}P_\infty = K_1KK_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής . ***Ο κύκλος Ταχυτήτων*** $[K_1, K_1K_2]$ απεδείχθη ότι είναι ένας Κύκλος - Καμπής πού ***κόβει τον Οριακό Κύκλο - Ανάκαμψης , Διαμέτρου*** K_1P_k στο σημείο R_{k-a} *και τούτο , διότι οι κύκλοι Ανάκαμψης Αντιστρέφονται* , η δε ευθεία $KR_{\cdot k-a}$ επεκτεινομένη περνά από το σημείο V9 , και κόβει τον κύκλο [O , OK] στο σημείο K_9 η δε χορδή KK_9 είναι η πλευρά του Κανονικού Ενιαγώνου .

Τούτο συμβαίνει στά Πολύγωνα πού ο Κύκλος Καμπής των ή και Ανάκαμψης κόβει τον άξονα O_k-O-K , οπότε η Αντιστροφή γίνεται στον Οριακό κύκλο Ανάκαμψης διαμέτρου K_1P_k .

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΕΝΔΕΚΑΓΩΝΟΥ

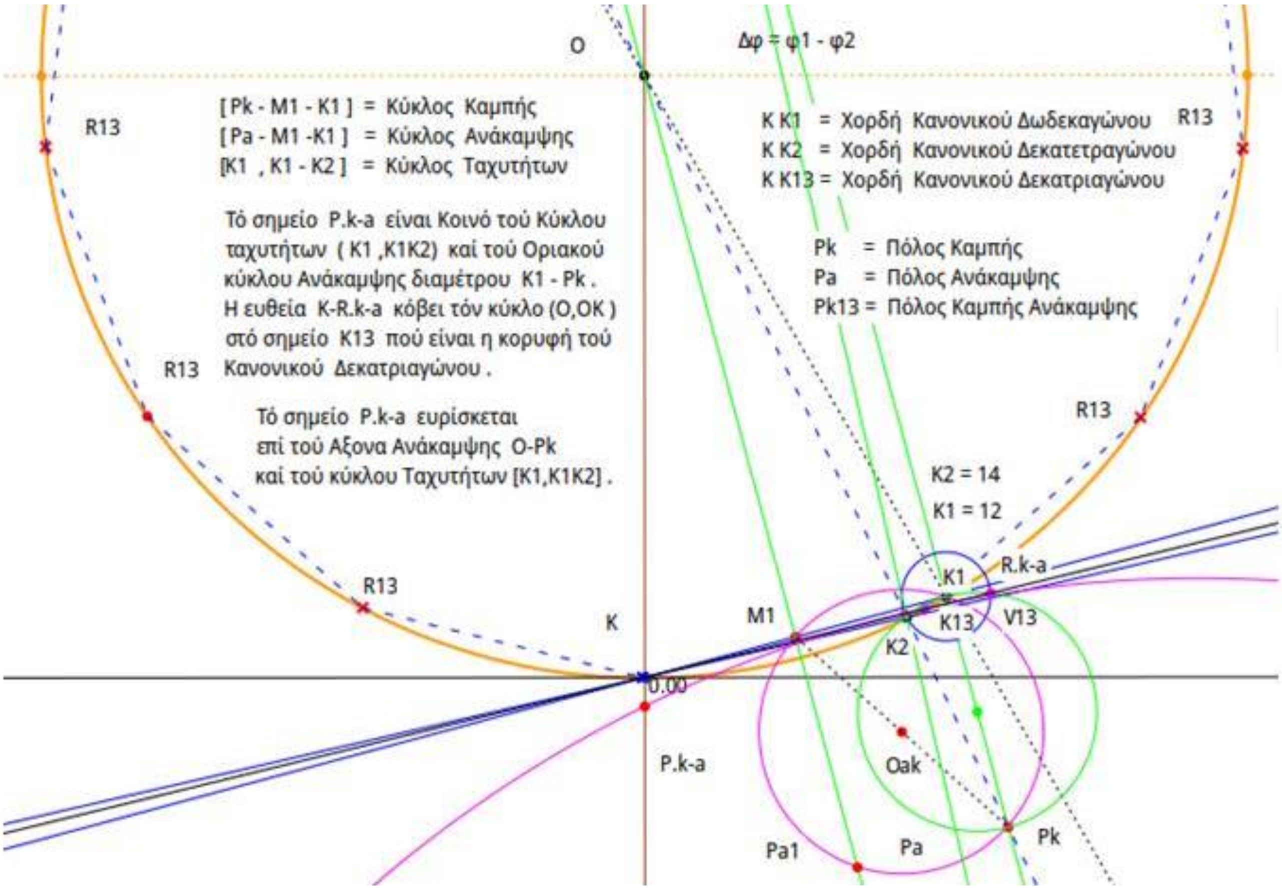


F.22 → Στον κύκλο (O , OK) , για **n = 10** , η Χορδή $K K_1$ είναι η πλευρά του Ζυγού - Κανονικού Δεκαγώνου , ενώ για , **n = n + 2 = 12** , η χορδή $K K_2$ είναι η πλευρά του Ζυγού - Κανονικού Δωδεκαγώνου η δε Χορδή $\mathbf{KK_{11}}$ του ***Κανονικού – Μονού - Εντεκαγώνου.*** **→**

Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1\,P_a$, *εντός του Κύκλου Ανάκαμψης* , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία < $P_{k1}K_1M_1 = K_1M_1P_a = 90$ °, Άρα και η χορδή $P_{k1}P_a$ // K_1M_1 η δε γωνία < $P_{k1}P_aP_{k11} = K_1K\,K_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία < $P_{k1}P_a\,P_{k11} = P_aP_{k1}P_\infty = K_1K\,K_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής . ***Ο κύκλος Ταχυτήτων*** [K_1 , K_1K_2] απεδείχθη ότι είναι ένας κύκλος Καμπής πού ***κόβει τον Οριακό Κύκλο - Ανάκαμψης , Διαμέτρου*** K_1P_k στο σημείο R_{k-a} *και τούτο , διότι οι κύκλοι Ανάκαμψης Αντιστρέφονται* , η δε ευθεία $K - R_{k-a}$, *η οποία και περνά από το Οριακό σημείο ταχυτήτων* V11 , επεκτεινομένη περνά και από το σημείο V11 , όπου και κόβει τον κύκλο [O , OK] στο σημείο K_{11} , η δε χορδή $K K_{11}$ είναι η πλευρά του Κανονικού Εντεκαγώνου .

Τούτο συμβαίνει στά Πολύγωνα πού ο Κύκλος Καμπής των ή και Ανάκαμψης κόβει τον άξονα O_k-O-K , οπότε η Αντιστροφή γίνεται στον Οριακό κύκλο Ανάκαμψης διαμέτρου K_1P_k .

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΔΕΚΑΤΡΙΑΓΩΝΟΥ



F.23 → Στον κύκλο (Ο , ΟΚ) , για **n = 12** , η Χορδή $K K_1$ είναι η πλευρά του Ζυγού - Κανονικού Δωδεκαγώνου , ενώ για , **n = n+2 = 14** , η χορδή $K K_{13}$ είναι η πλευρά του Ζυγού - Κανονικού Δεκατετραγώνου η δε Χορδή $\mathbf{KK_{13}}$ του ***Κανονικού –Μονού - Δεκατριαγώνου*** .

Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1 P_a$, *εντός του Κύκλου Ανάκαμψης* , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία $< P_{k1}K_1M_1 = K_1M_1P_a = 90°$, Άρα και η χορδή $P_{k1}P_a // K_1M_1$ η δε γωνία $< P_{k1}P_aP_{k13} = K_1K K_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία $< P_{k1}P_a P_{k13} = P_aP_{k1}P_\infty = K_1K K_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής . ***Ο κύκλος Ταχυτήτων*** $[K_1 , K_1K_2]$ απεδείχθη ότι είναι ένας κύκλος Καμπής πού ***κόβει τον Οριακό Κύκλο - Ανάκαμψης , Διαμέτρου*** K_1P_k στο σημείο R_{k-a} *και τούτο , διότι οι κύκλοι Ανάκαμψης Αντιστρέφονται* , η δε ευθεία $K - R_{k-a}$ επεκτεινομένη περνά από το σημείο V13 , και κόβει τον κύκλο [Ο , ΟΚ] στο σημείο K_{13} η δε χορδή $K K_{13}$ είναι η πλευρά του Κανονικού Δεκατριαγώνου .
Τούτο συμβαίνει στά Πολύγωνα πού ο Κύκλος Καμπής των ή και Ανάκαμψης κόβει τον άξονα O_k-O-K , οπότε η Αντιστροφή γίνεται στον Οριακό κύκλο Ανάκαμψης διαμέτρου K_1P_k .

