On Testability of "Relative State" Formulation of Quantum Mechanics

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Justifications of Everett's alternative interpretation of Quantum Mechanics are suggested.

KEYWORDS: Relative State, Everett, Quantum Mechanics, Quantum Synergy, Quantum Crossing Over.

Measurements in Quantum Mechanics – Revisited

Let us consider an act of measurement to be a process of identification of object of study via its comparison with each of some well-known *probe* objects. (1)

Structurally, Quantum Mechanics is fundamentally based on the notion that complete information about any given object is contained within its psi-function Ψ. (2)

Thus, keeping in mind (1) and (2), a process of measurement in Quantum Mechanics could have been presented as an act of consecutive comparison of unknown Ψ – function with each of the well-known probe functions φ_i until a perfect fit is found for some φ_k:

$$
\begin{aligned}
&\langle \varphi_1 | \Psi \rangle = A_1 \neq 1 \\
&\langle \varphi_2 | \Psi \rangle = A_2 \neq 1 \\
&\langle \varphi_3 | \Psi \rangle = A_3 \neq 1 \\
&\dots\dots\dots\dots\dots\dots \\
&\langle \varphi_i | \Psi \rangle = A_i \neq 1 \\
&\dots\dots\dots\dots\dots\dots \\
&\langle \varphi_k | \Psi \rangle = A_k = 1 \quad \Rightarrow \quad \varphi_k \equiv \Psi
\end{aligned} \tag{3}
$$

where A_i is a positive number from "0" to "1" that represents a degree of

overlapping-
fitting-
matching-
sameness- of functions φ_i and Ψ

However, a simple measuring procedure described above is not realizable in practice due to the sad fact that a psi-function Ψ does not have any direct explicit physical meaning. For this reason, in order to make any sensible comparison of

φ_i with Ψ one should consider not just a *pristine* overlap $\langle\varphi_i|\Psi\rangle$ but the overlap of φ_i with Ψ in terms of an operator $\hat{L}$, which corresponds to some physical entity L that can experimentally be measured:

$$\langle\varphi_i|\hat{L}|\Psi\rangle \qquad (4)$$

Necessity in obtaining a meaningful *numeric* experimental outcome L requires $\hat{L}$ and Ψ to satisfy an equation:

$$\hat{L}|\Psi\rangle = L|\Psi\rangle \qquad (5)$$

In general, not only one single pair of L and Ψ satisfies the equation for any given $\hat{L}$. That is why one should rewrite the equation as:

$$\hat{L}|\psi_i\rangle = L_i|\psi_i\rangle \qquad (6)$$

where L_i - one value from the *spectrum* –
set of *eigenvalues* –
numeric outcomes of the measurements –
ψ_i - one function from the *spectrum* –
set of *eigenfunctions* – that satisfies eq. (6)

Equation (6) allows one represent Ψ as a *superposition* (*i.e.* linear combination) of its eigenfunctions ψ_i:

$$|\Psi\rangle = \sum_i c_i|\psi_i\rangle \qquad (7)$$

where c_i - some numeric weighing coefficients

Experimental paradigm: "one experiment – one outcome" dictates eigenfunctions to be mutually orthonormal:

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} = \begin{cases} 1; i = j \\ 0; i \neq j \end{cases} \qquad (8)$$

Ideas behind eqs.(5,6,7) are also applicable to the probe functions φ_m *as* well. Indeed, an experimenter, instead of using only one single eigenfunction φ_m in every single experiment, may use at once a collection of all probe functions φ_m as their superposition within a composite probe function Φ :

$$\left|\Phi\right\rangle = \sum_m \chi_m \left|\varphi_m\right\rangle \tag{9}$$

where χ_m - some numeric weighing coefficients

Testability of Everett's Interpretation

Everett's alternative interpretation of Quantum Mechanics [1] is based on the assumption that an observer and the object system could have been considered and treated mathematically as being mutually *equal* at any point of the temporal sequence: before measurement ➔ measurement ➔ after the measurement.

