# A simple model of quantization: an approach from chaos
## Experimental consequences and uncertainty principle

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**Abstract.** There is a paradigm in Quantum Mechanics that explains quantization through normal vibration modes called *Eigenstates* that arise from Schrödinger wave equation. In this contribution we propose an alternative methodology of quantization by using basic concepts of mechanics and chaos from which a *Toy Model* is built.

## 1. Motivation

Let us assume that a pair of particles interact with quantum noise [3][11] such that they are perturbed in the form of *kicks* [1][10] and besides, these particles attract each another due to a central force. The Lagrangian that describes this phenomenon consists of one term associated to the central force acting on the total mass of the system and the *Ansatz* that models the complex interaction between the quantum noise and the particles:

$$\mathcal{L} = \frac{1}{2}\left(m\dot{r}^2 + mr^2\dot{\theta}^2\right) - V(r) - K\theta\sin\left(\Omega + \xi\dot{\theta}\right)\sum_{j=-\infty}^{\infty}\delta(t - jT)\,. \tag{1}$$

Where $j\epsilon\mathbb{Z}$, $-\pi \leq \Omega \leq \pi$ and, $K, \xi$ are parameters that will be defined later on, and $T$ is the perturbation period.

From Euler-Lagrange equations [4] we have

$$\frac{d}{dt}(m\dot{r}) - mr\dot{\theta}^2 + \frac{\partial}{\partial r}V(r) + \epsilon(K, \xi(m, r), T) = 0 \tag{2}$$

$$\frac{d}{dt}\left(mr^2\dot{\theta}\right) + K\sin\left(\Omega + \xi\dot{\theta}\right)\sum_{j=-\infty}^{\infty}\delta(t - jT) = 0 \tag{3}$$

with $\epsilon(K, \xi(m, r), T) \ll \frac{\partial}{\partial r}V(r)$.

Let us assume that $r$ is a constant, thus from equation (3) we have

$$\ddot{\theta} = -\frac{K}{mr^2}\sin\left(\Omega + \xi\dot{\theta}\right)\sum_{j=-\infty}^{\infty}\delta(t - jT) \tag{4}$$

$$\dot{\theta}_{j+1} = \dot{\theta}_j - \frac{K}{mr^2}\sin\left(\Omega + \xi\dot{\theta}_j\right). \tag{5}$$

Taking the fixed points [5] in (5), $\dot{\theta}_{j+1} = \dot{\theta}_j = \dot{\theta}^*$ then

$$\sin\left(\Omega + \xi\dot{\theta}^*\right) = 0.$$

Thus, the fixed points are

$$\dot{\theta}_n^* = \frac{n\pi - \Omega}{\xi} \quad n\epsilon\mathbb{Z}. \tag{6}$$

In order to obtain the stable fixed points we take [5]

$$\left|f'\left(\dot{\theta}^*\right)\right| < 1 \tag{7}$$

Where

$$f'\left(\dot{\theta}^*\right) = 1 - \frac{K\xi}{mr^2}\cos\left(\Omega + \xi\dot{\theta}^*\right). \tag{8}$$

Taking the $n$ even stable fixed points in equation (6) we have

$$\dot{\theta}_n^* = \frac{2n\pi - \Omega}{\xi} \quad n\epsilon\mathbb{Z} \tag{9}$$

And from (7), (8) and (9):

$$0 < \frac{K\xi}{mr^2} < 2. \tag{10}$$

Taking[1] $\frac{K\xi}{mr^2} = ϡ$ this is a stability parameter without units and $0 < ϡ < 2$, we can write equation (9) as:

$$\dot{\theta}_n^* = \frac{(2n\pi - \Omega)\mathrm{K}}{ϡ mr^2} \quad n\epsilon\mathbb{Z}. \tag{11}$$

Now, if $\Omega = 0$ and $K = ϙĦ$, $ϙ$ is a variation of Ħ without units, in equation (11) then

$$L_n = \left(\frac{2\piϙ}{ϡ}\right)nĦ$$

Finally, if $2\piϙ = ϡ$ and $0 < ϙ < \frac{1}{\pi}$

$$L_n = nĦ$$

## 2. Experimental consequences

An interesting consequence from the stability condition is that, if $K = ϙ\hbar$ in $\frac{K\xi}{mr^2} = ϡ$ where $\hbar$ is the Planck's constant, and taking $r$ as in Bohr's model [7][8][9] $r^2 = \frac{n^4\hbar^4}{m^2k^2e^4}$ where $k$ is the Coulomb constant, $\nu_\xi^s = \frac{1}{\xi}$ is the stable perturbation frequency, we obtain

$$\nu_\xi^s = \frac{4\piϙRc}{ϡ n^4} \tag{12}$$

Where $R = \frac{mk^2e^4}{4\pi c\hbar^3}$ is the Rydberg's constant, with a value of $R = 1.0972 \times 10^7\ m^{-1}$, $c$ is the speed of light.