Η Αναστροφή των κύκλων Καμπής $P_kK_1M_1$ *γίνεται διότι η Διάμετρος* K_1OM_{13} *του Κανονικού Δεκατριαγώνου είναι Μεσοκάθετος της έναντι πλευράς του στο μέσο σημείο* M_{13} *,εν αντιθέσει με την Διάμετρο* $K_2OM_2 \equiv OK_2 \rightarrow P_k$ *πού διέρχεται από την ορυφή του Κανονικού Δεκατετραγώνου.*

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΟΛΩΝ , ΤΩΝ ΚΑΝΟΝΙΚΩΝ – ΜΟΝΩΝ – ΠΟΛΥΓΩΝΩΝ ΜΕ ΤΗ ΜΕΘΟΔΟ ΤΩΝ ΤΡΙΩΝ ΚΥΚΛΩΝ .

Η ανωτέρω Γεωμετρική Απόδειξη επιλύει γενικά το πρόβλημα των Κανονικών – Πολυγώνων παρακάμπτοντας τούς μέχρι σήμερα περιορισμούς στην Αλγεβρική-θεωρία των Πρώτων προς αλλήλους αριθμούς . Η Αναστροφή γωνίας πέριξ άξονος ΟΑ { σχήμα F16.(3) } είναι όταν συμβαίνει ΟΧ ⊥ ΟΑ δηλαδή η γωνία < XOK = X\`OK = 90° . Τυχούσα γωνία XOC < XOA < 90° ισούται με την συμμετρική της X \`OC $_1$, εφόσον περάσει από την θέση ΟΑ όπου καί XOA = X \`OA = 90° (***Αναστροφή***) και η πλευρά OC περνά από το άπειρο ∞ . Στο σχήμα F-20

Το Σύστημα των Κύκλων - Καμπής – Ανάκαμψης σχηματίζεται από τον κύκλο μεγαλυτέρας διαμέτρου του κύκλου , και είναι το Ορθογώνιο Παραλληλόγραμμο $K_1M_1P_aP_{k1}$ είτε το $K_1M_1P_{a1}P_k$.
Ο Οριακός Κύκλος – Καμπής επί του τριγώνου $M_1K_1P_k$ έχει την κορυφή P_k επί της O_kP_k , ενώ ο Οριακός Κύκλος – Ανάκαμψης επί του τριγώνου $K_1M_1P_a$, έχει την κορυφή P_a επί της OM_1 παραλλήλου της O_kP_k .Επειδή δε οι χορδές O_kP_k , $O\ P_a$ είναι κάθετοι της KK_1 , *άρα είναι και παράλληλοι ,* και επειδή οι χορδές , OP_k , O_kP_a , είναι μεταξύ των παραλλήλων , *άρα και οι Εντός Εναλλάξ γωνίες των* < $P_aOP_k = OP_kO_k$ και η < $P_aO_kP_k = OP_aO_k = K_1KK_2 = \Delta\varphi = \varphi_1 - \varphi_2$.
Οι χορδές O_kP_k ,OP_a είναι παράλληλοι , ΑΡΑ , ***το Τετράπλευρο*** $\mathbf{OO_kP_kP_a}$ ***είναι Τραπέζιο με Ύψος*** K_1M_1 με τα Ανάστροφα τρίγωνα $P_kK_1M_1$, $P_aM_1K_1$. Οι κύκλοι επί των διαμέτρων P_kM_1 , P_aK_1 είναι ο Ακραίος Κύκλος –Καμπής και Ανάκαμψης αντίστοιχα και είναι ο ***Γεωμετρικός - Τόπος διάβασης των σε Θέση Μεγίστου – Ελαχίστου.***

Η Αναστροφή των κύκλων γίνεται διότι η Διάμετρος $\mathbf{K_7OM_7}$ ***είναι Μεσοκάθετος της έναντι πλευράς στο μέσο σημείο*** $\mathbf{M_7}$ ***, εν αντιθέσει με την Διάμετρο*** $\mathbf{K_2OM_2 \equiv OK_2 \rightarrow P_k}$ ***πού διέρχεται από κορυφή .***

Για να καταστούν οι γωνίες < $P_kO_kP_a$, OP_aO_k , *Εντός – Εναλλάξ* και ίσες της < K_1KK_2 , πρέπει η ευθεία O_kP_k να περιστρέφεται πέριξ του Πόλου O_k από το άπειρο (∞) μέχρι τη χορδή O_kP_a . Αυτή η περιστροφική κίνηση της ευθείας είναι Ισοδύναμη με την κίνηση του σημείου K_1 προς το σημείο K_2 επί του κύκλου [O , OK] , *με τα κάτωθι επακόλουθα* :

1.. Με την περιστροφή της χορδής O_kP_k πέριξ του πόλου O_k , η χορδή O_kK_1 έχει την κάθετο ταχύτητα K_1V_1 επί της επέκτασης της KK_1 . Το ίδιο συμβαίνει και διά την χορδή O_kK_2 πού έχει την κάθετο ταχύτητα K_2V_2 επί της επέκτασης της KK_2 , *Δηλαδή* , έκαστο σημείο K_7 μεταξύ των σημείων K_1 , K_2 έχει μίαν κάθετο ταχύτητα , *έστω την* K_7V_7 , επί του κύκλου ταχυτήτων [K_1 , K_1K_2] και με κατεύθυνση την $O\ K_7$, στην εκάστοτε θέση του σημείου .
Απεδείχθη προηγουμένως ότι η Αιχμή του Βέλους V_1 , ***διέρχεται διά Κύκλου Καμπής*** , (και τούτο διότι όταν το σημείο P_k είναι στο ∞ , τότε ο κύκλος (P_k, P_k ∞) προβάλλεται στο σημείο K_1 και γίνεται η εφαπτομένη του σημείου πού είναι η KK_1 .
όπως και κάθε άλλου βέλους V_7 έχοντας σχέση με την Θέση - Αναστροφής της Διαμέτρου .

2.. Με την περιστροφή της χορδής O_kP_k πέριξ του πόλου O_k , Άπειροι Κύκλοι – Καμπής από τα ορθογώνια τρίγωνα $P_kK_1M_7$ σχηματίζονται με διάμετρο την P_kM_7 ,(*όπου* M_7 *είναι η τομή της* O_kK_7 *και της* $K\ K_1$) , με Οριακό Κύκλο –Καμπής τον επί της διαμέτρου P_kM_1 , *ταυτόχρονα δε* , Άπειροι Κύκλοι – Ανάκαμψης σχηματίζονται από τα ορθογώνια τρίγωνα P_a , M_1 , M_7 με διάμετρο την P_aM_7 και με Οριακό Κύκλο Ανάκαμψης τον επί της μεγαλυτέρας διαμέτρου P_aK_1 ευρισκόμενο.
Η Αναστροφή των κύκλων Καμπής $P_kK_1M_1$ *γίνεται διότι η Διάμετρος* K_1OM_{n+1} *του Κανονικού* (n+1) *Μονού Πολυγώνου είναι Μεσοκάθετος της έναντι πλευράς του , στο μέσο σημείο* M_{n+1} *, εν αντιθέσει με την Διάμετρο* $K_2OM_2 \equiv OK_2 \rightarrow P_k$ *πού διέρχεται από την κορυφή του Ζυγού-Κανονικού* (n) , (n+2) *Πολυγώνου . Η κίνηση της Κορυφής* ,K, *στη θέση* O_k ,διατηρεί την Χορδή K_1K_2 σταθερή.

3.. ***Απεδείχθη*** ότι η εξίσωση , Σ (h) = n .OK *, δηλαδή το άθροισμα των Υψών* , h , *των κορυφών των Κανονικών* (n) Πολυγώνων από τυχούσα ευθεία (e) εφαπτομένη σε μίαν κορυφή του , είναι , n , φορές την ακτίνα του κύκλου . Όταν δε , n , n +2 , είναι οι Αριθμοί των Κορυφών δύο διαδοχικών Ζυγών Πολυγώνων , τότε μεταξύ των υπάρχει και το , n +1 , Μονό Πολύγωνο .
Η θέση του Μονού Πολυγώνου είναι κοινή του Κύκλου - Καμπής και του Κύκλου – Ανάκαμψης .
Επίσης απεδείχθη ότι , η Αιχμή του Βέλους επί του Κύκλου των Ταχυτήτων [K_1, K_1K_2] διέρχεται διά της ***Περιβάλλουσας των Κύκλων-Καμπής , οπότε η τομή τών Οριακών Κύκλων -Ανάκαμψης με Διαμέτρο τό Τμήμα*** K_1P_k , καθορίζει το σημείο $R._{k-a}$ καί την κατεύθυνση K_1V_7 , πού είναι αυτή τού n+1 Μονού –Κανονικού –Πολυγώνου .
Δηλαδή , η ευθεία KV_7 κόβοντας τον κύκλο [O , OK] στο σημείο K_7 , καθορίζει την χορδή KK_7 πού είναι ***η Πλευρά του Ενδιάμεσου Μονού - Πολυγώνου*** , και
Στην περίπτωση όπου ο Κύκλος Καμπής ή και Ανάκαμψης τέμνει τον άξονα O_k-O-K στο σημείο P_{k-a} , ή και έχοντας την μεγαλυτέρα διάμετρο τότε το Κοινό σημείο Καμπής ευρίσκεται επί του Οριακού κύκλου Ανάκαμψης διαμέτρου K_1P_k , και του κύκλου των Ταχυτήτων .