N.B.: Conventional interpretation (CI) is totally orthogonal to Everett's assumption, as it (CI) requires the original object's state function *collapse* at the very act of measurement just to one eigenstate. Indeed, the *collapse* would make the experimenter and the object distinguishable, i.e. *unequal*.

As long as there is nothing special about the act of measurement, an observer's state function together with an object's state function, both should undergo a continuous, deterministic change, as they would behave if each of them were an isolated system. To avoid the *"collapse issue"* which would have made Quantum Mechanics fundamentally incompatible with either of Relativities (see NB above), Everett further suggests the observer's state function together with the object's state function be treated as the states that are *relative* to each other within the *universal* observer-object state function. The scheme implies an existence of strong mutual correlation between the state functions of the observer and the object system, meaning any change in one subsystem should always come along with its *relative* change in the counterpart subsystem, pretty much like in the stamp-and-its-print case. In other words, Everett's interpretation suggests an observer's state function being playing an active role in the process of measurement. Thus, any observation of the experimenter's state function being affecting the object's state function / being affecting the outcome of the measurement, should have been considered as justification of Everett's interpretation.

Weak Justification – Quantum Synergy

Let us consider an act of measurement from both points of view.

<table>
<tr><th rowspan="2">Measurement Steps</th><th colspan="2">Quantum Mechanics</th></tr>
<tr><th>Relative State Interpretation</th><th>Conventional Interpretation</th></tr>
<tr><td>Assumption</td><td>observer's state function / probe function Φ is an active participant of the measurement</td><td>observer's state function has absolutely no effect on the outcome of the measurement</td></tr>
<tr><td rowspan="4">Before Measurement</td><td colspan="2">observer's state function / probe function</td></tr>
<tr><td>$\left|\Phi\right\rangle = \sum_m \chi_m \left|\varphi_m\right\rangle$ (9)</td><td>N/A</td></tr>
<tr><td colspan="2">object's state function</td></tr>
<tr><td colspan="2">$\left|\Psi\right\rangle = \sum_i c_i \left|\psi_i\right\rangle$ (7)</td></tr>
<tr><td>During the Measurement</td><td>interaction / superimposion / overlapping of Φ with Ψ in terms of $\hat{L}$ is happening:
$\left\langle\Phi\middle|\hat{L}\middle|\Psi\right\rangle$ (10)
$\left(\sum_m \left\langle\varphi_m\right|\chi_m^*\right)\hat{L}\left(\sum_i c_i\left|\psi_i\right\rangle\right)$ (12)
object's wave function branches to each and all of the eigenstates that are identical with eigenstates of the probe function</td><td>observer "measures" Ψ in terms of $\hat{L}$:
$\left\langle\Psi\middle|\hat{L}\middle|\Psi\right\rangle$ (11)
$\left(\sum_i \left\langle\psi_i\right|c_i^*\right)\hat{L}\left(\sum_i c_i\left|\psi_i\right\rangle\right)$ (13)
object's wave function collapses only to one of the eigenstates</td></tr>
<tr><td rowspan="6">After the Measurement</td><td>all eigenstates keep on existing</td><td>only one eigenstate exists</td></tr>
<tr><td colspan="2">probability ρ_k of …</td></tr>
<tr><td>…being on the branch with ψ_k</td><td>… collapsing to ψ_k</td></tr>
<tr><td>$\varphi_n \equiv \psi_k$ (14)
$\frac{1}{L_k}\left(\left\langle\varphi_n\right|\chi_n^*\right)\hat{L}\left(c_k\left|\psi_k\right\rangle\right)$ (15)
$\chi_n^* c_k \left\langle\psi_k\middle|\psi_k\right\rangle$ (17)</td><td>$\frac{1}{L_k}\left(\left\langle\psi_k\right|c_k^*\right)\hat{L}\left(c_k\left|\psi_k\right\rangle\right)$ (16)
$c_k^* c_k \left\langle\psi_k\middle|\psi_k\right\rangle$ (18)</td></tr>
<tr><td colspan="2">probability of getting an outcome L_k</td></tr>
<tr><td>$\chi_n^* c_k$ (19)</td><td>$c_k^* c_k$ (20)</td></tr>
</table>

As one can see, the probability of getting any given outcome in conventional formulation is governed by the Ψ-function only and is not depended on the probe function the observer uses. On the contrary, according to Everett's interpretation, the weighing coefficients of eigenfunctions within the probe function should synergistically affect the probability of getting any given outcome. In other words, an observer's ability to affect the probability of getting any certain outcome via the usage of different probe functions should have been considered as justification of Everett's interpretation.