Then, if $2\piϙ = ϡ$

$$\nu_\xi^s = \frac{2Rc}{n^4} \tag{13}$$

When we choose a specific Ħ, we are selecting the scale that we want. In this case, we choose the Planck's scale. Now, we can analyze the next table.

Table 1

| $n$ | $\frac{2Rc}{n^4}\ [Hz]$ |
|---|---|
| 1 | $6.580 \times 10^{15}$ |
| 2 | $4.112 \times 10^{14}$ |
| 3 | $8.123 \times 10^{13}$ |
| 4 | $2.570 \times 10^{13}$ |
| 5 | $1.053 \times 10^{13}$ |
| 6 | $5.077 \times 10^{12}$ |
| 7 | $2.740 \times 10^{12}$ |
| 8 | $1.606 \times 10^{12}$ |
| 9 | $1.003 \times 10^{12}$ |
| 10 | $6.580 \times 10^{11}$ |
| 11 | $4.494 \times 10^{11}$ |
| 12 | $3.173 \times 10^{11}$ |
| 13 | $2.304 \times 10^{11}$ |
| 14 | $1.712 \times 10^{11}$ |
| 15 | $1.299 \times 10^{11}$ |

We can observe in the Table 1, $n = 8$, $1.606 \times 10^{12}\ Hz$, this value is about 10 times the Cosmic Microwave Background Radiation CMB or $\nu_{CMB} = 1.602 \times 10^{11}\ Hz$.

An interesting observation is that the value of $\frac{Rc}{12^4} = 1.586 \times 10^{11}\ Hz$ is approximately equal to $\nu_{CMB}$ with 1% of error.

From Rydberg formula, we have $\nu_R = \frac{Rc}{n^2}$ at n=143.27 is about $\nu_{CMB}$. In figure 1, we can compare $\nu_{CMB}$, $\nu_R = \frac{Rc}{n^2}$, $\nu_\xi^s = \frac{2Rc}{n^4}$.

The intersection between the curves $\nu_R = \frac{Rc}{n^2}$, $\nu_\xi^s = \frac{2Rc}{n^4}$ is at $n = \sqrt{2}$.

The intersection between $\nu_{CMB}$ and $\nu_\xi^s = \frac{2Rc}{n^4}$ is at $n = 14.27$.

An important issue is when $n \to \infty$, $\nu_\xi^s \to 0$ very fast $\propto \frac{1}{n^4}$, for that reason $\nu_{CMB}$ could be considered a stability border for the system.

[1] ϡ, ϙ are archaic Greek letters named sampi and koppa.