ο.ε.δ. Μάρκος 16 /06/2017 .

THE GEOMETRICAL CONSTRUCTION OF ALL THE ODD - REGULAR - POLYGONS USING THE THREE CIRCLES METHOD

The above Geometric Proof , solves the problem of the Odd- Regular - Polygons by surpassing the limitations to the theory of Algebraic numbers and to the Unsolvability of the Greek problems using the Wrong Theory of Constructible Numbers .

In figure F16 (3) is shown the Inversion of an angle through an axis where is holding $OX \perp OA$, i.e. angle < XOK = X `OK = 90°. Any other angle XOC < 90° is equal to the symmetric $X`OC_1$, when it passes from OA line , *Inversion of angle through* OA , where angle < XOA = X`OA =90° and where OC side passes through infinite ∞ . *In Figure F.20 – A ,*

The system of Coupler curves , *the Inflection and the Inverted Reflection circles* , is formatted in the rightangled Parallelograms $K_1M_1P_aP_{k1}$ or $K_1M_1P_{a1}P_k$. The circumscribed Inflection circle lying on $M_1K_1P_k$ triangle , defines vertices P_k on O_kP_k line , while the circumscribed Reflection circle on $M_1P_aK_1$ triangle , defines vertices P_a on OM_1 line parallel to O_kP_k formation .
Segments O_kP_k ,OP_a are parallel therefore , ***Quadrilateral*** $\mathbf{OO_kP_kP_a}$ ***is Trapezium*** of height K_1M_1.
Because chords O_kP_k , OP_a are perpendicular to KK_1 chord , *so these are parallels* , and because chords , OP_k , O_kP_a , are *in cross* between the parallels , *therefore* the two *Alternate Interior angles* < $P_aOP_k = OP_kO_k$ and angle < $P_aO_kP_k = OP_aO_k = K_1KK_2 = \Delta\varphi = \varphi_1 - \varphi_2$.
Presupposition for these ***Alternate Interior angles*** , is the Inversion (*Rotation*) of O_kP_k line through pole O_k , starting from Infinite (∞) and limiting to chord O_kP_a . It is an Extrema case (maximum or minimum) applied between K_1 , K_2 points where Parallels are inverted .

This type of Rotation is equivalent to the motion of point K_1 to point K_2 on circle [O , OK] ,

with the followings ,

1… During Rotation of chord O_kP_k through pole O_k , establishes the velocity direction K_1V_1 to chord $K K_1$ extended , or on KV_1 line . The same happens for chord O_kK_2 which establishes the velocity direction K_2V_2 perpendicular to chord $K K_2$ extended also . *Generally for* , Any point K_7 between the points K_1 , K_2 occupies a perpendicular to chord O_kK_7 velocity , *say the Velocity* K_7V_7 , on the Inflection -Velocity - Circle [K_1 , K_1K_2] directed on OK_7 line for every Position of point V_7 . It was proved before , *that the edge of arrow* V_1 ***, passes through an Inflection circle*** , *Inversion to maxima* , and the same is happening for any other arrow V_7 .

2.. The Rotation of line O_kP_k with the greater diameter through pole O_k , formulates *Infinite Inflection - Circles* circumscribed in the rightangled triangles $P_kK_1M_7$ with diameter P_kM_7 , (*where* M_7 *is the intersection of line* O_kK_7 *and line* $K K_1$) , limiting to the Inflection – circle of P_kM_1 diameter ,
But Simultaneously , are formulated *Infinite Reflection - Circles* circumscribed in the rightangled triangles $P_aM_1M_7$ with diameter P_aM_7 , limiting to the Reflection – circle of P_aK_1 diameter .

Inversion of the circles happens because Diameter $\mathbf{K_7OM_7}$ ***is Mid-perpendicular to the opposite Side in the middle point*** $\mathbf{M_7}$ ***in contradiction to Diameter*** $\mathbf{K_2OM_2}$ ***which passes only through the vertices of Polygon . It is the Geometrical Locus between points*** K_1 , K_2 ***where exists Maxima*** (maximum or minimum) **.**

3.. ***It was proved*** the equation Σ (h) = n .OK , *the Summation of heights* h , *of the vertices of any* (n) *Polygon from any (e) line tangential to any vertices* , is equal to , n , times the radius OK . When , n , n+2 , are the numbers of the vertices of any two sequent and Even Polygons , then exists the In-between , n+1, Odd -Polygon . The position of this Odd-Polygon is common to the Inflection and Reflection circles . ***It was proved also*** , *that the edge of arrow* V_1 ***passes through the Inflection circle*** [K_1, K_1K_2] and through the Envelope of Inflection circles where then , the point of intersection , R.k-a , defines the direction K_1V_7 , which belongs to the n+1 Odd – Regular –Polygon . **i.e.** line KV_7 intersecting the circle [O , OK] at point K_7 defines chord $K K_7$ ***which is the Side of the intermediate Odd – Regular – Polygon***. i.e.

In circle [O , OK] of diameter $K - O_k$, any two chords $K K_1 , K K_2$ and the circle [K_1 , K_1K_2] , Formulate the Trapezium $\mathbf{OO_kP_kP_a}$ and $\mathbf{K_1M_1P_aP_k}$, such that the two circles on the Diamesus and diameters M_1P_k , K_1P_a , intersect the circle [K_1 , K_1K_2] at the point S_{ka} such that , this tobe the common Inversion point of the two Inverted circles . (q.e.d) .

Remark :
Maxima ***in Geometry*** , *is the maximum or minimum Magnitude between two Positions* , Maxima ***in Mechanics*** , *is the Inflection or Deflection for Coupler curves* , Maxima ***in Calculus*** , *is the local maximum or minimum between two points* called critical point .

Geometrical Inversion , is the Mechanism where Extrema in a closed-bounded-interval , as this is arc $\overset{\frown}{K_1K_2}$ between points K_1 , K_2 , is the Critical – Point S_{ka} common to [K_1 , K_1K_2] circle and to Inversion circles of , diameters M_1P_k , K_1P_a , transfered on circle (O ,OK) as P_{1-2} point , and as , $P_{1-2}O$, diameter , *axis* , which is applied 90° to critical - chord .

6.3. The Methods :

Preliminaries : The Subject , F.16(3).

Any circle (O , OK) can be divided **into** ,

a.. ***Two*** equal parts by the diameter KA [It is the Dipole AK] with angle < AOK = 180 ^{o} .

b.. ***Four*** equal parts by the Bisector of 180^{o} which is the perpendicular and second diameter X `X .

c.. ***Eight*** equal parts by the Bisector of the four angles which are 90 ^{o} .

d.. ***Sixteen*** equal parts by the Bisector of the Eight angles which are 45 ^{o} , and so on .

e.. The circle having 360 ^{o} = 2π radians , can be divided **into** ,

Three equal parts as 360^{o} / 3 = 120^{o} and which is possible [The Equilateral triangle] ,

Six equal parts as 360^{o} / 6 = 60^{o} and which is possible by the bisectors of the triangle [The Regular Hexagon] ,

Twelve equal parts as 360^{o} / 12 = 30^{o} and which is possible by the bisectors of the Hexagon [The Regular Dodecagon] , and so on , to 15^{o} , 7,5^{o} ……...

Remark :

a… The series of Even Numbers is 2 , 4 ,6 , 8 ,10 ,12 ,14 ,16 ,18 ,20 ,…………….