Strong Justification – Quantum Crossing Over

Thesis: Everett's interpretation allows for exchange with eigenstates between the probe function of the observer and the object's state function, *i.e.* allows for *Quantum Crossing Over.*

Proof: To provide one with the same degree of complexity for the object system and the observer, let us consider the most extreme and, at the same time, the most general case of the measurement, namely:

1) an observer M, who is completely described by the state function:

$$\left|M\right\rangle = \sum_i m_i \left|\mu_i\right\rangle \equiv \left\{\left|\mu_i\right\rangle\right\} \tag{21}$$

where $\left\{\left|\mu_i\right\rangle\right\}$ means complete set of $\left|\mu_i\right\rangle$

2) an observer N, who is completely described by the state function:

$$\left|N\right\rangle = \sum_j n_j \left|\nu_j\right\rangle \equiv \left\{\left|\nu_j\right\rangle\right\} \tag{22}$$

where $\left\{\left|\nu_j\right\rangle\right\}$ means complete set of $\left|\nu_j\right\rangle$

3) an act of mutual probing of $\left|N\right\rangle$ by M and $\left|M\right\rangle$ by N in one single act of measurement.

Let us now suppose that observer M has got a record of the result of the measurement. That would mean that original function $|M\rangle$ underwent a transformation to $|M\rangle'$:

$$|M\rangle \quad \rightarrow \quad |M\rangle' \tag{25}$$

before measurement after the measurement

$$|M\rangle' \equiv \big|\, |M\rangle + record \,\big\rangle \neq |M\rangle \tag{26}$$

energy:

$$E_M \quad \neq \quad E'_M \tag{26}$$

As long as the system (M, N) is isolated, any change in M *must* cause a complementary / compensative / relative change in N:

$$|N\rangle \quad \rightarrow \quad |N\rangle' \tag{27}$$

energy:

$$E_N \quad \neq \quad E'_N \tag{28}$$

in order to secure the fulfillment of energy conservation law:

$$E_{(M+N)} \quad = \quad E'_{(M+N)} \tag{29}$$

As soon as any state function can equally be represented as a complete set of the corresponding eigenstates (see eqs. 21, 22), any change / transformation of the state function would inevitably imply the change of content of the original set of eigenstates. *"The change of content"*, on the other hand, can only happen either as a result of the loss of some original eigenstates or the acceptation of some new eigenstates, or both. Since the system is isolated and limited to the observers M and N, their respective state functions can undergo the transformations only via the *crossing over* between the sets $\{|\mu_i\rangle\}$ and $\{|\nu_j\rangle\}$. *Q.E.D.*

N.B.: It is interesting to note that any type of successful measurement may arguably be considered as justification of Everett's interpretation, provided similar train of reasoning as above.

Concluding Remarks

One may expect the testability of justifications described above can be done using the arsenal of techniques physicists nowadays routinely use in the area of correlated states. In a nutshell, an experimenter P prepares an object system to be then observed by an experimenter O. The key things for the experimental set up are:

1. P knows exactly only the *"eigenstate content"* of the initial state function of the object.
2. O knows exactly only the *"eigenstate content"* of the initial probe function he uses.

Results of the measurements then are getting compared with the original separated informations from 1, 2 in order to reveal whether the anticipated effects happened or not.

References

1. Hugh Everett, III, *"Relative State" Formulation of Quantum Mechanics*, Reviews of Modern Physics, Vol. 29, No. 3, July, 1957, p.454-462