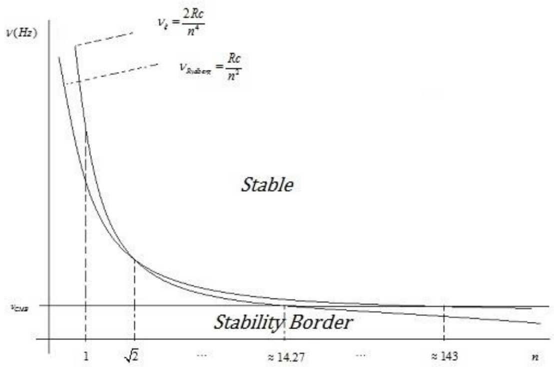


**Figure 1**

Also we can analyze the unstable points

$$L_n = (n + \tfrac{1}{2})\hbar$$

Using again Bohr's model, we can obtain,

$$\nu_\xi^u = \frac{2Rc}{(n + \frac{1}{2})^4} \qquad (14)$$

Table 2

| $n$ | $\frac{2Rc}{(n+\frac{1}{2})^4}$ $[Hz]$ |
|---|---|
| 0.5 | $1.053 \times 10^{17}$ |
| 1.5 | $1.299 \times 10^{15}$ |
| 2.5 | $1.684 \times 10^{14}$ |
| 3.5 | $4.384 \times 10^{13}$ |
| 4.5 | $1.604 \times 10^{13}$ |
| 5.5 | $7.189 \times 10^{12}$ |
| 6.5 | $3.685 \times 10^{12}$ |
| 7.5 | $2.079 \times 10^{12}$ |
| 8.5 | $1.260 \times 10^{12}$ |
| 9.5 | $8.077 \times 10^{11}$ |
| 10.5 | $5.412 \times 10^{11}$ |
| 11.5 | $3.761 \times 10^{11}$ |
| 12.5 | $2.695 \times 10^{11}$ |
| 13.5 | $1.981 \times 10^{11}$ |
| 14.5 | $1.488 \times 10^{11}$ |

Now, we will construct the table 3, with harmonics $M$ of $\nu_{CMB}$, versus an approximation of unstable and stable points of the system by using $n = \sqrt[4]{\frac{2Rc}{M\nu_{CMB}}}$.

Table 3

| $M$ | $\sqrt[4]{\frac{2Rc}{M\nu_{CMB}}}$ |
|---|---|
| 2 | 11.97 |
| 4 | 10.07 |
| 5 | 9.52 |
| 10 | 8.01 |
| 13 | 7.50 |
| 17 | 7.01 |
| 23 | 6.50 |
| 32 | 5.99 |
| 45 | 5.50 |
| 66 | 4.99 |
| 100 | 4.50 |
| 161 | 4.00 |
| 273 | 3.50 |
| 510 | 3.00 |
| 1059 | 2.50 |
| 2592 | 2.00 |
| 8192 | 1.50 |
| 41472 | 1.00 |
| 663552 | 0.5 |

We can observe an interesting approximation in the first harmonics. It suggests a feedback process. Last harmonics could be approximated very well with almost any values of radiation because they are large harmonics.

A stability diagram for the previous points of the system can summarise tables 1 and 2, it includes the border points.

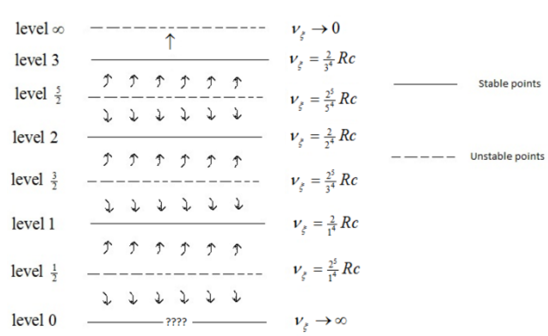


**Figure 2**

In figure 2, level 0 shows an attractor point, physical intuition say us, we need to invert the order of the stable points, in order to forbid that point, see figure 3. That is physically likely?

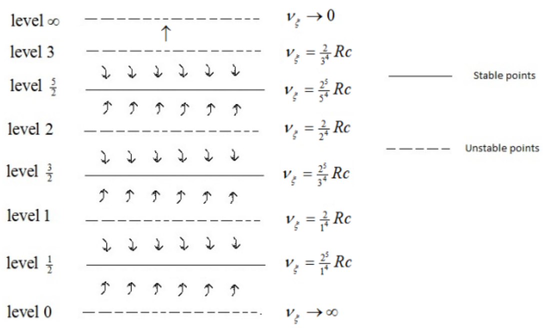


**Figure 3**

**Uncertainty principle**

Also known as Heisenberg's indeterminacy principle, it is an important element of the quantum theory. From eq uation (10) we have

$$0 < \frac{\varrho\hbar}{mr^2\nu_\xi} < 2 \qquad (15)$$

If we define $\omega_\xi = 2\pi\nu_\xi$ then

$$0 < \frac{2\pi\varrho\hbar}{2} < m\omega_\xi r \cdot r$$

$$0 < \frac{2\pi\varrho\hbar}{2} < p_\xi \cdot r$$

Now, we substitute $r = r_{min} = \Delta r$ and $p_\xi = p_{max} = \Delta p$

$$0 < \frac{2\pi\varrho\hbar}{2} < \Delta p \, \Delta r$$

Finally, if $\varrho = \frac{1}{2\pi}$ and $\hbar = 1$, this principle has imposed values for $\varrho, \hbar$ , then

$$0 < \frac{\hbar}{2} < \Delta p \, \Delta r$$

Therefore,

$$\frac{\hbar}{2} < \Delta p \, \Delta x$$

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