The series of Odd Numbers is 1 ,3 ,5 ,7,9 ,11 ,13 ,15 ,17,19 ,21 , ……………

Becoming from the Arithmetic - mean between two Adjoined - Even numbers , as for example , Number five $5 = \frac{4+6}{2} = \frac{10}{2} = 5$. The logic of addition issues in Geometry in its moulds which is the logic of Material – Point , which is Zero (0 = Nothing) and exists as the Addition of Positive + Negative (→ + ←) . [See , Material Geometry 58 – 60 – 61]

b… In previous paragraph 5.5(Case c) was proved (1) Σ(h) = (2k) . h = n .h = n .OK , *where*

Σ = The Summation of Heights , h , of the Vertices (n) – in the Regular Polygon from the vertices K_n , projected to tangential (e) at the initial point K ,

h = OK , The height of center , O, measured on (e) tangent ,

n = The number of Sides of the Regular Polygon …… *and which* Changes the Sum of heights from the Tangential line (e) to a Linear and Integer number of the radius of the circle , ***and which is directly related to angles , φ_n , and vertices of sides , KK_n .***

c... On any Chord KK_1 of circle (O ,OK) , the central angle < KOK_1 , is twice the Inscribed and equal to < $K O_K K_1 = KOM_1$. The mid - perpendicular OM_1 , is parallel to the Perpendicular line $O_K K_1$, therefore cut each other to infinite (∞) . Because the two perpendiculars pass from O and O_K points , these consist the Poles of their rotation .

In F.18 -A **, any Point K_2** on circle , formulates the second chord **KK_2 ,** while the perpendicular $O_K K_2$ projected cuts OM_1 , *the parallel to* $O_K K_1$ at a point P_1 , which is the Pole of rotation of the two chords , or angles , and this because point P_2 is moving on OM_1 from infinite to KP_1 diameter .

On diameter KP_2 of circle (O_2 , $O_2P_2 = O_2K$) , and center O_2 , *are formulated the same angles* φ_1 , φ_2 by chords P_1M_1 , P_2K_2 , such that angles are equal < $M_1P_1K_2 = K_1KK_2 = OP_1O_k$, That is , ***on any two chords*** $\mathbf{KK_1 , KK_2}$ ***, of circle*** **(O , OK)** ***, with common vertices*** **K** ***, the Mid - Perpendicular*** $\mathbf{OM_1}$ ***of the first , and the Perpendicular*** $\mathbf{O_K K_2}$ ***of the second , cut each other at a point*** $\mathbf{P_1}$ ***, which defines its conjugate circle*** **(O`$_1$ , O`$_1$P$_1$)** , { it is the Circle of equal angles with circle (O ,OK) } ***. The same happens with circle*** **(** $\mathbf{O_2 , O_2P_2 = O_2K}$ **) .**

d... From relation Σ = (2k) . h = n .h = n .OK , For n = 2 then Σ = 2.h = 2.OK that is diameter KO_K . For n = 3 then Σ = 3.h = 3 .OK and for n = 4 then Σ = 4.h = 4 .OK . Because the Odd - numbers are the Arithmetic - mean between two Adjoined - Even numbers so for 3.OK is (2.OK +4.OK)/2 . The difference of heights is $\Delta h = h_{K1} - h_{K2} = K_1K`_1$ and it is between the parallels through points $K_1$ , $K_2$ , and line (e) . Circle ( $K_1$ , $K_1K`_1$) is the circle of *Hypsometric differences* of the chords K K_1 , K K_2 , and changes according to point K`$_1$ or the same with point K $_2$. That is ,

The circle of the Hypsometric differences **(** $\mathbf{K_1 , K_1K`_1}$ **)** ***is correlated with chords*** **[** $\mathbf{KK_1 ,KK_2}$ **] , [** $\mathbf{O_KK_1 ,O_KK_2}$ **]** ***of circle*** **( O , OK )** ***through the corresponding vertices*** **K ,** $\mathbf{O_K}$ ***and with that of Equal angles circle*** **( O`$_1$,O`$_1P_1$)** ***through the mid - perpendicular*** $\mathbf{OM_1}$ ***of the first chord*** **K** $\mathbf{K_1}$ **,** ***and the mid - perpendicular*** $\mathbf{O_KK_2}$ ***of the second chord*** $\mathbf{KK_2}$ **.**

This co relation of this Formation between these four circles ,

$$\{ \ (\mathbf{O , OK}) - (\mathbf{K_1 , K_1K`_1}) - (\mathbf{O`_1 , O`_1P_1}) - (\mathbf{O_2 , O_2P_2}) \ \}$$

and Perpendicular to line (e) , ***Allows to Any circle*** (O , OK) ***to define their in between motion*** through the two chords K K_1 , K K_2 , or and angles φ_1 , φ_2 , ***that is ,*** From the relation of Heights Σ (h) = (2k) . h = n .h = n .OK , becomes that the Summation of heights of any two *Adjoined - Even* Regular Polygons , n , n+2 is → $\frac{\Sigma\ 2(h1)}{2} + \frac{\Sigma\ 2(h2)}{2} = [\frac{n_1}{2} + \frac{n_2}{2}].OK = [\frac{n_1+n_2}{2}].OK = n_3 .OK$, where $n_3 = [\frac{n_1+n_2}{2}]$ is the number of vertices between the two Even n_1 , n_2 ,

The Odd – Number - Vertices Regular – Polygon .

On the Hypsometric difference $\Delta h = O_1K`_1$ and on the perpendicular to line (e) are kept all properties of the addition .From the Instaneous position of angles φ_1,φ_2 , to the two circles the chords are defined.

e... Because chords K K_1 , K K_2 , are perpendicular to OP_1 ,O_KP_1 , lines , ***Therefore point*** **K** ***is the Orthocenter*** of all perpendicular and rightangled triangles , as well as their common chord K_1M_1 , of the two circles (O_2 , O_2P_2) , (O , OK) . Because the Geometric locus of chords K K_1 , K K_2 , ***of the Common Orthocenter*** **K** ***is*** → for circle (O ,OK) the arc K_1K_2 , and for circle (O_2 ,$O_2K = O_2P_2$) arc M_1 K_2 , and for circle (O_1 , $O_1P`_1$) arc (1)-(2) with the points of the chords intersection , ***Therefore*** points (1) , M_1 are limit points of these circles such that exists K $M_1 \perp P_1M_1$.

The above logics result to the , ***Mechanical and Geometrical solution*** , which follows .

The new Mechanical Approach :

In F. 18 - A. **is** the circle (O ,OK) with the tangential line (e) at point K , and the diameter KO_K . Define on the circle from vertices , K , The vertices K_1 , K_2 corresponding to the edges of sides of two *Adjoined Even - Regular Polygons* and the corresponding angles φ_1 , φ_2 , between sides K K_1 , K K_2 , and the tangent line (e) .

Draw the parallels from vertices K_1 , K_2 , to (e) line and from vertices K_1 perpendicular to (e) , such that cuts the parallel from point K_2 , at point K`$_1$ , and draw the perpendicular $K_1K`_1$ as the radius the circle (K_1 , $K_1K`_1$) .

Draw $O_K K_1$ produced which cuts OK_2 extended (from point O) at point P_2 and from point O_2 (the middle of diameter K P_2) draw the circle (O_2 ,$O_2 K = O_2 P_2$) .

Extend sides $O_k K_1$, $O_k K_2$, so that they cut circle (O_1 ,$O_1 K`_1$) at points 1 ,1`, and 2 , 2`, and draw chords 1 - 2` και 2 -1` respectively .

Define the common point , T , of chords 1 - 2` και 2 - 1` and produce , $O_k T$, such that cuts circle (O , OK) at point K_5 . ***OR*** , *with the* Harmonic Mean ,

Draw from point $K`_1$ the perpendicular , $K`_1 A = (K`_1 K_1)/2$ and the circle ( A , $AK`_1$) cutting the chord $O_1 A$ at point B .

Draw from point K_1 the circle (K_1 , $K_1 B$) such that intersects the perpendicular $K_1 K`_1$ at point , C , and from this point C the parallel to (e) so that cuts circle (O, OK) at point K_5 .

The chord* K K_5 *is the side of the Regular - Odd - Polygon , and this because

The circle (O_4 ,$O_4 K = O_4 O$) is the circle of the middle of chords KK_1 , KK_2 ***so*** and for KK_5.

Angles < $KM_1 O_2 = KM_2 O`_1 = 90^o$, < $KM_1 P_1$ = $KM_1 O$ = 90^o, < $K K_2 P_1$ = $KK_2 O_\kappa = 90^o$,

Therefore point K is the Orthocenter of the triangles KOM_2 , KOP_1 , $KO_k P_2$, $K O_k O_1$.

Angles < $K_1 KK_2$, $K_1 O_k K_2$, $OP_1 O_k$, $OP_2 O_k$, $P_2 OP_1$ are equal between them ,

Because these are α) Inscribed to the same arc , $K_1 K_2$, of circle (O, OK) ,

β) Their sides $P_1 M_1$, $P_1 K_2$, and being perpendicular to KK_1 , KK_2 are in circle ($O`_1$ , $O`_1 K = O`_1 P_1$) ,

γ) Alternate Interior angles between the parallels , OP_1 , and $O_k P_2$ of the circles (O_4 , $O_4 K = O_4 O$) , (O_2 , $O_2 K = O_2 P_2$) .

