Simple Relation Between Proton Radius, Bohr Radius and Fine Structure Constant

Raji Heyrovska

Private Research Scientist (present), Academy of Sciences of the Czech Republic

(Emer.) Email: rheyrovs@hotmail.com

Abstract:

The radius of the proton has aroused a lot interest in recent years, since the reported values vary from 0.8751 fm to 0.8335 fm, where fm = 10^{-15}m. Here the author shows that the former CODATA value obtained for hydrogen can be related to the ground state Bohr radius for hydrogen and the fine structure constant by a simple equation. The latter lower value obtained using muons could be due to the differences in the reduced masses of hydrogen and muon.

1. Introduction

For an introduction to the proton radius puzzle, see [1-4]. The CODATA 2014 value [1] of the proton radius, r_p = 0.8751(61) fm, whereas the more recent value [2] is 0.8335 (95) fm, and hence the puzzle. Here, using classical equations, it is shown that the CODATA value of the proton radius can be related to the ground state Bohr radius and the fine structure constant as described below.

2. Ground state energy, Bohr radius and the fine structure constant.

The ground state ionization energy of hydrogen (E_H) is related to the Bohr radius (a_H = distance between the electron and proton at ionization) and the fine structure constant (α) by the known relation:

$$E_H = \mu_H c^2 \alpha^2/2 = e^2/8\pi\varepsilon_o a_H \quad (1)$$

where, $\mu_H = m_e m_p/(m_e+m_p)$ is the reduced mass, c is the speed of light in vacuum, e is the elementary charge and ε_o is the electric constant and α is the fine structure constant. From the CODATA [1] values for the constants, the ground state Bohr radius is obtained as,

$$a_H = (e^2/4\pi\varepsilon_o\alpha^2)/\mu_H c^2 = 5.29461 \text{ x } 10^{-11}\text{m} \quad (2)$$

It can be seen from Eq. 1 that E_H attains the maximum value, $E_{max,H}$ given by $\mu_H c^2/2$

$$E_{max,H} = \mu_H c^2/2 = E_H/\alpha^2 = [e^2/8\pi\varepsilon_o\alpha^2 a_H] \quad (3)$$

when the distance between the electron and proton is minimum [5],

$$a_{H,min} = \alpha^2 a_H = 2.819474998 \text{ x}10^{-15} \text{ fm} \quad (4)$$

The CODATA value of the proton radius,

$r_p = 0.8751\text{fm} = 0.3104a_{H,min}$ (5)

Thus, Eq. 5 relates the proton radius to the Bohr radius and the fine structure constant. Note that the ratio,

$r_p/a_{H,min} = 0.310$ ($\sim 1/2\phi = 0.309 = 0.8713/a_{H,min}$) (6)

where ϕ is the Golden ratio, which divides the ground state Bohr radius into two sections pertaining to the electron and proton [6,7].

References

1. P. J. Mohr, D. B. Newell, B. N. Taylor, J. Phys. Chem. Ref. Data **45**, 043102 (2016); doi: 10.1063/1.4954402

2. A. Byer et al, Science, **358**, 79 (2017).

3. J. C. Bernauer, R. Pohl, Scientific American, 310(2), 32 (2014).

4. D. Castelvecchi, Nature News, 5 October 2017, Nature doi:10.1038/nature.2017.22760

5. R. Heyrovska, Chapter 8, pp. 203 - 215 in: "Frontiers of Fundamental Physics (Vol. 3)", ISBN 978-81-7271-577-8; ed. B. G. Sidharth; Proceedings of the Fifth International Conference on Frontiers in Fundamental Physics, Hyderabad, January 2003. Universities Press (Division of Orient Longman), 2007.

6. R. Heyrovska, Molecular Physics, 103 (2005) 877 - 882. Special Issue of in honor of Nicholas Handy

7. R. Heyrovska, 113th Statistical Mechanics Conference, Rutgers University, Hill Center, May 10-12, 2015, Program of Conference, Abstract B2, p 22.
https://www.researchgate.net/publication/281270357 Rutgers conf