Chords $O_k K_1$, OM_1 are perpendicular to chord KK_1 , ***Therefore*** are parallels ,

Chords $O_k K_2$, OM_2 are perpendicular to chord KK_2 , ***Therefore*** are parallels ,

The Geometrical locus of point K_1 , ***from Point* K_1 *to point* K_2** , and on circle (O , OK) is arc $K_1 K_2$ of the circle , ***while*** on circle (O_1 ,$O_1 K`_1$ ) arc 1 , 2` of the circle .

The Geometrical locus of point K_2 , ***from Point* K_2 *to point* K_1** , and on circle (O , OK) is arc $K_2 K_1$ of the circle , ***while*** on circle (O_1 ,$O_1 K`_1$ ) arc 2 , 1` of the circle .

***The Geometrical locus from point* , O ,** of the parallels to chord $O_k O_1$, are the chords OP_1 , $O_4 O`_1$ , ***and from Pole*** , $O_k$ , section , T , between chords 1 , 2` and 2 , 1` respectively .

Because angle < $O_k O_1 K$ = $O_k K_2 K$ = 90^o , ***Therefore*** section , T , moves parallel to line $O_1 K$, and it is the common point of the two ***Geometrical loci .***

Because points* K_1 , K_2 *are the two Adjoined - Even Regular Polygons of circle (O ,OK) ***and simultaneously points* O_1 , P_2 *, the corresponding extreme Poles on circles*** (O_1 ,$O_1 K`_1$),(O_2 ,O_2 K), following the common joint for point K , **to be the Orthocenter and the Pole of Polygons ,** and point , T , **the constant and common Pole of the System ,** ***Therefore*** line $O_k T$, is constant and cuts circle (O ,OK) , ***at point* K_5 *which is the vertices of the intermediate Regular – Odd – Polygon ??***
OR , ***because of the Harmonic relation* (1) *and* (4)** as $(K_1 K`_1)^2 = ( K_1 C ).( K_1 C + K_1 K`_1)$ is ***defined the harmonic height* $K_1 C$ *and from parallel chord*** CK_5 , **point K_5** , on circle (O ,OK) such that corresponds the ***above Harmonic relation , Therefore chord*** K K_5 *is also of the inner and* ***The between Odd –Regular- Polygon*** *q.e.d*

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The new Geometrical Approach :

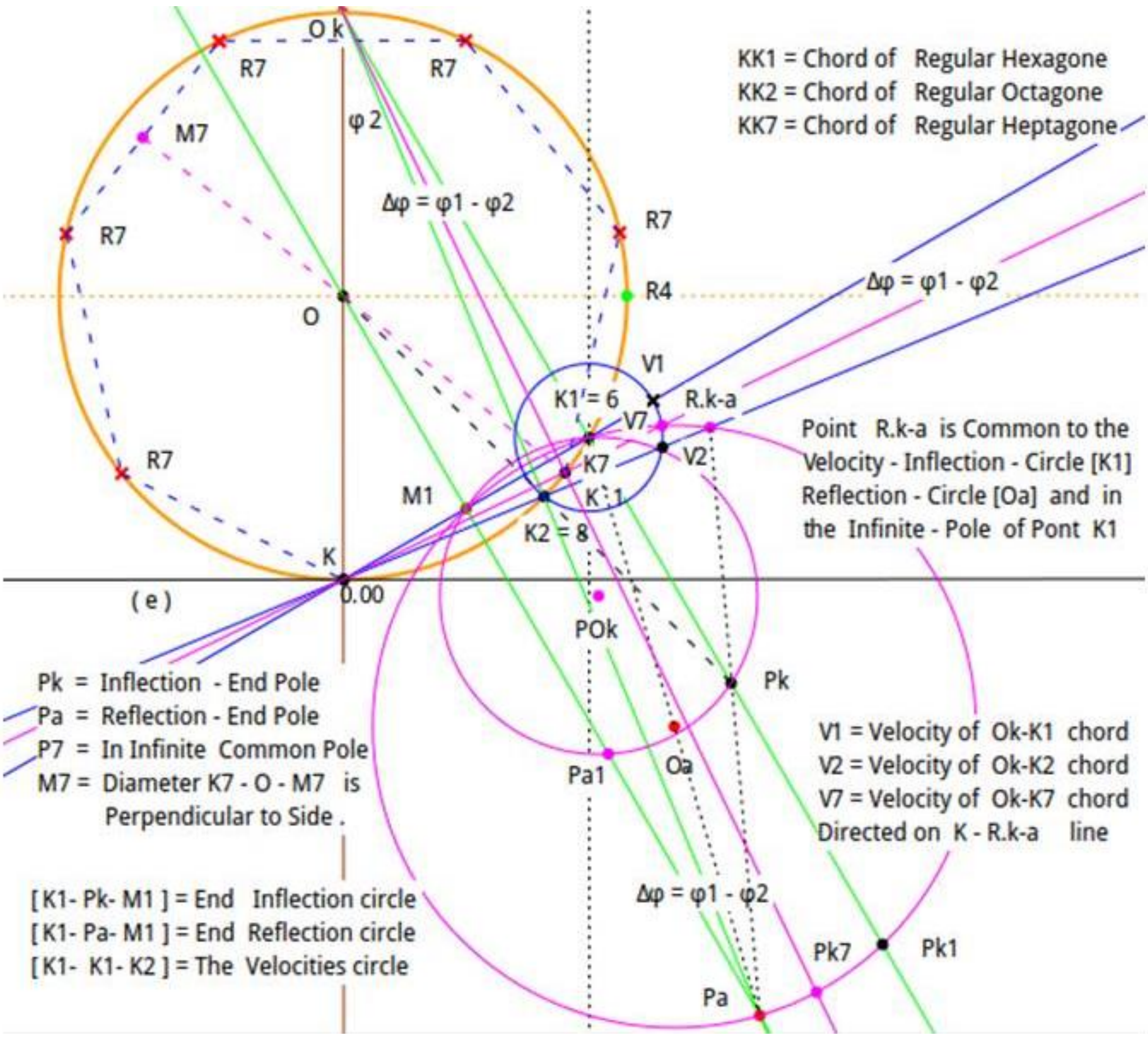
In F. 18 - A. of circle (O ,OK) , since the sides P_1O_k , P_1O are perpendicular to KK_2 , KK_1 respectively ***So*** angle < $OP_1O_k = K_1KK_2$, and since also P_2O chord is between the parallel lines P_1O , P_2O_k , ***Therefore*** angles < OP_1O_k , OP_2O_k are equal , either on the constant Poles of the vertices O , O_k , or on the movable Poles of vertices P_1 , P_2 . Since angles < OP_1O_k , OP_2O_k , are equal ***So*** lie on a circle of chord OO_k .Since also exist on the same circle the Poles O_k ,O, P_1 , P_2 ***Therefore*** lie on a circle of center the intersection of the mid-perpendicular of chords O O_k , O P_2 , and is point O_3
The point K of line (e) is common to the infinite (∞) Regular – Polygons of the circles with center the point ,O, and radius KO = 0 → ∞ , ***Therefore*** the ***Infinite*** Regular Polygon becomes line (e) , the ***Regular Polygons*** lie on circle (O , OK) and the ***Zero*** Regular Polygon is point K .
Since the movable Poles P_1 , P_2 , *of the two **Adjoined - Even Regular Polygons*** lie on circle [O_3 ,O_3O] ***The Anti-Space circle*** [12] , ***So*** the inter and movable pole of the Odd – Regular – Polygon passes from the infinite , ∞ , and which is the intersection of line (e) and this circle and it is the common point P_5 . The same happens with angle of 90 ^{o} with two lines passing from infinite .
Chord OP_5 corresponds to the Reflection chords of the ***Reflection - circle*** [O_2 ,O_2P_2] with center in infinite and which is in point P_5 .The two intersecting pairs P_4 , $P`_4$ and $P_6$ , $P`_6$, converge to the one pair such that $P_5 = P`_5$, where the two points coincide . ***q.e.d.***

Remarks **:**

In F. 18 – B , chords O_kK_1 , O_kK_2 , are perpendicular to KK_1 , KK_2 , therefore angle < $K_1O_kK_2 = K_1KK_2$. Chord O_kK_1 is parallel to OM_1 , OP_a and since chord P_aO_k is between the two parallels then the *Alternate Interior angles* < OP_aO_k , $P_aO_kK_1$ are equal . In order that point P_k reaches to P_a , which means from *Inflection* - *Envelope* to the *Reflection* - *Envelope* , line O_kP_k must move from point K_1 to point M_1 perpendicularly .This motion presupposes that the point K_1 is lying on Inflection circle which happens because the perpendicular velocities of O_kK_1 chord are always directed on KK_1 chord . ***i.e. the Velocity - circle [K_1 , K_1K_2] is an Inflection circle .***
Since the ***End –Inflection –Circle passes through K_1 , P_k points ,*** and the ***End –Reflection –Circle passes through K_1 , P_a points*** , with point K_1 always common , ***then Passes*** also through the outer ***Common - Inflection - Reflection –Point which lies on the Velocity –circle*** , where for point K_1 the Pole of Rotation is in infinite and the *Alternate Interior angles* reversible .
Because the Diameters through the vertices K_1 , K_2 *pass through the corresponding* ***, n , and , n+2 , Odd – Regular – Polygons ,*** *the Diameter through the vertices* $K_{7 = n+1}$ *passes through the center of the Opposite Side ,* ***Therefore*** *it is Mid-perpendicular between the Inflation and to the Reflation point* .

The Exact Geometrical Solution of the Odd – Regular – Polygons follows **:**

THE GEOMETRICAL CONSTRUCTION OF THE REGULAR HEPTAGON



F.20 - A → In circle (O , OK) **For n = 6** , then $K K_1$ is the Side of the Even – Regular - Hexagon while for **n = 8** , then KK_2 is the Side of the Even - Regular –Octagon .

$\mathbf{K K_1}$ is the Side of the Odd - Regular – Hexagon ,

$\mathbf{K K_2}$ is the Side of the Odd - Regular – Octagon ,

Exists Circle of Heights $\Delta h = h_{K1} - h_{K2} = K_1K`_1$ and Velocity Inflection circle $\Delta V = K_1K_2$

Straight - Line { O_k , K_1 , P_k } is parallel to { O , M_1 , P_a } and the Alternate Interior angles equal , $< O P_aO_k = P_kO_k P_a = K_1KK_2$. The same for angle $< O O_k P_k = P_kOP_a$

The Inflection Circle [PO_k , PO_k - K_1] or the Reflection circle [O_a , O_a - K_1] cut the Inflection Velocity - Circle [K_1 , $\Delta V = K_1K_2$] at Edge point , $R._{k-a}$.

Line K $R._{k-a}$ intersects the circle (O ,OK) at point K_7 which is the vertices of the n+1 = 7 Regular Odd Polygon , and which is the Regular –Heptagon .

$\mathbf{K K_7}$ is the Side of the Odd - Regular - Heptagon ,

The Geometrical Proof :

In circle (O ,OK) of F.20-A(B) , the points K_1, K_2 are the ***Vertices*** and KK_1 , KK_2 are the ***Sides*** of two ***Adjoined - Even Regular Polygons*** . Chords O_kK_1 , O_kK_2 are perpendicular to the sides KK_1 , KK_2 because lie on diameter KO_k . The mid-perpendicular OM_1 of KK_1 side , is parallel to O_kK_1 chord because both are perpendicular to KK_1 side . Line OK_2 produced intersects O_kK_1 line at point P_k and since Segment OP_k lies between the two parallels , the ***Alternate - Interior angles*** < OP_kO_k , P_kOP_a are equal .

Line O_kK_2 produced intersects OM_1 line at point P_a and since Segment O_kP_a lies between the two parallels then the , *Alternate Interior angles* < OP_aO_k , $P_aO_kP_k$ are equal , and since angle < $K_1O_kK_2 = K_1KK_2$, then also angle < $OP_aO_k = P_aO_kP_k = K_1KK_2$.

Segments O_kP_k , OP_a are parallel therefore , ***Quadrilateral*** $\mathbf{OO_kP_kP_a}$ ***is Trapezium*** of height K_1M_1 . Since the right angle triangles , $P_kK_1M_1$, $P_aM_1K_1$ occupy the common segment $K_1M_1 = M_1K_1$ therefore are Inverted (*either Inflection or Reflection*) Triangles and their Hypotenuses P_aK_1 , P_kM_1 , formulate the ***Reflection*** [$P_aM_1K_1$] and the ***Inflection*** [$P_kK_1M_1$] ***Circles*** on $K_1M_1 = M_1K_1$ common segment . [*This terminology of ,* Inflection and Reflection circle *, becomes from Mechanics* . Inversion is the case of *Maxima* where is not happening *maximum* or *minimum* , but a change of direction].

Remark : *Trapezium* $OP_aP_kO_k$ *is a Geometrical mechanism with its Alternate Interior angles equal to the angle* < K_1KK_2 *of Sides* . When triangle OO_kK_1 changes from K_1 to K_2 position then , the right angled triangles KK_1O_k , KK_2O_k are directed on KK_1 , KK_2 , lines and in the (K_1 ,K_1K_2) circle as K_1V_1 , K_2V_2 , segments , because these lie on perpendicular Segments , while the ***Inverted*** (*Backing Formation*) ***circles*** [O_a, $O_aK_1 = O_aP_a$] , [O_{ak} , $O_{ak}M_1 = O_{ak}P_k$] are constant for all combinations .

The End –Inflection circle is of Diameter M_1P_k ***and is Inverted*** to (K_1 , K_1K_2) circle . The End –Reflection circle is of Diameter K_1P_∞ ***and is Inverted*** to (K_1 , K_1K_2) circle since the Infinite circles passing Tangentially from K_1 and K_1V_1 .

Inversion of circles happens in infinite through the Trapezium , in where ,

a.. Triangles O_kP_kO , $O_kP_kP_a$ are of equal area , because lie on the common Segment O_kP_k , and the common height K_1M_1 . Since triangle $O_kP_kK_2$ is common to both triangles therefore the remaining triangles K_2O_kO , $K_2P_aP_k$ are of equal area , and ***point*** $\mathbf{K_2}$ is a ***constant*** point to this mechanism . Since also ***triangles*** K_2O_kO , $K_2P_aP_k$ lie on opposites of line $O_kK_2P_a$ position then ***are Inverted*** on this line . (*the Alternate Inverted triangles*)

The Inversion of the circles happens because Diameter K_7OM_7 *is the Mid - perpendicular to the opposite Side of the Odd in the middle point* M_7 *in contradiction to Diameter* $K_2OM_2 \equiv OK_2 \rightarrow P_k$ *which passes through the vertices of the Even-Regular-Polygon forming angle* < $K_1OK_2 = 2.\ K_1KK_2$

b.. ***Because*** at point K_1 of chord $O_kK_1 \perp KK_1$, *infinite points* P_k exist on O_kK_1 for all points $K_2 \equiv K_1$ and circle of radius $K_1K_2 = 0$ **, *Therefore separately must issue*** and for chord O_kK_2 . But since is $K_1K_2 \neq 0$ then Chords KK_1 ,KK_7 ,KK_2 are ***all projected on the*** **(** $\mathbf{K_1}$ **,** $\mathbf{K_1K_2}$ **)** ***circle*** , and Diameter P_kM_1 ***is Inverted*** to Diameter P_aK_1 with their circles .The edges of Segments K_1V_1 , K_2V_2 , are on

KK_1 , KK_2 lines , so all triangles of Parallel sides of Trapezium , occupy the ***point*** **K** , as the same ***Orthocenter*** for all the Regularly-Revolving triangles KO_kP_k , $KO_kK_{\infty \to 7}$, KO_kP_a ,with the Sides $O_kP_k \to O_kP_7 \to O_kP_a$, and the ***Inverted*** Circles [O_a , $O_aK_1 = O_aP_a$] , [O_{ak} , $O_{ak}M_1 = O_{ak}P_k$] .

c.. ***That Inverted circle*** [O_a , $O_aK_1 = O_aP_a$] , [O_{ak} , $O_{ak}M_1 = O_{ak}P_k$] with the greater diameter ,when intersects the circle (K_1 , K_1K_2) ***between the points*** V_1 , V_2 defines the Inverted Position V_c , i.e. that of the ***Odd - Regular - Polygon*** . q.e.d

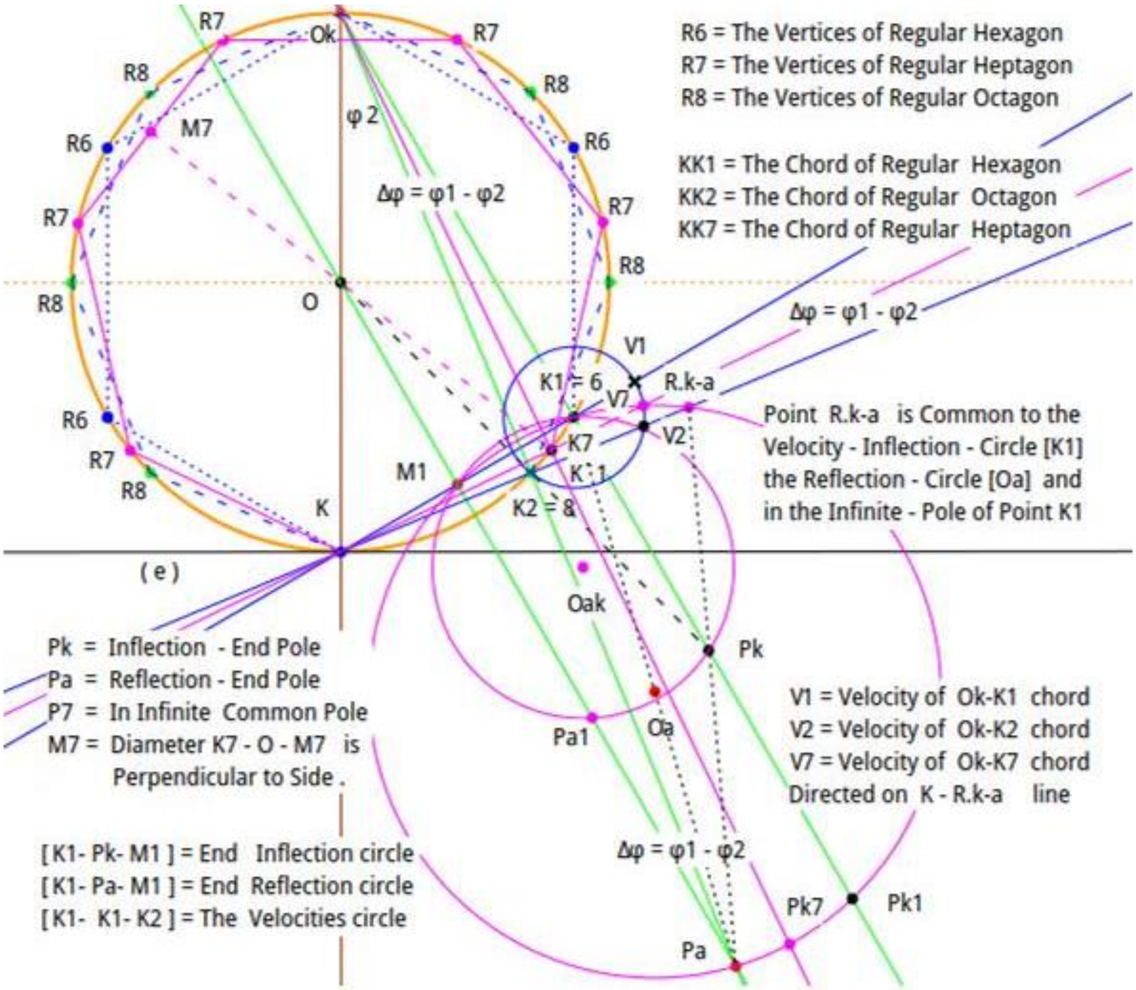
In case that Inverted circles intersect axis O_kOK , ***Then*** The - Inverted - Position is the Common - Point of the circle (K_1 , K_1K_2) and the circle of diameter K_1P_k , and this is because , the Tangential Inflection circle becomes the End Inflection circle on K_1P_k .

In all cases the Trapezium [$\mathbf{OO_kP_kP_a}$] is , the New Regular Polygons Mechanism and exhibits *The How* (By Scanning Chord KK_1 to KK_2) *and Where* (In the Inverted triangles OO_kK_2 ,$K_2P_kP_a$) *Work* (Energy → Kinetic or Dynamic) *produced from any Removal , is Stored* .
A wide analysis for the Energy - Storages in [64] .

In F.20 - A , **For n = 6** , then KK_1 is the Side of the Even - Regular – Hexagon
For n = 8 , then KK_2 is the Side of the Even - Regular – Octagon .
For n = 7 , then KK_7 is the Side of the Even - Regular – Heptagon . q.e.d

THE REGULAR - POLYGONS

In F.19 – (Page 69) , Is shown the Geometrical construction of the *Regular – Triangle* , Through the Regular → Digone and Tetragon .
In F.18-B – (Page 67) , Is shown the Geometrical construction of the *Regular – Pentagon* , Through the Regular → Tetragon and Hexagon .
In F.20 – (Page 70) , Is shown the Geometrical construction of the *Regular – Heptagon* , Through the Regular → Hexagon and Octagon .
In F.21 – (Page 71) , Is shown the Geometrical construction of the *Regular – Ninegone* , Through the Regular → Octagon and Decagon .
In F.22 – (Page 72) , Is shown the Geometrical construction of the *Regular – Endekagone* , Through the Regular → Decagon and Dodecagon .
In F.23 – (Page 73) , Is shown the Geometrical construction of the *Regular – Dekatriagone* , Through the Regular → Dodecagon and Dekatriagone .



F.20 - B → In circle (O ,OK)=(O ,OO_k) *and* [O_a, O_aK_1= O_aP_a] , [PO_k, PO_kM_1= PO_kP_k] , (K_1 ,K_1K_2)

For n = 6 , then **KK_1** is the Side of the Odd - Regular – Hexagon ,
For n = 8 , then **KK_2** is the Side of the Odd - Regular – Octagon ,
For n = 7 , then **KK_7** is the Side of the Even - Regular – Heptagon . 5 / 8 / 2017

The Physical notion of the Regular and Not - Polygons **:**

Segment **M_1K_1** or chord **KK_1** is the locus of the infinite circles on **OM_1**,**O_kK_1** parallels of Trapezium [**$OO_kP_kP_a$**] which intersect **(K_1 , K_1K_2**) circle . Chord **KK_1** revolving (*Scanning*) through point K , to **KK_7** and to **KK_2** produces , *Work* , when the Trapezium System passes through infinite .
Since triangles **KK_1O_k** , **KK_2O_k** are rightangle triangles , then **$KK_1 \perp O_kK_1$** , **$KK_2 \perp O_kK_2$** , and for any removal of point **K_1** to **K_2** the Work produced is zero .
In all Odd and Even - Regular -Polygons , AND in Any – Non- Regular –Shape , *The Area* of the *Space triangle* , **K_2O_k** O , is equal to the Area of the *Anti – Space triangle* **$K_2P_kP_a$** .

Generally by Scanning **Any Space-Monad KK_1 to a Space –Monad KK_2 of the circle , the Work produced is conserved in the first Space - triangle of the circle , and in the Outside of the Equal area triangle .**The area of the first triangle denotes the , *Work Produced* [i.e. *Energy as Electricity , as Vibration as Frequency ,as Thermal , as Movement , as any other Alteration e.t.c*] , while the area of the second triangle denotes the , *Work Quantized in the Plane – Stores of Anti-Space* . [61C]

Epiloque :

In Material Geometry [58-61] , Zero - point 0 = ∅ ={⊕+⊖} = The Material-point = *The Quantum* = The Positive Space and the Negative Anti-Space , between Opposites =The equilibrium of opposite →←
Point O , is nothing and maybe anywhere .
Point K , is nothing and maybe anywhere .
Segment $\overline{OK}$, is the ***Monad* OK** , ⊕ , and maybe on circle [O ,OK] where OK is , ***the* ⊕ *Space*** .
Point O_k ,is nothing and this is in Opposite Position of point O such that *Segment* $\overline{OO_k}$≡*The Quantum* **≡ *Anti-Monad* (OO_k)** = - (OK) = ⊖ , and Opposite direction (OO_k) → = - (OK) ← is , ***the Anti-Space*** .
Any Point K_1 , is nothing also and maybe on circle [O , OK] .
Segment $\overline{KK_1}$ is the monad KK_1 and it is the chord on circle [O, OA] , where KK_1 *is the* ⊕ *Space* .
Segment $\overline{O_kK_1}$ is the monad O_kK_1 = *is the* ⊖ *Space* and it is the perpendicular chord on circle [O ,OA] , where , since O_kK_1 is perpendicular to KK_1 then No-Work is produced ,therefore the velocities of chords are also perpendicular . *Here Velocity is the change of direction of the Space* KK_1 and always on K_1O_k .

Any Point K_2 , is nothing and maybe on circle [O , OK] also , and which occupies all above .
Angle < $K_1K\,K_2$ is the *Inbetween-Space* of chords KK_1 ,KK_2 on triangle $K_1K\,K_2$, *the Space triangle* , which locus is the constant circle (O , OK) and Triangle $K_1O_kK_2$ is , *the Anti-Space triangle* .

Chord K_1K_2 remains constant during the Removal of point K , *the* ⊕ *Space* , in order to reach point O_k *the Anti-Space* ⊖ , and this because arc $\overparen{K_1K_2}$ of the circle is constant . Since K_1K_2 Segment is constant therefore point K_2 lies on (K_1 , K_1K_2) circle which we call , *Velocity circle* .

Conclusion 1 :

On monad [OK] , *The Quantum* , exists the equilibrium and the opposite *Anti-monad* [OO_k] = - [OK] and from points K ,O_k are formed Infinite monads either as couple of chords $K\,K_1$,O_kK_1- $K\,K_2$,O_kK_2 , or as the angles < $K_1K\,K_2$, $K_1O_kK_2$ which have common their velocity circle (K_1,K_1K_2) . On this velocity circle any *motion of Space* $K\,K_1$, $K\,K_2$ *lies on Anti-space* O_kK_1 , O_kK_2 and the opposite .

This is the equilibrium of , ⊕ , Space $K\,K_1$ ***and , ⊖ , Anti-space*** O_kK_1 ***in Material Geometry .***

It was shown [12] that Space K_1O ≡ ⊕ is in equilibrium with the Anti-space K_1O_k ≡ ⊖ through the area of triangle K_1O_kO , and it is the Work embedded in point K_1 of Space .
The case of the Space K_2O is the same as in K_1O infront .
In case of simultaneous *Spaces* K_1O ≡ ⊕ ≡ K_2O then line OK_2 produced , intersects O_kK_1≡ ⊖ at point P_k which is called *the Inflection Pole* , and this because point K_2 is Inflected on circle (O , OK) .
Line O_kK_2 produced , intersects OM_1 line produced **,** *the parallel to* O_kK_1 *passes through the center* M_1 *of the chord* KK_1 **,** at the point P_a , which is called the *Reflection Pole* , and this because point M_1 is Reflected on triangle K_1OK .

Since lines OP_a , O_kP_k are parallels , and this because are both perpendicular to $K\,K_1$ chord , then quadrilateral $OP_aP_kO_k$ is Trapezium , and since Segments OP_k , O_kP_a are between the parallels then , the *Alternate Interion angles* < OP_aO_k , $P_aO_kP_k$ are equal , and both equal to angle < $K_1K\,K_2$ and this because of angles equality < $P_aO_kP_k$ ≡ $K_2O_kK_1$ ≡ $K_1K\,K_2$.
The same also for the *Alternate Interior angles* < OP_kO_k = P_kOP_a .

Since triangles P_kO_kO , $P_kO_kP_a$, occupy the common segment P_kO_k and common height K_1M_1 , so are equal , and therefore the Area of triangles P_kO_kO , $P_kO_kP_a$ equal , and since also triangle $P_kO_kK_2$ is common to them , then the Remaining triangles K_2O_kO , $K_2P_kP_a$ are also equal .
Since the Area [S] of triangle K_2O_k O represents the Work embedded in Point K_2 therefore the Work is conserved in triangle $K_2P_kP_a$ of this trapezium .
It was found that when λ_a= the length of the side of the Regular Polygon and R = OK is the radius of the circle then , the Area $S = \frac{\lambda_a}{4}.\sqrt{4R^2 - \lambda^2{}_a}$ and Polygon`s Length $\lambda_a = \sqrt{2.R^2 \pm \sqrt{R^4 - 4S^2}}$
A wide analysis for the nature of Polygon`s length λ_a in [64] .

Conclusion 2 :

Any relative motion of ,Space ≡ ⊕ monad KK_1 to KK_2 , *it is an alterating Chord - Scanning* , and is defined in the Outer Space K_2P_a as the Area of triangle $K_2P_aP_k$, *and it is the conserved Work* , and equal to K_2O_kO Area , *it is the Work* . i.e.
The Work produced in any Removal of Space is conserved in the Plane triangle of Anti-Space .

This is the Conservation of Work , in Material Geometry , for monads either as Segments or Angles through the Area of the Space triangle , K_2O_kO ***, to the Area of the Anti - Space triangle ,*** $K_2P_kP_a$ **.**

The circles of diameters K_1P_a , M_1P_k , are called the , *Reflection and the Inflection circle alternately* because these lie on common height of Trapezium , *the Segment* K_1M_1 , and are reflected at point K_1 which pass from the removable P_a , P_k , Poles of this Quadrilateral .
On the *Anti - space chord* K_1O_k , Infinite Inflection circles exist on the diameters K_1P_k , for point $P_k \equiv K_1 \rightarrow \infty$ and for $P_k \equiv \infty$ then all parallels to K_1O_k , lie on the Space - Chord K_1K_∞ with the Infinite Inflection circles passing from $K_1 \equiv V_1$ point . The same also for Anti-Space Chord K_2O_k where velocity at $K_2 \equiv V_2$ point .
Since circle (K_1 , K_1K_2) lies on K_1O_k line with center at point K_1 , then is the *End-Inflection -circle* , and since also K_1P_a diameter is equal to zero , then is also , and the *End Reflection circle* .
For both Anti - Space - Chords K_1O_k , K_2O_k corresponds the Intermediate - Space - Chord O_kK_7 on KV_7 line with the Reflection - Circle of diameter K_1P_a passing from V_7 common point , and to the End Inflection Velocity circle (K_1 , K_1K_2) .

Conclusion 3 :

On any Anti - Space - Chord K_1O_k and the corresponding Space - Chord K_1K , the Work done from any Removal is equal to the Area of triangle K_1O_k O and is spread on line $K_1P_k \rightarrow \infty$, and in case of a , *simultaneously* , second Anti-Space – Chord K_2O_k , then *Work is gothered to* $K_2P_aP_k$ triangle .
The Reflection circle of diameter K_1P_a intersects the End-Inflection-Velocity Circle of diameter M_1P_k at a point $R._{k-a}$, between the two points V_1, V_2 such that line KV_7 intersecting the circle (O,OA) at point K_7 , and the Work produced is equal to the Area of triangle K_7O_kO , which is conserved .
The above Geometrical Mechanism Constructs , Points K_7 , Chords KK_7 , Triangles K_7O_kO inwhere Work for any Removal is conserved . Since the Area of the triangles can be transformed to Equal Area of any other Shape then this *Shape* consists the Conservation -Work-Stores in Material – Geometry .

In case that Points K_1 , K_2 , consist the Vertices of any ***Two Sequent - Even - Polygons*** , then K_7 is the Vertices of ***The Inbetween - Odd - Regular - Polygon*** with the Produced ***and Conserved Work*** the ***Area of the Triangle*** K_7O_kO .

This is the the Quantization of Work in Monads , Either - as , Odd - Regular - Polygons ***and their Interior Angle , OR – as , of Any - Shape - Area equal to the Space triangle ,*** K_2O_kO ***, and equal also to the Area of the Anti - Space triangle ,*** $K_2P_kP_a$ **.**

By Scanning The Space-Monad KK_1 ***to Space –Monad*** KK_2 ***of the circle , The Work produced is conserved in*** **[** OO_kK_2] ***Space - triangle , and in the equal area triangle*** $K_2P_kP_a$ ***of the Anti – Space .***

The above relation of Work , Quantization in Geometry – Shapes ***, in Area – Stores of Anti-Space , is the Unification of Geometry - monads with the Energy monads*** **(** The How in [61] **,** ≡ where → *The How Energy from Chaos Becomes Discrete Monads*) .

Conclusion 3 : The Physical meaning .

In article was shown the Geometrical construction of all the - Regular - Polygons in a circle and for Odd , between any two Sequent Even Polygons . Any two Chords KK_1 , K_1O_k at the Ends of a diameter are perpendicular each other , and consist the Space and Anti-Space monads respectively and since are Perpendicular each other , these do not produce Work (Stored Work = Area of triangle OK_1O_k) .

In case of a Removal of any two chords the Work Produced between them is equal to the Central triangle Surface which consists the Quantization of Work in Monads .

For , Odd - Regular - Polygons and their Angle , OR - for , Any - Shape of Area equal to the Space triangle , K_2O_kO , *Work Quantization* [*Energy as Electricity ,as Vibration ,as Frequency ,as Thermal , as Movement , as any Alteration e.t.c*] , is equal also to the Area of the Anti - Space triangle $K_2P_kP_a$. It was also proved that , *By Scanning* Any Space-Monad KK_1 to a Space – Monad KK_2 of the circle , the Work produced is conserved in a Space - triangle in the circle , ***The Store*** , and in one of the equal area triangle outside the circle , which is the Anti-Space triangle , meaning that ,

The above relation of this Plane Work , *is The Quantization of Geometry – Shapes into the Plane – Stores of Anti-Space* ***, consists the Unification of the*** *Geometry – monads* ***with those of*** *Energy monads* ***, and which were analyzed and have been fully described .*** *markos 20/8/2017*

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